# STUDIES OF DROPLET BURNING AND EXTINCTION 

F.A. Williams<br>Center for Energy and Combustion Research<br>Department of Applied Mechanics and Engineering Sciences<br>University of California San Diego, La Jolla, California 92093

## Introduction

A project on droplet combustion, pursued jointly with F.L. Dryer of Princeton University, has now been in progress for many years. The project involves experiments on the burning of single droplets in various atmospheres, mainly at normal atmospheric pressure and below, performed in drop towers (e.g., ref. 1) and designed to be performed aboard space-based platforms such as the Space Shuttle or the Space Station and currently manifest for Spacelab in the MSL-1 flight of the Space Shuttle in April of 1997. It also involves numerical computations on droplet burning, performed mainly at Princeton, and asymptotic analyses of droplet burning, performed mainly at UCSD. The focus of the studies rests primarily on time-dependent dropletburning characteristics and on extinction phenomena.

The presentation to be given here concerns the recent research on application of asymptotic methods to investigation of the flame structure and extinction of alcohol droplets. These theoretical studies are relevant to the second of the proposed space-flight tests and are currently investigating the extent to which combustion of alcohols can be described by four-step reduced chemistry similar to that which has achieved a good degree of success for alkane flames. These studies have progressed to a point at which a number of definite conclusions can now be stated. These conclusions and the reasoning that led to them are outlined here.

## Asymptotic Descriptions of Alcohol Diffusion Flames

A mixture fraction $Z$, defined to be zero in the oxidizer and unity in the fuel, serves as an attractive independent variable for analyses of diffusion-flame structures because its adoption enables the problem to be formulated in a universal manner that does not depend on the specific geometrical configuration. In the mixture-fraction formulation, the simplest asymptotic description is the Burke-Schumann reaction-sheet structure. This limit corresponds to an infinite Damköhler number for one-step, irreversible chemistry and provides the outer, transport-zone solutions to which the reaction-zone solutions are matched. In the reaction zone there is a oneparameter family of solutions, the parameter being $\chi_{\text {st }}$, the value at the stoichiometric mixture fraction $Z_{s t}$ of the scalar dissipation rate

$$
\begin{equation*}
\chi=2 D|\nabla Z|^{2}, \tag{1}
\end{equation*}
$$

where $D$ is the diffusion coefficient of the outer solutions. As $\chi_{s t}$ increases, the peak temperature decreases, and eventually an abrupt extinction is approached, at a critical value. For droplet burning, it is found that

$$
\begin{equation*}
\chi=\frac{8 D}{d_{l}^{2}}\left\{\frac{[\ln (1-Z)]^{2}(1-Z)}{\ln (1+B)}\right\}^{2}, \tag{2}
\end{equation*}
$$

where $d_{l}$ is the droplet diameter, and $B$ is the transfer number for the droplet combustion process, given in an approximation of constant specific heat $c_{p}$ by

$$
\begin{equation*}
B=\left\{c_{p}\left(T_{\infty}-T_{l}\right)+2 Q Y_{\mathrm{O}_{2 \infty}} /\left(3 n W_{\mathrm{O}_{2}}\right)\right\} / L, \tag{3}
\end{equation*}
$$

where $T, W$, and $Y$ denote temperature, molecular weight, and mass fraction, the subscript $\infty$ identifies the ambient atmosphere, $L$ is the energy required to vaporize a unit mass of fuel, $Q$ stands for the heat released per mole of gaseous fuel in combustion, and $n$ represents the number of carbon atoms in the alcohol. By use of equation (2) at $Z=Z_{s t}$, the droplet diameter at extinction $d_{l e}$, can be obtained from $\chi_{s t e}$, the value of $\chi_{s t}$ at extinction, which has been calculated without reference to the specific geometrical configuration. The calculation employs the reduced chemistry described below.

## Reduced Chemistry for Methanol

The starting chemical-kinetic mechanism and the associated rate parameters for methanol are listed in Table 1. The elementary steps and rate parameters given here differ somewhat from those of other studies, since they are based on an evaluation of rate data performed specifically for the present program. In Table $1, \mathrm{CH}_{2} \mathrm{OH}$ stands for the Boltzmann-equilibrated isomers $\mathrm{CH}_{3} \mathrm{O}$ and $\mathrm{CH}_{2} \mathrm{OH}$, and the last two sets of rate parameters refer to this mixture. The table represents a minimal set, in the sense that all of the elementary steps shown there are essential.

Introduction of steady-state approximations for all radicals except the important H atom results in a four-step reduced mechanism for the alcohols that can be written as

$$
\begin{gather*}
\mathrm{C}_{n} \mathrm{H}_{2 n+1} \mathrm{OH}+\gamma H+\left[\beta-\left(n+1+\frac{1}{2} \gamma\right)\right] \mathrm{H}_{2} \mathrm{O}+\frac{1}{2}\left[3 n+\frac{1}{2} \gamma\right.  \tag{I'}\\
-\beta(1+\alpha)] \mathrm{O}_{2} \rightarrow \beta\left(\mathrm{H}_{2}+\alpha \mathrm{CO}\right)+(n-\alpha \beta) \mathrm{CO}_{2} \\
\mathrm{CO}+\mathrm{H}_{2} \mathrm{O} \leftrightarrows \mathrm{CO}_{2}+\mathrm{H}_{2}  \tag{II"}\\
2 \mathrm{H}+\mathrm{M} \rightarrow \mathrm{H}_{2}+\mathrm{M}  \tag{III"}\\
3 \mathrm{H}_{2}+\mathrm{O}_{2} \leftrightarrows 2 \mathrm{H}+2 \mathrm{H}_{2} \mathrm{O} \tag{IV"}
\end{gather*}
$$

where $\alpha$ is the number of CO molecules produced per $\mathrm{H}_{2}$ molecule produced, $\beta$ is the number of $\mathrm{H}_{2}$ molecules produced per fuel molecule consumed, and $\gamma$, a quantity of critical importance, is the number of radicals destroyed for each fuel molecule consumed. For methanol, this four-step description involves introducing steady-state approximations for $\mathrm{CH}_{2} \mathrm{OH}, \mathrm{CH}_{2} \mathrm{O}, \mathrm{CHO}, \mathrm{HO}_{2}$, OH and O in Table 1 and results (ref. 2) in $\gamma=2$, giving a reduced mechanism that can be written as $\mathrm{CH}_{3} \mathrm{OH}+2 \mathrm{H} \rightarrow 3 \mathrm{H}_{2}+\mathrm{CO}, \mathrm{CO}+\mathrm{H}_{2} \mathrm{O} \leftrightarrows \mathrm{CO}_{2}+\mathrm{H}_{2}, 2 \mathrm{H}+\mathrm{M} \rightarrow \mathrm{H}_{2}+\mathrm{M}$ and $3 \mathrm{H}_{2}+\mathrm{O}_{2} \leftrightarrows$ $2 \mathrm{H}+2 \mathrm{H}_{2} \mathrm{O}$, with corresponding rates $k_{14} \mathrm{C}_{F} \mathrm{C}_{\mathrm{H}},\left(k_{9} / K_{3}\right) \mathrm{C}_{\mathrm{H}}\left(\mathrm{C}_{\mathrm{CO}} \mathrm{C}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{C}_{\mathrm{H}_{2}}-\mathrm{C}_{\left.\mathrm{CO}_{2} K_{3} / K_{9}\right)}\right.$, $k_{5} \mathrm{C}_{M} \mathrm{C}_{\mathrm{H}} \mathrm{C}_{\mathrm{O}_{2}}$ and $k_{1 f} \mathrm{C}_{\mathrm{H}}\left[\mathrm{C}_{\mathrm{O}_{2}}-\left(\mathrm{C}_{\mathrm{H}^{2}}^{2} \mathrm{C}_{\mathrm{H}_{2} \mathrm{O}}^{2} / \mathrm{C}_{\mathrm{H}_{2}}^{3} K_{1} K_{2} K_{3}^{2}\right)\right]$, respectively, where $\mathrm{C}_{i}$ denotes the concentration of species $i$, subscript $F$ representing the fuel, $\mathrm{CH}_{3} \mathrm{OH}$, and the $k$ 's and $K$ 's are specific reaction-rate constants and equilibrium constants. Here approximations of partial equilibria have been introduced for steps 2 and 3 , and the third-body concentration $C_{M}$ is weighted with the efficiencies given in Table 1, assuming that only $\mathrm{N}_{2}, \mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ are present in significant concentrations in the reaction zone.

Full numerical computations have shown that water-gas equilibrium is an excellent approximation in the reaction zones of these flames, much more accurate than for hydrocarbon
flames, for example. Therefore, the second of the four steps is put into equilibrium, giving $\mathrm{C}_{\mathrm{CO}}$ $=\alpha \mathrm{CH}_{2}$, where $\alpha \equiv\left(\mathrm{C}_{\mathrm{CO}_{2} K_{3}}\right) /\left(\mathrm{C}_{\mathrm{H}_{2} \mathrm{O}} K_{9}\right)$. There results a three-step mechanism, which can be written as

$$
\begin{gather*}
\mathrm{CH}_{3} \mathrm{OH}+2 \mathrm{H}+\frac{3 \alpha-1}{\alpha+1} \mathrm{CO}_{2} \rightarrow \frac{4}{\alpha+1}\left(\mathrm{H}_{2}+\alpha \mathrm{CO}\right)+\frac{3 \alpha-1}{\alpha+1} \mathrm{H}_{2} \mathrm{O}  \tag{I'}\\
2 \mathrm{H}+\frac{\alpha}{\alpha+1} \mathrm{CO}_{2} \rightarrow \frac{1}{\alpha+1}\left(\mathrm{H}_{2}+\alpha \mathrm{CO}\right)+\frac{\alpha}{\alpha+1} \mathrm{H}_{2} \mathrm{O} \tag{II'}
\end{gather*}
$$

and

$$
\begin{equation*}
\mathrm{O}_{2}+\frac{3}{\alpha+1}\left(\mathrm{H}_{2}+\alpha \mathrm{CO}\right) \rightarrow 2 \mathrm{H}+\frac{2-\alpha}{\alpha+1} \mathrm{H}_{2} \mathrm{O}+\frac{3 \alpha}{\alpha+1} \mathrm{CO}_{2} \tag{III'}
\end{equation*}
$$

with rates $k_{14} \mathrm{C}_{F} \mathrm{C}_{\mathrm{H}}, k_{5} \mathrm{C}_{M} \mathrm{C}_{\mathrm{H}} \mathrm{C}_{2}$ and $k_{1 f} \mathrm{C}_{\mathrm{H}}\left[\mathrm{C}_{\mathrm{O}_{2}}-\mathrm{C}_{\mathrm{H}}^{2} \mathrm{C}_{\mathrm{H}_{2} \mathrm{O}}^{2} /\left(\mathrm{C}_{\mathrm{H}_{2}}^{3} K_{1} K_{2} K_{3}^{2}\right)\right]$, respectively.
If, in addition, a steady-state approximation for H is introduced, then there results the two-step reduced mechanism

$$
\begin{gather*}
\mathrm{CH}_{3} \mathrm{OH}+\mathrm{O}_{2} \rightarrow\left(\frac{1+2 \alpha}{1+\alpha}\right) \mathrm{H}_{2} \mathrm{O}+\left(\frac{1}{1+\alpha}\right) \mathrm{CO}_{2}+\left(\frac{1}{1+\alpha}\right)\left(\mathrm{H}_{2}+\alpha \mathrm{CO}\right)  \tag{I}\\
\left(\frac{1}{1+\alpha}\right)\left(\mathrm{H}_{2}+\alpha \mathrm{CO}\right)+\left(\frac{1}{2}\right) \mathrm{O}_{2} \rightarrow\left(\frac{1}{1+\alpha}\right) \mathrm{H}_{2} \mathrm{O}+\left(\frac{\alpha}{1+\alpha}\right) \mathrm{CO}_{2} \tag{II}
\end{gather*}
$$

The steady-state approximation for H results in

$$
\begin{equation*}
\mathrm{C}_{\mathrm{H}}=\left(K_{1} K_{2}\right)^{1 / 2} K_{3} \mathrm{C}_{\mathrm{O}_{2}}^{1 / 2}\left(\mathrm{C}_{\mathrm{H}_{2}}^{3 / 2} / \mathrm{C}_{\mathrm{H}_{2} \mathrm{O}}\right)\left[1-\left(k_{14} \mathrm{C}_{\mathrm{F}}\right) /\left(k_{1 f} \mathrm{C}_{\mathrm{O}_{2}}\right)\right]^{1 / 2} \tag{4}
\end{equation*}
$$

where $k_{5} \mathrm{C}_{M} \mathrm{CO}_{2}$ has been neglected in comparison with $k_{14} \mathrm{C}_{F}$ and $k_{1 f} \mathrm{C}_{\mathrm{O}_{2}}$ because it is quite small in this expression at the pressures considered. The rates of steps I and II are found by reduction of the preceding rates to be

$$
\begin{equation*}
\omega_{I}=k_{14} C_{F} C_{\mathrm{H}}, \quad \omega_{I I}=2 k_{5} \mathrm{C}_{M} \mathrm{C}_{\mathrm{O}_{2}} \mathrm{C}_{\mathrm{H}} \tag{5}
\end{equation*}
$$

in which equation (4) can be used for $\mathrm{C}_{\mathrm{H}}$. This serves to relate the rates to the temperature and the concentrations of the species appearing in steps I and II, as needed in the analysis. In the three-step mechanism, equation (4) is discarded because $\mathrm{C}_{\mathrm{H}}$ becomes an additional dependent variable described by an independent differential equation.

## The Flame Structure

The flame structure is analyzed by rate-ratio asymptotics (RRA). In the RRA description, there is a thin fuel-consumption zone on the rich side of a broader but still thin layer of $\mathrm{H}_{2}$ and CO oxidation. This structure is illustrated in figure 1 , where $\delta$ and $\varepsilon$ are small parameters that measure, in the mixture-fraction coordinate, the thicknesses of the fuel-consumption zone and of the oxidation layer, respectively. The fuel-consumption zone, having peak temperature $\mathrm{T}^{0}$, is located at $Z=Z_{0}$, a value displaced from $Z_{\text {st }}$ by the small amount $\varepsilon \xi_{0}$, where $\xi_{0}$ is calculated in the analysis. This structure is quite similar to that previously determined in the present program for alkane flames, such as heptane; essentially the only difference is the chemistry that occurs in the fuel-consumption zone. The methanol flame is in fact simpler than the alkane flames in that water-gas equilibrium is a much better approximation, so that a three-step description suffices.

The RRA analysis yields the peak temperature as a function of $\chi_{s t}^{-1}$, as shown in figure 2. These results are essentially the same for both two-step and three-step descriptions. The minimum value of $\chi_{s t}^{-1}$ identifies the extinction condition. From these minima, equation (2) can
be used to calculate extinction diameters as functions of the oxygen mole fraction in the atmosphere, as shown in figure 3 for three different values of the atmospheric pressure $p$. Figure 3 applies for pure methanol droplets that have not absorbed water during burning. In fact, water absorption is significant and modifies extinction conditions, as indicated below.

## Effects of Water Dissolution

Water is produced in the flame and dissolves in the liquid fuel as it burns. We have completed an analysis of this effect, treating the gas phase as quasisteady. Two limiting cases were analyzed, namely time-dependent liquid-phase diffusion and perfect liquid-phase mixing. Figure 4 shows results of these analyses for the square of the droplet diameter as a function of time.

It is seen from figure 4 that with time-dependent liquid-phase diffusion the $d^{2}$ plots are nearly straight lines. This is because the low diffusion coefficient for water in methanol leads to a relatively small amount of water absorption; the water is contained only in a very thin boundary layer at the liquid surface. On the other hand, with perfect liquid-phase mixing, the $d^{2}$ plots are curved, illustrating the reduced vaporization rate as water builds up inside the liquid. These curved lines are seen to be in better agreement with experiment. It is therefore concluded that the experiments involved an appreciable amount of liquid-phase mixing. This same conclusion was reached independently by Dryer and co-workers, on the basis of numerical integrations. The agreement in the conclusion from the two different approaches is a strong indication that the conclusion is correct.

The water absorption also influences flame extinction, a further point of agreement with Dryer et al. In the present approach, since the gas phase is quasisteady, calculating extinction conditions entails calculating extinction diameters for droplets consisting of water-methanol mixtures with different percentages of water. For example, figure 5 shows predicted extinction diameters for droplets consisting of $50 \%$ water and $50 \%$ methanol on a mass basis. These diameters are seen to be appreciably larger than those in figure 3. Figure 6 illustrates how to determine extinction conditions with time-dependent water absorption. Basically, the droplet diameter and the extinction diameter are plotted as functions of time, and extinction occurs when these two diameters become equal. The resulting extinction predictions are in good agreement with experiment.

## Future Plans

The research by asymptotics on methanol droplet combustion is essentially completed. It is intended to spend a little time looking at the kinetics of fuel consumption for higher alcohols, to determine the associated values of $\gamma$; the kinetic information needed to do a reasonable job of this has just become available, and so it is a timely investigation. However, most of the future research will involve studying heptane droplet combustion in helium-oxygen atmospheres. This is the system that will be employed in the first flight, MSL-1, and data on this system already are available from preliminary drop-tower tests. Most of the future work, for the next year or two at least, will therefore concentrate on this system. Data on histories of droplet and flame diameters and on extinction conditions will be reduced and compared with our previous theoretical predictions. Data on intensity profiles of emissions in OH bands also will be addressed, to determine what information that data can provide on flame structure, to be compared with theory. The general intent will be to improve understanding of heptane droplet combustion.

## References

1. Choi, M.Y., Dryer, F.L, Card, J.M., Williams, F.A., Haggard, J.B., Jr., and Brorowski, B.A., "Microgravity Combustion of Isolated $n$-Decane and $n$-Heptane Droplets," AIAA Paper No. 92-242, January 1992.
2. Zhang, B.L., Card, J.M., and Williams, F.A., "Application of Rate-Ratio Asymptotics to the Prediction of Extinction for Methanol Droplet Combustion," Combustion and Flame, to be submitted, 1995.

Table 1
The Starting Chemical-Kinetie Mechanism and the Associated Rate Parameters in the form $k=. A I^{n} \exp \left(-E / R^{0} T\right)$

| No. ${ }^{\text {a }}$ | Reactions | $A^{\text {a }}$ | $\mathrm{n}^{\text {e }}$ | $E^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Hydrogen-Oxygen Chain |  |  |  |  |
| 11 | $\mathrm{H}+\mathrm{O}_{2} \rightarrow \mathrm{OH}+\mathrm{O}$ | $3.52 \times 10^{26}$ | -0.7 | 17070 |
| 16 | $\mathrm{OH}+\mathrm{O} \rightarrow \mathrm{H}+\mathrm{O}_{2}$ | $1.15 \times 10^{14}$ | -0.324 | -175 |
| $2 f$ | $\dot{\mathrm{H}}_{2}+\mathrm{O} \rightarrow \mathrm{OH}+\mathrm{H}$ | $5.06 \times 10^{4}$ | 2.67 | 6290 |
| 2b | $\mathrm{OH}+\mathrm{H} \rightarrow \mathrm{H}_{2} \div \mathrm{O}$ | $2.22 \times 10^{4}$ | 2.67 | 4398 |
| 3 f | $\mathrm{H}_{2}+\mathrm{OH} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{H}$ | $1.17 \times 10^{9}$ | 1.30 | 3626 |
| 3 b | $\mathrm{H}_{2} \mathrm{O} \div \mathrm{H} \rightarrow \mathrm{H}_{2} \div \mathrm{OH}$ | $6.65 \times 10^{9}$ | 1.30 | 4352 |
| 41 | $\mathrm{OH} \div \mathrm{OH} \rightarrow \mathrm{H}_{2} \mathrm{O} \div \mathrm{O}$ | $k=5.46 \times 10^{12} \mathrm{exp}(0.001497)$ |  |  |
| 4 b | $\mathrm{H}_{2} \mathrm{O}+\mathrm{O} \rightarrow \mathrm{OH}+\mathrm{OII}$ | $k=4.19 \times 10^{7} T^{.58} \mathrm{exp}(0.003 T)$ |  |  |


| Hydroperoxyl Formation and Consumpion |  |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
| $5^{6}$ | $\mathrm{H}_{4}+\mathrm{O}_{2}+\mathrm{M}-\mathrm{HO}_{2}+\mathrm{M}$ | $6.76 \times 10^{25}$ | -1.4 | 0 |
| $6-$ | $\mathrm{HO}_{2}+\mathrm{H} \rightarrow \mathrm{OH}+\mathrm{OH}$ | $1.70 \times 10^{14}$ | 0.0 | 874 |
| 7 | $\mathrm{HO}_{2}+\mathrm{H} \rightarrow \mathrm{H}_{2}+\mathrm{O}_{2}$ | $4.28 \times 10^{13}$ | 0.0 | 1411 |
| 8 | $\mathrm{HO}_{2}+\mathrm{OH} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{O}_{2}$ | $2.89 \times 10^{13}$ | 0.0 | -497 |


|  |  |  |  |  | Wiater-Ges Shift |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Of | $\mathrm{CO}+\mathrm{OH} \rightarrow \mathrm{CO}_{2} \div \mathrm{II}$ | $4.40 \times 10^{\circ}$ | 1.5 | .741 |  |
| $9 b$ | $\mathrm{CO}_{2}+\mathrm{IH} \rightarrow \mathrm{CO}+\mathrm{OH}$ | $4.97 \times 10^{8}$ | 1.5 | 21446 |  |


| - | Formyl Formation and Consumption |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
| 10 | $\mathrm{CHO}+\mathrm{H} \rightarrow \mathrm{CO} \div \mathrm{H}_{2}$ | $1.00 \times 10^{14}$ | 0.0 | 0 |
| $11^{\mathrm{c}}$ | $\mathrm{CHO}+\mathrm{M} \rightarrow \mathrm{CO}+\mathrm{H}+\mathrm{M}$ | $2.85 \times 10^{14}$ | 0.0 | 16795 |
| 12 | CH | $\mathrm{O}+\mathrm{H} \rightarrow \mathrm{CHO} \div \mathrm{H}_{2}$ | $2.26 \times 10^{20}$ | 1.05 |

Fuel Consumption and Formaldehyde Formation

| Fuel Consumption end Formalden |  |  |  |  |  |
| :--- | :--- | :--- | :--- | ---: | :---: |
| 13 | $\mathrm{CH}_{2} \mathrm{OH}+\mathrm{H} \rightarrow \mathrm{CH}_{2} \mathrm{O}+\mathrm{H}_{2}$ | $3.00 \times 10^{13}$ | 0.0 | 0 |  |
| 14 | $\mathrm{CH}_{3} \mathrm{OH}+\mathrm{H} \rightarrow \mathrm{CH}_{2} \mathrm{OH}+\mathrm{HI}_{2}$ | $4.00 \times 10^{13}$ | 0.0 | 6092 |  |

- Units: $\mathrm{mol} / \mathrm{cm}^{3}, \mathrm{~K}, \mathrm{cal} / \mathrm{mol}$.
( Chaperon Efficiencies: $\mathrm{N}_{2}, \mathrm{O}_{2}$ : $1.0, \mathrm{CO}: 1.9, \mathrm{CO}_{2}: 3.8, \mathrm{H}_{2}: 2.5, \mathrm{H}_{2} \mathrm{O}: 12.0$.
c Chaperon Efficiencies: same as $b$ except $\mathrm{H}_{2} \mathrm{O}: 16.3$


Figure 1. Schematic diagram of the diffusion-flame structure in the mixture-fraction coordinate; dashed lines give the Burke-Schumann structure and solid carves the structure in the two-step approximation.


Figure 3 The predicted extinction diameters of pure methenol droplets, burning in oxygen-nitrogen mixpures at 300 K , as functions of the ambient oxygen mole fraction, for three different pressures.


Figure 5 The predicied extinction diameters of droplets composed of $50 \%$ methanol and $50 \%$ water on a mass basis, burning in oxygen-nitrogen mixtures at 300 K , as functions of the ambient oxygen mole fraction, for three different pressures.


Figure 2 The predicted temperature at the fuelconsumption zone, as a function of the reciprocal of the rate of scalar dissipation at the stoichiomerric mixpure fraction, for pure methanol droplets buming in air at 300 K , for three different pressures.


Figure 4 The square of the droplet diameter as a function of time for droplets initially 1 mm in diamerer, burning in air at $p=1 \mathrm{am}$ in a helium-oxygen mixnure with $\mathrm{XO}_{2} 2=$ 0.5 at $p=1 \mathrm{~atm}$, and for a droplet initially 0.7 mm in diameter, buming in air at $p=0.25 \mathrm{~atm}$; solid lines are for perfect liquid-phase mixing with the present theory, dashed lines are for time-dependent liquid-phase diffusion with the present theory, and points are data from the literature.


Figure 6 The droplet diameier, the extinction diameter, and the water flux fraction $\varepsilon_{H_{2}} O$ as functions of time with timedependent water absorpion for perfect liquid-phase mixing illustrating how extincion conditions are determined.

