Correction Factor for Determining the London Penetration Depth from Strip Resonators

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ABSTRACT

The strip resonator technique is a popular way to measure the magnetic penetration depth $\lambda(T)$ in superconducting thin films. The temperature dependence provides fundamental information about the superconducting energy gap and hence insight into the pairing mechanism. There has been much controversy regarding the actual form of the temperature dependency, with some researchers reporting a weak-coupled BCS-like behavior and others favoring a Gorter-Casimir type fit. This paper shows that the disagreement can be at least partially attributed to a temperature sensitive term traceable to stray susceptance coupled into the resonator. The effect is inherent to the technique but a simple procedure to compensate for it can be used and is presented here.

INTRODUCTION

The resonant frequency of a high-Q microwave transmission line is inversely proportional to the square root of the sum of kinetic and magnetic inductance per unit length. The kinetic inductance ($L_k(T)$), associated with the inertial mass of the charge carriers, is strongly dependent on the penetration depth. Hence, the shift in resonant frequency with temperature can, in principle, yield a sensitive measure of $\lambda(T)$. However, extracting the zero temperature penetration depth ($\lambda(0)$) generally requires the assumption of a particular theoretical model to which the data is curve fit. The situation is exasperated by the complex interdependency among variables such as film thickness (t), circuit geometry including strip width (W) and substrate thickness (h), critical temperature (Tc), and $\lambda(0)$. The penetration depth is also sensitive to the quality of the film, especially near its surface, as well as the transition width $\Delta(T)$, which is an indicator of phase purity. Some studies have focussed only on extremely low impedance lines [1] or strictly low temperature (i.e. $T < Tc/2$) $\lambda(T)$ dependence [2]. For most practical microwave applications, line impedances will be in the neighborhood of 50 Ω, and film thickness will be of the same order as the penetration depth. Experimental investigations using strip transmission lines near Tc have invariably revealed a strong deviation from theory [3-5] when $t \approx \lambda$. This short paper shows that the disagreement can be attributed, at least in part, to the susceptance coupled into the resonator from the gap discontinuity as well as the feed line of electrical length βl. The coupled susceptance is modified by the temperature dependent characteristic impedance of the resonator. When the effect is taken into account, the natural resonant frequency of the resonator is shown to increase as T approaches Tc, and the $\lambda(T)$ profile changes accordingly. The situation when the strip characteristic impedance is not matched to the generator is included.

DERIVATION OF THE PERTURBATION OF THE NATURAL RESONANT FREQUENCY DUE TO LOADING

A lumped equivalent circuit model representing the excited resonator is shown in figure 1. A transmission line gap is more often depicted as a capacitive pi network [6]. But it is mathematically convenient to model it as shown here, and the transformation is straightforward. It is well known that the measured resonant frequency (ω_o') of an inductively or capacitively coupled resonator is pulled from the actual resonant frequency (ω_o) of the isolated circuit because of the reactance or susceptance associated with the coupling mechanism. A good estimate of the unperturbed resonant frequency can be obtained by considering the coupled susceptance in the calculation of ω_o. The total susceptance of the loaded resonator is

$$X = j \, (\omega C - 1/(\omega L) - n^2 B) \tag{1}$$

where B is the susceptance of the network left of the transformer. Since $\omega^2 LC = 1$ at resonance and $Q_o = R\omega_o C$, it follows that

$$\omega_o = \omega_o' \, (1 + n^2 RB/(2Q_o)) \tag{2}$$

and finally, using the approximation $R = 2Z_o Q_o/\Pi$ from [7],

$$\omega_o = \omega_o' \, (1 + n^2 BZ_o/\Pi) \tag{3}$$

Equation 2 essentially agrees with the graphical derivation of Kajfez [8] with the approximation $(1 + \xi)^{-1} \approx (1 - \xi)$ where $\xi \ll 1$.

In the case of a superconductor Z_o in (3) is implicitly taken as a function of temperature because of the kinetic inductance. For a low-loss line

$$Z_o \approx Z(0)\{(L_k(T) + L_e)\varepsilon(0)/[(L_k(0) + L_e)\varepsilon(T)]\}^{1/2} \tag{4}$$

where $\varepsilon(T)$ is the temperature dependent effective permittivity, L_e is the magnetic or geometrical inductance, and $Z(0)$ is the characteristic impedance of the transmission line at $T = 0$. Equation 4 is markedly different than the equation derived in [9] which expressed Z_o as being proportional to the ratio $\lambda(T)/\lambda(0)$. That expression was derived specifically for kinetic inductance delay lines, where $h \ll W$, and $t \ll \lambda$. In the situation considered here, L_k is not $\gg L_e$. Wheeler's incremental inductance rule, commonly used to characterize planar quasi-TEM transmission lines, only applies to shallow field penetration. Here, the inductance was derived from the imaginary part of the impedance calculated from the phenomenological loss equivalence method [10]. This method has been shown to provide accurate results for both attenuation and phase velocity for quasi-TEM, normal and superconductor, transmission lines.

Determining $\varepsilon(T)$ is not so straightforward but it can be estimated from [3] where resonant frequency versus temperature data was provided for a metallic conductor on $LaAlO_3$. $\Delta\varepsilon(T)$ was taken as -550 ppm/K which is an order

of magnitude more severe than results disclosed in [4]. Still, the effect is subtle and the correction factor of (3) is dominated by L_k for resonators studied herein.

The susceptance B can be evaluated as follows, for the general case when Z_0 is not equal to Z_g. It is easy to show that $C_s = C_g + C_p$, $C_f = C_p[(C_p/C_g)^2 + 3C_p/C_g + 2]$, and $n^2 = C_s/(C_s + C_f)$, where C_g and C_p are the elements of the equivalent capacitive-pi representation from [11]. A series network can be made equivalent to a parallel network, and vice versa, at one frequency. Since we are interested in the behavior of the circuit of figure 1 over a very narrow frequency range, the immittance looking towards the generator from the transformer was closely approximated by performing such a transformation. Let $K_1 = Z_0^2 Z_g [1 + \tan^2(\beta l)]/[Z_0^2 + Z_g^2 \tan^2(\beta l)]$, $K_2 = Z_0(Z_0^2 - Z_g^2)\tan(\beta l)/[Z_0^2 + Z_g^2\tan^2(\beta l)]$, $K_3 = (\omega C_s K_2 - 1)/(\omega C_s)$, and finally $K_4 = K_3[1 + (K_1/K_3)^2]$. Then $B \approx \omega C_f - K_4^{-1}$, and was found to be only a weak function of temperature up to $T = 0.99$ Tc for a wide range of microstrip geometries. It should be noted that the strip transmission line gap parameters are assumed to be static. It will merely be mentioned that following the above approach and solving for the conductance, one can show that the Q of the resonator will depend on the feedline characteristics.

SAMPLE CALCULATION

In order to illustrate the impact of the correction factor on a practical resonator, an example is presented here. Considering the ring resonator presented in [5], with $\beta l \approx 1.3$ radians, $Zg = 50\ \Omega$, $t = 800$ nm, $h = 500\ \mu m$, $W = 160\ \mu m$, $Tc = 105$ K, and taking $Z_0(0) \approx 53\ \Omega$, $\lambda(0) = 1200$ nm, the normalized correction factor obtained from (3) is shown in figure 2. C_s and C_f were estimated to be 0.034 pF and 0.012 pF, respectively. Applying this term to the experimental data in figure 11 of [5], the corrected normalized resonant frequency data is shown in figure 3.

CONCLUSIONS

A corrective term has been presented which slightly modifies the shape of the resonant frequency versus temperature curve of strip resonators, used to evaluate the London penetration depth. The term is temperature sensitive primarily because of the kinetic inductance associated with the superconducting resonator, and is not expected to be as significant when t is thick compared to λ. But, some earlier resonator based measurements may have overestimated $\lambda(0)$ because of this effect. This may help explain some of the disagreement in measurements of λ determined by other methods. An implication of the approach presented herein is that the observed resonant frequency will depend on the characteristics of the feed line.

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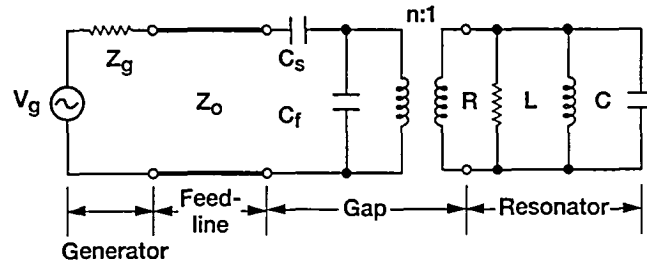
Figure 1.—Lumped equivalent circuit model of a strip
transmission line resonator near resonance, coupled to
a feedline of arbitrary impedance across a narrow gap,
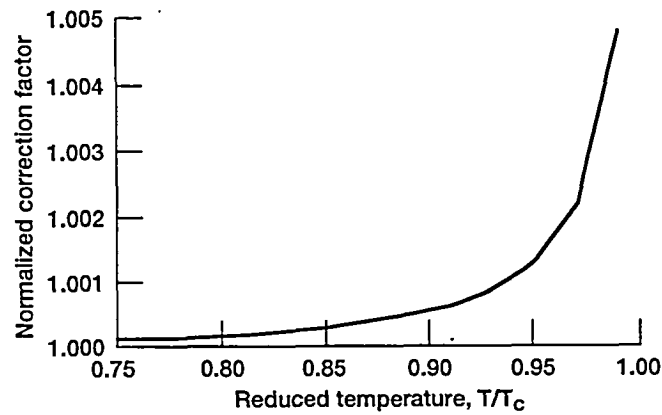which is in turn connected to a source.

Figure 2.—Sample calculation of the normalized correction
factor ($\xi(T)/\xi(0)$) as a function of reduced temperature
(T/Tc). Data corresponds to a Tl-Ba-Ca-Cu-O thin film
(0.8 μm) ring resonator ($Z_0(0) \approx 53\ \Omega$) on 500 μm thick
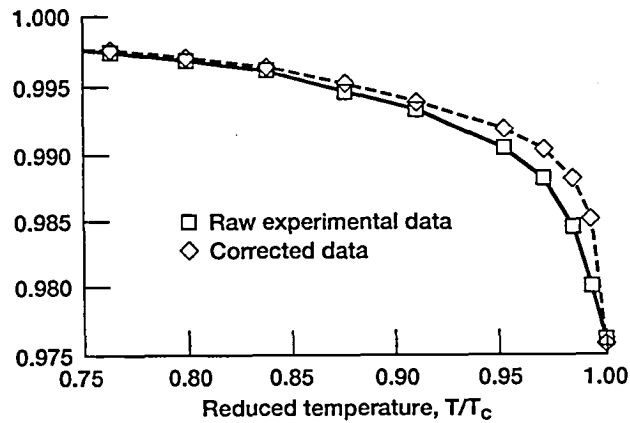LaAlO3. The zero temperature London penetration depth
was taken as 1200 nm.

Figure 3.—Illustration of the effect of the correction factor on
normalized resonant frequency versus reduced temperature
data. The effect tends to steepen and raise the knee of the
curve and suggests a smaller value for the zero temperature
London penetration depth than might otherwise be predicted.

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A significant disagreement is often seen between the theoretical temperature dependent magnetic penetration depth profile and experimentally derived calculations based on stripline type resonators. This short paper shows that the disagreement can be attributed to the susceptance coupled into the resonator from the gap discontinuity as well as the feed line. When the effect is taken into account, the natural resonant frequency of the resonator is increased, and the frequency shift due to kinetic inductance can be calculated much more accurately. While it is necessary to include this effect to determine the penetration depth, it is shown that the impact on unloaded quality factor is generally negligible. The situation when the strip characteristic impedance is not matched to the generator is included.

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