

NASA Technical Memorandum 107035

# An Optimized Integrator Windup Protection Technique Applied to a Turbofan Engine Control

Stephen R. Watts  
*United Technologies Pratt & Whitney*  
*West Palm Beach, Florida*

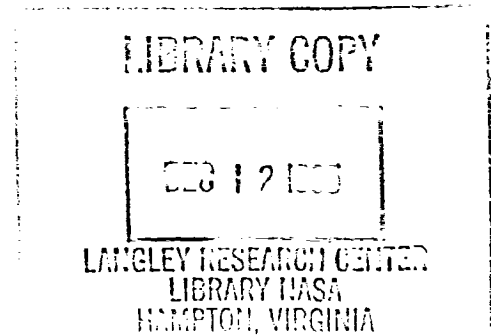
and

Sanjay Garg  
*Lewis Research Center*  
*Cleveland, Ohio*

October 1995



National Aeronautics and  
Space Administration







3 1176 01423 5064

# An Optimized Integrator Windup Protection Technique Applied to A Turbofan Engine Control

**Stephen R. Watts**

United Technologies Pratt & Whitney  
Government Engine and Space Propulsion  
West Palm Beach, Florida

**Sanjay Garg**

Advanced Control Technology Branch  
NASA Lewis Research Center  
Cleveland, Ohio

## Abstract

This paper introduces a new technique for providing memoryless integrator windup protection which utilizes readily available optimization software tools. This integrator windup protection synthesis provides a concise methodology for creating integrator windup protection for each actuation system loop independently while assuring both controller and closed loop system stability. The individual actuation system loops' integrator windup protection can then be combined to provide integrator windup protection for the entire system. This technique is applied to an  $H^\infty$  based multivariable control designed for a linear model of an advanced afterburning turbofan engine. The resulting transient characteristics are examined for the integrated system while encountering single and multiple actuation limits.

## Introduction

Increasing performance requirements for propulsion systems have resulted in the introduction of multiple input multiple output multivariable control designs. These multivariable control designs can result in degraded performance when the system encounters an actuation system range limit or rate limit if integrator windup protection is not available. Numerous ideas for integrator windup protection and bumpless transfer have been previously published in literature.[1,2]

Simply stated, integrator windup protection (IWP) must be included in control system design for controllers which attempt to drive steady-state errors to zero. The effort to drive steady-state errors to zero is accounted for through integral action on the part of the controller. If at any point in time the controller outputs are limited and controller inputs are non zero and of the same numerical sign (+/-), the integral action of the controller will attempt to increase the magnitude of the already limited controller output, thus the controller's integrator(s) will wind up. Following this period of integrator wind up, the controller response to command inputs might be very poor because the controller's integrators must first unwind prior to

attempting to calculate the new controller outputs for the new controller inputs.

The IWP design requirements discussed in reference [3], are summarized below:

- 1) IWP should be memoryless, and should not contribute to the nominal control system when actuation system limits are not encountered.
- 2) IWP should provide smooth transfers between the unlimited and limited actuator, while providing accurate tracking of the limited actuator when the limit is encountered
- 3) IWP should be closed loop stable for all possible actuator limitation combinations within the system's operating envelope.
- 4) IWP should attempt to maintain system performance when limits are encountered. If system performance cannot be maintained, IWP should provide a smooth, stable transition to some minimally degraded operating point.

The previous approaches described in references [1,2] and reviewed for a typical application in reference [3], require a specific form of the controller for the integrator windup protection implementation and provide partial guarantees for controller stability but do not provide guarantees for the entire closed loop system stability. Thus a new methodology for defining the IWP which provides both controller stability and closed loop stability, when the IWP control loops are active, is being introduced. These IWP design requirements, listed above, will be referred to and the means for meeting each of the design requirements will be discussed in the IWP design methodology

This paper will describe the new integrator windup protection (IWP) technique in the following manner. The basic requirements for the optimized IWP will be defined in terms of a generalized Integrator Windup Protection Overview. The detailed methodology for implementation will then be described. The IWP

design technique will then be applied to a turbofan engine control. The linear model for the turbofan engine will be introduced and defined. The engine control mode and corresponding  $H_\infty$  based controller will be defined. The IWP design process will then be applied for a single actuation system loop. The design process will be repeated for each of the remaining actuation system loops independently.

The IWP for each of the actuator loops will then be incorporated into the system's integrator windup protection scheme. The resulting system with integrator windup protection will then be examined for closed loop stability and exercised while encountering single and multiple actuation loop limits.

### Integrator Windup Protection Design Methodology Overview

The evolution of the Integrator Windup Protection Design Methodology is very similar to that of a command tracking disturbance rejection methodology. The problem will be solved as a white noise covariance optimization problem. The generalized optimization design plant is illustrated in Figure 1.

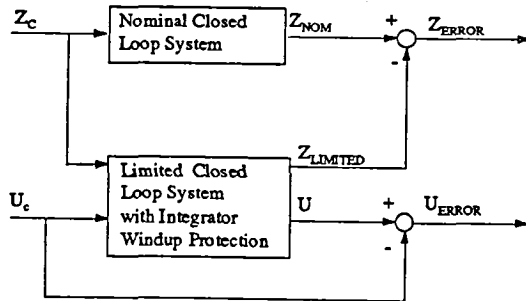


Figure 1. Generalized Optimization Design Plant

This problem definition allows for the calculation of errors between the nominal closed loop, or "Ideal", system and the limited system with IWP for the performance loops and the actuator command tracking error. The performance errors,  $Z_{ERROR}$ , are defined as the difference between the response of controlled variables for the nominal system,  $Z_{NOM}$ , and the response of the controlled variables for the limited system with IWP,  $Z_{LIMITED}$ . The controlled actuator error,  $U_{ERROR}$ , is defined as the difference between the commanded actuator position,  $U_c$ , and the controller with IWP commanded actuator position,  $U$ . The optimization will be solved using Gaussian white noise

as the excitation for the controller commands,  $Z_c$ , and the limited actuator command,  $U_c$ .

The integrator windup protection gains will be implemented in the controller architecture as shown in Figure 2. This controller architecture is identical to the controller architecture in reference [1]. This implementation of the controller meets IWP design requirement 1, the requirement that the IWP be memoryless and not affect the system when the controller outputs are not being limited.

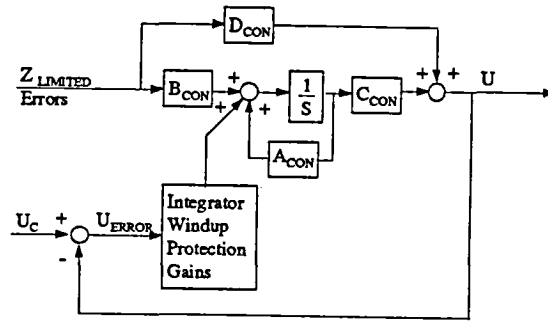


Figure 2. Integrator Windup Protection Implementation in Controller

In Figure 2,  $Z_{LIMITED ERROR}$  represents the error between the commands,  $Z_c$ , and the feedback variables,  $Z$ , and  $A_{CON}$ ,  $B_{CON}$ ,  $C_{CON}$ , and  $D_{CON}$ , represent the matrices for the controller of form:

$$\begin{aligned}\dot{X} &= A_{CON} * X + B_{CON} * Z_{LIMITED ERROR} \\ U &= C_{CON} * X + D_{CON} * Z_{LIMITED ERROR}\end{aligned}$$

The optimization will then calculate the Integrator Windup Protection gains to minimize the following performance index:

$$J = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (Z_{ERROR}^2 + U_{ERROR}^2) dt \right\}$$

This type of optimization problem is easily solved using commercially available software analysis tools. For this particular engine control application, MATRIXx with Optimization Module [4] was used.

### Detailed Integrator Windup Protection Methodology Description

The general framework for the optimization problem has been described and is now ready for the specific application. An addition to the generalized optimization design plant is required to solve the IWP

optimization problem. This modification is the addition of input and output signal conditioning to the generalized optimization design plant. The IWP optimization design plant is shown in figure 3.

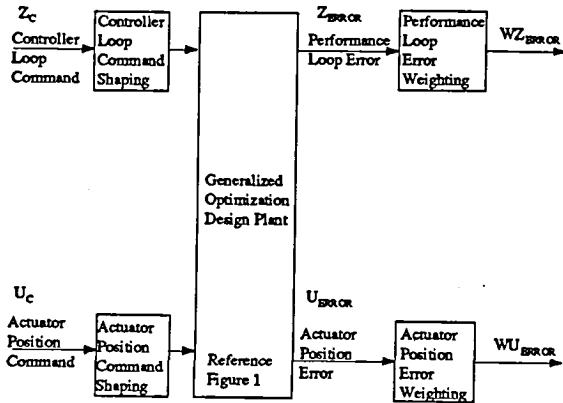


Figure 3. IWP Optimization Design Plant

The IWP optimization design plant uses the generalized optimization design plant, but it also includes command shaping for the controller loop command and the actuator position command, and weighting for the performance loop errors and the actuator position error. The command shaping and output weighting are defined so that each loop in the design plant has appropriate "weighting" and appropriate frequency spectrum in the optimized IWP gain calculation.

The controller command loop shaping consists of two pieces: a loop scale factor and first order lag, for each of the controller loops. For the engine model, which will be discussed in a later section, the scale factors were chosen as the maximum variation in the controlled variables when the engine model inputs were varied ten percent about the operating point. The first order lag time constant was chosen to be equivalent to the controller loop bandwidth specification.

The actuator command loop shaping consists of two pieces: a loop scale factor and first order lag. The scale factor was chosen as ten percent of the operating point for the actuator loop being evaluated. The first order lag time constant was chosen to be one half of the smallest controller loop bandwidth specification. The actuator command loop bandwidth should be defined at a lower frequency than the smallest controller loop bandwidth specification to prevent the optimization solution from providing actuator position tracking while simultaneously penalizing the primary controller loops. IWP design requirement 2 dictates tracking the

limited actuator command but this need not be accomplished at the expense of the primary controller loops.

The performance loop error weighting also consists of two pieces: a scale factor and a frequency weighting function. The performance loop error scale factor is simply the inverse of the controller command loop scale factor. The performance loop error frequency weighting function provides a sensitivity specification for the controller loop error in the optimization design plant. The frequency weighting function should contain a large magnitude at very small frequencies, to satisfy the zero steady state error requirement, and transition to a small magnitude at high frequencies where it is not critical to maintain the same performance as the nominal closed loop, "Ideal", system.

The actuator position error weighting also consists of two pieces: a scale factor and a frequency weighting function. The scale factor is simply the inverse of the actuator command loop scale factor. The actuator position error frequency weighting function also provides a sensitivity specification for the weighted actuator position error in the optimization design plant and is defined with the same limitations as the performance loop frequency weighting functions.

The optimization routine will then calculate the performance index using the weighted errors from the IWP Optimization Design Plant as shown below.

$$J = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (WZ_{ERROR}^2 + WU_{ERROR}^2) dt \right\}$$

Figures 2 and 3 represent the IWP optimization design plant, and the controller architecture for the IWP gain calculation. In this setup, the optimization is performed with respect to the IWP gains. The parameter optimization software tool [4] requires the designer to generate a "cost" function for the optimization. The cost function for this optimization was defined in the following manner. The cost function will first verify controller stability. It will then verify the closed loop system stability, refer to IWP requirement 3. If an unstable controller or closed loop system results, the cost function places a large penalty, or "cost", on that solution, and the optimization attempts to form another solution. If the controller and the closed loop system are stable, the cost is calculated as the performance index evaluated for that particular set of IWP gains. The optimization routine iterates on the IWP gains until the solution

provided meets the convergence specifications for the optimization routine.

This optimization methodology is completed for each actuator loop independently. The entire system can be evaluated in a similar method by including additional actuator position requests and making the appropriate changes to include all actuator loops in the optimization design plant.

IWP gains are calculated for each actuator loop in the same manner. The results of the individual optimizations for each actuator loop are then combined into a single matrix to form the system's IWP gains, where the columns of the matrix are the individual IWP gains for each actuator loop.

It is now incumbent upon the designer to perform several tests to insure the IWP requirements are met. IWP requirement 3 requires that the system be closed loop stable for all possible actuator limitations combinations within the systems operational envelope. This is accomplished by defining a system exactly like Figure 2, except that all the actuator loops are included and the entire system IWP gains are used. The inputs are the controller loop commands and all actuator position commands. The outputs are the controller commanded actuator positions. The eigenvalues of the total system with IWP are then evaluated to insure that the system is closed loop stable for all possible combinations of actuator limitations.

The review of IWP requirements 2 and 4 are somewhat more qualitative in nature. The closed loop system with actuator limits and IWP is exercised while encountering single and then multiple actuation limits. The overall system control loop responses and limited actuator position tracking in terms of overshoot and settling time are reviewed to verify acceptable system behavior. Additionally, the designer must realize that maintaining overall system performance while encountering actuation limits may not be realizable. In fact, without redundant actuation capability the overall systems performance will always be somewhat degraded. It is therefore the designer's responsibility to use sound engineering judgment in determining acceptable degraded system response. This engineering judgment could include the modification of the cost function to allow for a relaxed response on one or more of the control loops whenever certain actuator limits are encountered. This is, of course, very specific to the design plant and should be carefully reviewed prior to implementation.

Even though this optimized method for calculating the IWP gains may provide excellent results for a the continuous system, this is not necessarily the case when the discrete system is reviewed. The discrete system with IWP gains may result in unstable operation. While the IWP gains might provide excellent response to the actuator command tracking requirement in continuous system, this result could be achieved by calculating IWP gains which result in very large eigenvalues. When the system is discretized these large values eigenvalues may result in unstable discrete controllers.

As a result of the unstable discrete controllers, the generalized optimization design plant was reviewed. The generalized optimization design plant optimizes a system which calculates the errors for the controller loop commands and the actuator position command. These two sets of errors are dynamically different. The controller command loops are first order responses of the appropriate bandwidth, while the actuator position commands are zeroth order responses. The optimization of these two different types of errors resulted in the large IWP gains which invariably resulted in an unstable discrete controller. This analysis led to a modification to the generalized optimization design plant.

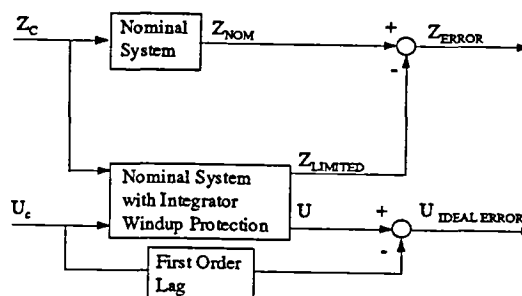


Figure 4. Modified Generalized Optimization Design Plant

The generalized optimization design plant was modified to allow the designer to specify the desired response for the actuator tracking error, which would now be a first order response. The new actuator tracking error would be the difference in the actuator ideal response and the limited controller commanded actuator response. This was accomplished by adding a first order lag filter on the commanded actuator position,  $U_c$ . The modified generalized optimization design plant is illustrated in Figure 4. The

performance index was modified to substitute the weighted actuator ideal error in place of the weighted actuator error.

### Turbofan Engine Control IWP Application

#### Example

The IWP optimization methodology was applied to an existing turbofan engine model with  $H_\infty$  based control [5]. The turbofan engine control IWP application example requires the introduction of an engine model, actuator model and the turbofan engine control mode. Each piece is briefly described in the following.

#### Engine Model

The test system used in the IWP design is a linear model of an advanced afterburning turbofan engine. The engine model is represented in the following state space form:

$$\begin{aligned}\dot{X} &= A_{ENG} X + B_{ENG} U \\ Y &= C_{ENG} X + D_{ENG} U\end{aligned}$$

where the state vector is:

$$X = [N1, N2, TMHPT]^T$$

and the states are defined as:

N1 = Low Pressure Compressor Speed (RPM)  
N2 = High Pressure Compressor Speed (RPM)  
TMHPT = Metal Temperature of the High Pressure Turbine (Degrees Rankin).

The control input vector is:

$$U = [WFGG, AJ, CIVV, RCVV]^T$$

where the inputs are defined as:

WFGG = Main Burner Fuel Flow (LB/HR)  
AJ = Nozzle Exit Area (Square Inches)  
CIVV = Low Compressor Inlet Variable Guide Vanes (Degrees)  
RCVV = Rear Compressor Inlet Variable Guide Vanes (Degrees).

The output vector is:

$$Y = [OPR, EPR, N1, N2]^T$$

and the outputs are defined as:

OPR = Ratio of Burner Pressure/Inlet Pressure (Dimensionless)

EPR = Ratio of Nozzle Pressure/Inlet Pressure (Dimensionless)

N1 = Low Pressure Compressor Speed (RPM)

N2 = High Pressure Compressor Speed (RPM).

OPR, EPR and N2 are the sensed outputs for the control loops and N1 is used for the inner loop scheduling for the Low Compressor Inlet Variable Vanes (CIVV). This model is a perturbation model and the inputs and outputs are deltas from the nominal operating conditions. CIVV is scheduled open loop as a function of N1. The scale factor for CIVV/N1 is 0.01244.

The numerical values for the system matrices:  $A_{eng}$ ,  $B_{eng}$ ,  $C_{eng}$ ,  $D_{eng}$ , and the initial conditions for the engine model are listed in the appendix.

#### Actuator Model

The actuator dynamics are represented as first order lags with minor loop gains of 25 radians/second for the WF, CIVV, and RCVV loops. The AJ actuator loop is represented by a second order system with  $\zeta = 0.45$  and  $\omega_n = 55.8$  in series with a first order lag with a minor loop gain of 15 radians/second. See Figure 5 for the schematic view of the integrated engine/actuation system.

#### Control Mode

The Control Mode was selected because of inherent properties for directly controlling the engine operating line during transient operation [6], while providing rapid precise control of engine thrust. The three control loops are the ratio of burner pressure to inlet pressure (OPR), the ratio of nozzle pressure to inlet pressure (EPR), and high rotor speed (N2). The controller outputs are main burner fuel flow (WFGG), nozzle exit area (AJ) and high compressor inlet variable guide vanes (RCVV). The control specification was to track the input commands while maintaining zero steady state error in a de-coupled manner.

An  $H_\infty$  controller was designed to meet the desired bandwidth specifications and loop de-coupling requirements. The resulting controller was 8<sup>th</sup> order in size. The achieved bandwidths for the control loops was 10 (radians/second) for each loop. The matrices

for the control are  $A_{CON}$ ,  $B_{CON}$ ,  $C_{CON}$ ,  $D_{CON}$  and are listed in the appendix.

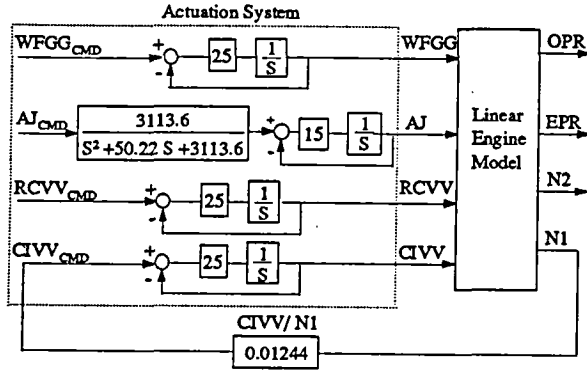


Figure 5. Integrated Engine/Actuation System Overview

### IWP Gain Calculation

The MATRIX<sub>x</sub> software optimization tool allows the user to provide initial values for the IWP gains along with upper and lower bounds for the IWP gains to bound the search space. As a starting point, the initial values for the IWP gains were chosen to be 0.0 and the upper and lower bounds were chosen to be +/- 100.0. The optimized gains were then computed. The gains were then compared to the upper and lower bounds. If any of the gains were limited by either the upper or lower bound, the bounds were increased or decreased as necessary, and the optimized gain calculation was repeated using the previously calculated gains as the initial values. This process was iterated on until all the IWP gains were no longer limited by the upper or lower bounds.

The next step was to insure the optimization had not stopped as the result of a local minimum. The minimum was checked by doubling the previously optimized IWP gains, and using these values as the initial predictions for the IWP gains. The upper and lower bounds were modified as necessary and the optimization was repeated. If the new optimized gains returned to the values of the previous optimization, then a true minimum was declared. Otherwise the process was repeated until a true minima was achieved.

At this point, the eigenvalues of the controller with IWP protection should be checked to verify that reasonable gains have been calculated to prevent unstable controllers when the system is discretized.

### Performance Analysis

In order to provide a more accurate replication of the turbofan engine controller problem, the controller command loop structure was slightly modified. The EPR command was generated as a function of the N1 feedback, while OPR and N2 commands were scheduled independently, see Figure 6. The scale factor for delta EPR to delta N1 is 0.00324 (1/RPM).

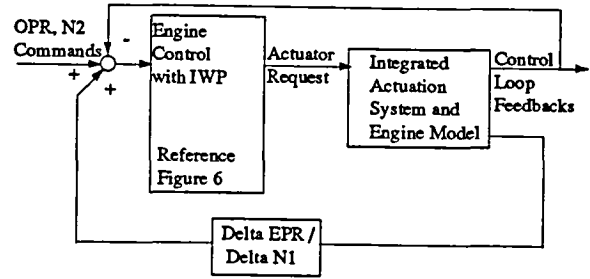


Figure 6. Performance Analysis System Setup

Performance Analysis of the integrator windup protection is completed by comparing the nominal system without actuator limits to the modified system with actuator limits and IWP. The systems are exercised for small command loop steps and the resulting overall system performance is compared.

Analysis of the system with actuator limits and IWP includes rise time, overshoot and tracking of the limited actuator, and steady state errors in the control loops. For the sake of brevity, all loops and combinations of actuator limits will not be reviewed. Instead a single actuator loop will be limited and then two actuator loops will be limited simultaneously and the results compared.

The magnitude of the step input to the control loop commands was equal to the scale factor that was used in the optimization design plant. The step magnitude is a very important consideration in this analysis. The range of OPR is 1-30.0, the range for EPR is 1-3.5 and the range for N2 is 0 -14000.0 rpm. Obviously a High Rotor Speed command step change of 1 rpm for N2 has very little impact on the system, while a step command of 1 unit of EPR has a very large impact, and is unrealistic for the linear model being used. Utilizing the appropriate magnitudes for the step commands provides meaningful data for analysis. The



magnitude of the OPR step was 1.6843 (Dimensionless).

The analysis was performed with the commanded input scaled to the specified magnitude and the outputs normalized to the inverse of the commanded input scale value. Hence, a unit command input will result in a unit output with the normalized system. This will result in normalized control loop parameters while retaining physical engineering units for the controller outputs (WF, AJ, RCVV).

For the purposes of this examination a perturbation model was used so that the delta about the operating point would be generated, and thus the controller outputs will be limited to a delta value about the operating point. The Ideal Response was generated without including any actuator limitations. Two additional tests were completed by limiting the RCVV actuator to  $\pm 2.0$  degrees for the single limited actuator case and then limiting WFGG to  $\pm 1000$  lbm/hour while retaining the RCVV limitation of  $\pm 2$  degrees for the two limited actuator case. The response for the limited actuator without any type of IWP was also reviewed but is not included because of the poor overall performance response of this system. The objective of including IWP is to maintain as closely as possible the Ideal System response. Therefore, only the Ideal System and the system with IWP will be compared.

The system excitation was provided by stepping the OPR Command for each of the three test cases: Ideal Response, RCVV Limited, and RCVV and WFGG limited. The OPR command step was initiated 0.1 seconds into the transient. The step command was then removed 2.5 seconds into the transient.

The transient responses for the test cases are shown in Figures 7-14. Figure 7 shows the transient response of normalized OPR. The ideal response shows a minimal amount of overshoot and settles at 1.0 second. Both limited actuator test cases show somewhat degraded performance in that the steady state value of OPR was 0.96, a four percent steady state error.

The normalized response of the EPR controller loop is shown in Figure 8. The normalized EPR settles at a value of approximately 0.12 for both the single and two limited actuator test cases, while the ideal normalized EPR settles at a value of 0.088. The limited actuator test cases resulted in a steady state error of 0.032, due to the limited RCVV actuator.

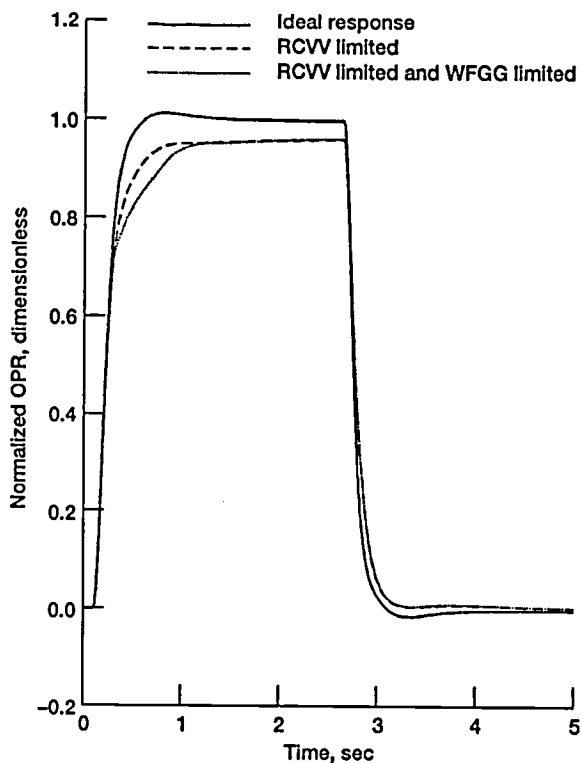


Figure 7. Normalized OPR Response to OPR Command Step.

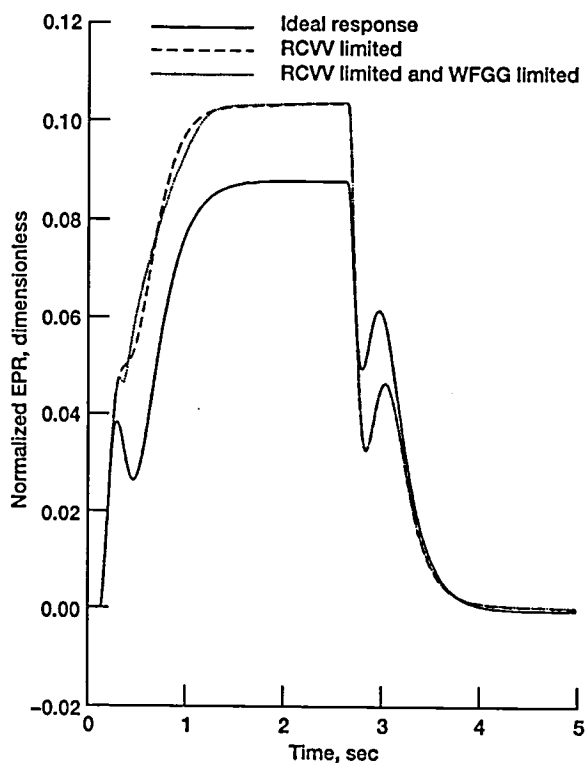


Figure 8. Normalized EPR Response to OPR Command Step.

The normalized response of N2 controller loop is shown in Figure 9. The normalized N2 settles at a value of approximately 0.41 while the ideal response remains at 0.0. The normalized N2 steady state error for both limited actuator test cases is 0.41, again due to the limited RCVV actuator.

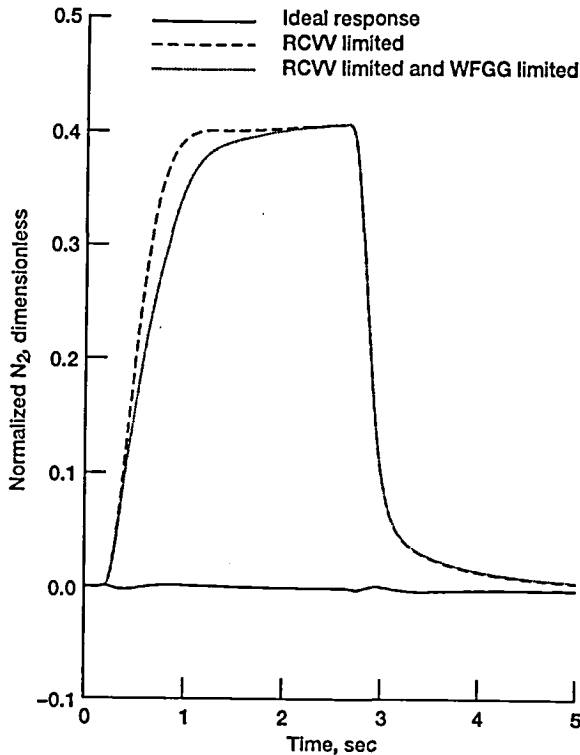


Figure 9. Normalized N<sub>2</sub> Response to OPR Command Step.

The RCVV limit of 2.0 degrees was encountered, see Figure 12, and the system remained limited until the step command was removed for both the single and dual limited actuator test cases. The WFGG limit was encountered, see Figure 10, but only momentarily, for approximately 0.5 seconds, during the transient for the two limited actuator test case. Both limited actuator test cases exhibited smooth stable transitions to degraded operating point, thus meeting IWP requirement 4.

Detailed analysis of the figures showed that all three transient test cases appear identical until 0.2 seconds into the transient. At this point in the transient, both the limited actuator test cases encounter the RCVV limit at 2.0 degrees, see Figure 12. As the transient continues, both the limited actuator test cases diverge from the ideal response and remain identical until 0.3 seconds into the transient. At 0.3 seconds into the

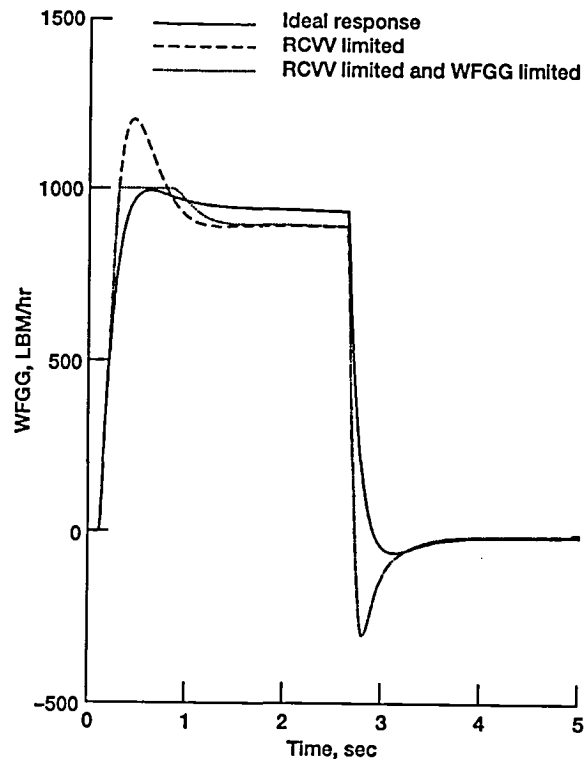


Figure 10. WFGG Response to OPR Command Step.

transient the limited actuator test cases diverge because the two limited actuator test case, RCVV and WFGG limited, encounters the 1000 (LBM/Hour) WFGG Limit and remains limited until 0.87 seconds into the transient (See Figure 10). After this point in time the limited actuator test cases converge and the system attains steady-state.

These values for steady state errors for the normalized EPR and N2 controller loops appear to be significant. However, remembering that the normalization factors were chosen for a ten percent variation in the design plant inputs, simple calculations show that the steady states errors are in fact relatively small when compared to the original operating point. Both the EPR and N2 steady state error are less than two percent of the operating point. The limited systems did however show a degraded response to the OPR Command steps. This was the result of the controller loop errors and the limited actuator position error canceling each other out in the controller calculations (see figure 2). This is simply a function of the controller loop commands. The engine controller being examined is a square system (3 inputs and 3 outputs) and does not contain any redundant actuation system capability. Therefore, encountering any actuation system limit will result in

degraded performance. If the coupling in the N2 response to the OPR command is considered too high, IWP gains for the RCVV limited actuator can be resynthesized with an increased weighting for N2.

Review of controller actuator requests for WFGG and AJ, figures 10, and 11, provide the control designer with valuable insight into how the integrator windup protection works when limits are encountered. The integrator windup protection increases the rate of WFGG when the RCVV limit is encountered, as indicated in Figure 10 by the slopes of the actuator limited test cases. IWP also modifies the AJ scheduling such that AJ does not open as much transiently when the WFGG limit is encountered (see Figure 11).

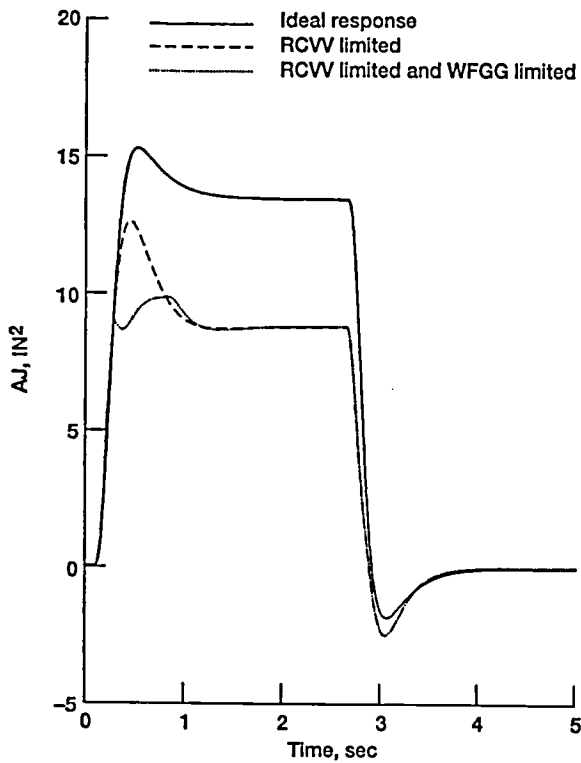


Figure 11. AJ Response to OPR Command Step.

Finally, the controller commanded actuator positions during limited operation are shown in Figure 13 and 14. Figure 13 shows the controller commanded RCVV request during both the single and two actuator limited actuator test cases. The single actuator limited controller commanded RCVV position exhibits a slight overshoot prior to settling at a steady state value of 2.55 degrees. The two limited actuator controller commanded RCVV position does not overshoot and settles at the same steady state value of 2.55 degrees. When the OPR step

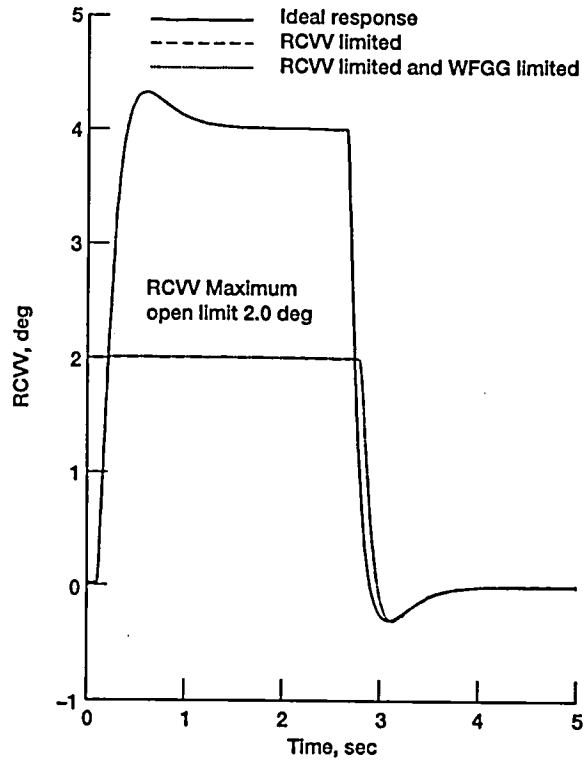


Figure 12. RCVV Response to OPR Command Step.

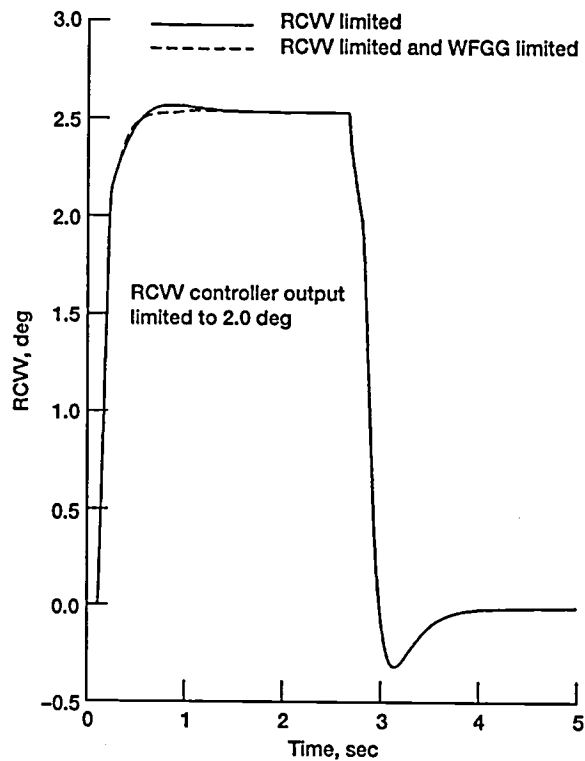


Figure 13. Controller RCVV Request Response to OPR Command Step.

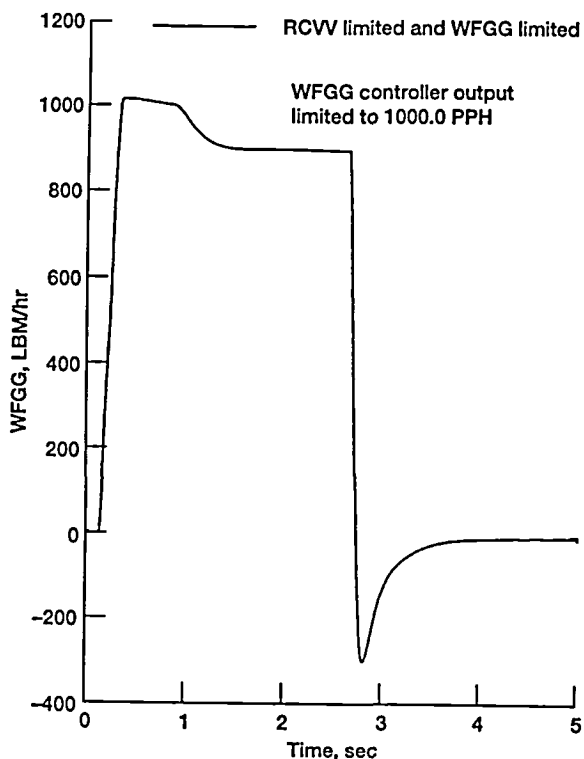


Figure 14. Controller WFGG Request Response to OPR Command Step.

command is removed both actuators return to the same steady state value as the ideal response.

Figure 14 shows the controller commanded WFGG actuator position during the two actuator limited transient test case. The controller commanded WFGG actuator position overshoots the actuator limit of 1000 LBM/Hour and settles at the limit before releasing from the limit to settle at the steady value.

For the test cases examined here, each of the IWP specifications were met.

### Conclusions

A new Integrator Windup Protection methodology was introduced in a general framework. The detailed description for the IWP methodology was then defined in terms of the optimization design process. This new IWP design methodology was then applied to a turbofan engine control problem.

The application of this integrator windup protection technique to the jet engine control problem provided acceptable system response to the limited actuator

transient response. Because this system does not have redundant control capability, degraded system response was to be expected for encountering any actuator limit.

It should be noted that the entire IWP design process is based upon designing the system to track an actuator position request. It does not however indicate how this actuator position request is generated. The actuator position request can be the result of range of motion physical hardware limits, an actuator rate capability limit, or any other type of limit, such as a WFGG, AJ or RCVV transient operating limit to insure adequate compression system surge margin. The calculation of the limited actuator position request is not important. Tracking the limited actuator position, while maintaining the highest possible overall system performance was the objective of this design process.

Furthermore, this technique provides the designer with the flexibility to modify the optimization procedure so that a specific degraded performance hierarchy will be followed.

### References

- [1] Campo P.J., Morari M., Nett C.N., "Multivariable Anti-Windup and Bumpless Transfer - A General Theory", 1989 American Control Conference, Pittsburgh, Pa, TP9-5:00, p. 1706
- [2] Hanus R., Kinnaert M., "Control Of Constrained Multivariable Systems Using The Conditioning Technique", 1989 American Control Conference, Pittsburgh, Pa, TP9-5:30, p. 1712
- [3] Mattern D., "A Comparison of Two Multi-Variable Integrator Windup Protection Schemes", NASA TM 194436, AIAA-93-3812, presented at the 1993 AIAA Guidance Navigation, and Control Conference, Monterey, Ca.
- [4] MATRIX<sub>x</sub> "Optimization Module, Edition 2", Integrated Systems, Inc., Santa Clara, CA, October 1989
- [5] Watts, S.R. and Garg, S., "A Comparison of Multivariable Control Design Techniques for A Turbofan Engine Control", NASA TM to be published
- [6] Larkin, L., "Pressure Based Closed Loop Thrust Control in a Turbofan Engine ", Patent Number 5,303,545

## Appendix

### Engine Model Matrices

$A_{ENG} =$

```
[-2.5764E+00 1.7038E+00 4.3646E-01
 2.1345E-02 -1.5592E+00 3.4403E-01
 1.7610E-02 1.4938E-02 -3.8463E-01]
```

$B_{ENG} =$

```
[9.4963E-01 2.3631E+01 -4.9595E+01 2.1515E+02
5.4757E-01 7.0019E+00 -6.9243E+00 -1.4309E+02
1.8770E-03 -6.7840E-02 6.5963E-02 9.1799E-01]
```

$C_{ENG} =$

```
[-2.5000E-04 2.5630E-03 5.8438E-04
 9.0446E-05 1.9460E-05 3.4843E-05
 1.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00 1.0000E+00 0.0000E+00]
```

$D_{ENG} =$

```
[9.1118E-04 -3.6610E-02 3.6595E-02 2.7484E-01
8.0736E-05 -7.9718E-03 6.5511E-03 1.4147E-03
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00]
```

### Engine Initial Conditions

#### Engine Inputs

WFGG = 8477.4 LBM/HR

AJ = 412.36 Square Inches

RCVV = 4.998 Degrees

CIVV = -3.2644

#### Engine Outputs

OPR = 29.7811

EPR = 3.3207

N1 = 10072.0

N2 = 12934.0

### Continuous Engine Control Matrices

$A_{CON} =$

```
[-4.3292E-02 1.6149E-01 -4.4758E-02 -1.9082E-02
-4.9261E-02 1.7718E-01 -1.3522E-02 -1.4203E-04;
```

```
4.2306E+00 -3.2123E+01 4.5686E+00 9.7397E-01
2.9011E+00 -9.9265E+00 7.7580E-01 -8.0083E-05;
1.6532E+00 1.2558E+01 -1.8890E+00 -3.1531E-01
-1.4378E+00 5.0617E+00 -3.9120E-01 -2.5933E-03;
9.5029E-02 -1.6824E-02 -4.7431E-01 -9.6873E-02
-1.4104E+00 5.5724E+00 -1.6838E-01 -2.4034E-02;
-1.7567E-01 4.5004E-01 -2.5185E-01 1.4600E+00
-1.2836E+00 5.1425E+00 -5.4465E-01 -4.07656E-02;
-1.7857E+00 1.8669E+01 -2.0910E+00 -8.3963E+00
3.5368E+00 -1.5721E+01 2.1437E+00 3.4084E-02;
8.7670E-02 -1.0829E+00 3.0569E-02 4.4022E-01
-5.8495E-01 2.1681E+00 -5.2122E-01 8.2662E-02;
7.4756E-02 8.9065E-01 -2.5186E-01 -2.9232E-01
2.6316E-01 -9.7410E-01 4.0254E-01 -2.5564E+01;]
```

$B_{CON} =$

```
[-2.5634E+03 5.7873E+03 6.1087E-01
 3.4000E+01 6.3507E-02 -4.1495E+00
 2.5597E+03 -6.8426E+03 1.8448E+00
 -5.1609E+02 2.3261E+02 1.3056E-01
 9.7177E+02 3.9223E+03 2.0604E-01
 3.6780E+02 8.6427E+02 2.2009E+00
 6.3838E+01 -8.1247E+02 -1.2134E-01
 4.0334E+01 2.5279E+03 1.1738E-01];
```

$C_{CON} =$

```
[3.1659E+00 -4.1167E+01 4.7435E+00 6.2546E-01
1.8765E+00 -5.8657E+00 4.9001E-01 2.1750E-03;
-4.2180E-02 -3.9587E-01 -2.2622E-02 -2.3789E-01
-2.2869E-01 1.0279E-01 9.2571E-02 2.2917E-01;
-9.4694E-02 7.3880E-02 -8.5972E-02 -1.4396E-02
2.8710E-02 3.5235E-02 1.3314E-02 1.6992E-04;]
```

$D_{CON} =$

```
[0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00]
```

### IWP Gains

WFGG	AJ	RCVV
-14.5710	-65.5204	-367.1137
-3.7220	-5.2604	485.3209
15.8385	64.4019	35.2803
-5.6320	-32.0446	-158.7061
1.0615	32.4970	-196.3278
8.7912	71.4423	104.3308
-8.9439	-35.0047	-152.2947
2.0274	-33.6911	134.0121

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE October 1995		3. REPORT TYPE AND DATES COVERED Technical Memorandum
4. TITLE AND SUBTITLE  An Optimized Integrator Windup Protection Technique Applied to a Turbofan Engine Control			5. FUNDING NUMBERS  WU-505-62-50	
6. AUTHOR(S)  Stephen R. Watts and Sanjay Garg				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191			8. PERFORMING ORGANIZATION REPORT NUMBER  E-9855	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Washington, D.C. 20546-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA TM-107035	
11. SUPPLEMENTARY NOTES Stephen R. Watts, United Technologies Pratt & Whitney, Government Engine and Space Propulsion, West Palm Beach, Florida (work funded by NASA Contract NAS3-26618) and Sanjay Garg, NASA Lewis Research Center. Responsible person, Sanjay Garg, organization code 2550, (216) 433-2355.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Categories 63 and 07  This publication is available from the NASA Center for Aerospace Information, (301) 621-0390.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  This paper introduces a new technique for providing memoryless integrator windup protection which utilizes readily available optimization software tools. This integrator windup protection synthesis provides a concise methodology for creating integrator windup protection for each actuation system loop independently while assuring both controller and closed loop system stability. The individual actuation system loops' integrator windup protection can then be combined to provide integrator windup protection for the entire system. This technique is applied to an $H^\infty$ based multivariable control designed for a linear model of an advanced afterburning turbofan engine. The resulting transient characteristics are examined for the integrated system while encountering single and multiple actuation limits.				
14. SUBJECT TERMS  Multivariable control; Engine control; Integrated windup protection			15. NUMBER OF PAGES 13	
			16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	



**National Aeronautics and  
Space Administration  
Lewis Research Center  
21000 Brookpark Rd.  
Cleveland, OH 44135-3191**

**Official Business  
Penalty for Private Use \$300**

**POSTMASTER: If Undeliverable — Do Not Return**

