Antibunching effect of k-component q-coherent states

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Abstract

We introduce the antibunching effect for the q-electromagnetic field, and study this kind nonclassical properties of k-component q-coherent states given by Kuang et al.[Phys.Lett.A173(1993)1]. The results show that all of them show antibunching effect.

Recently, coherent states of quatum algebras[1−3] (q-CSs) have attracted a lot of attention due to their maybe applications in many fields of physics and mathematical physics[4−5]. The Glauber-typed q-CSs[6−7] have been studies in great detail and applied widely to various concrete physical problems[8]. In the refrences[9−10], the even and odd q-CSs representations were constructed and the squeezing properties of them were discussed. The even and odd q-CSs are defined to be the eigenstates of the square (a_q^2) of the q-annihilation operator. More recently,k-component q-CSs were introduced and their properties were investigated by Kuang et al.[11]. On the basis of this work, in this paper, we study the antibunching effect of them, because of this effect has the typical nonclassical property. Squeezing properties of them were investigated by us[12].
The k-component (k is an integer and $k \geq 3$) $q - CSs$ were given by [11]

$$\mid z,k,i >_q = A_1^{-1/2}(\mid z \mid^2, k) \sum_{n=0}^{\infty} \frac{z^{kn+i}}{\sqrt{[kn+i]_q!}} \mid kn+i >_q, (i = 0,1,\cdots,k-1), \qquad (1)$$

$$A_i(\mid z \mid^2, k) = \sum_{n=0}^{\infty} \frac{\mid z \mid^{2(kn+i)}}{\sqrt{[kn+i]_q!}}, (i = 0,1,2,\cdots,k-1) \qquad (2)$$

where z is a complex number, the q-fractorial $[n]_q! = [n]_q[n-1]_q \cdots [1]_q$ with the q-number $[X]_q = (q^X - q^{-X}/(q - q^{-1})$. Their actions on the basis vectors are

$$a_q^+ = \sqrt{[n+1]_q} \mid n+1 >_q, a_q \mid n >_q = \sqrt{[n]_q} \mid n-1 >_q . \qquad (3)$$

It is easy to prove that the k states of (1) are all the eigenstates of the operator $a_q^k (k \geq 3)$ with the same eigenvalue z^k.
It is well known that, when the second-order correlation function of a light field[13]$g^{(2)}(0) < 1$, one says that the light field exhibits an antibunching effect. In a similar way, we introduce a second-

order q-correlation function for the q-light field,

$$g_q^{(2)} = \frac{_q <\mid a_q^{+2}a_q^2 >_q}{_q <\mid a_q^+ a_q \mid >_q^2}, \tag{4}$$

If the second-order q-correlation function of the q-light field $g_q^{(2)}(0) < 1$, we say that the q-light field exhibits the antibunching effect. Now, we study the antibunching effect of the k states given by (1).

Using the relations[11],

$$a_q^m \mid z, k, 0 >_q = z^m A_0^{-1/2} A_{k-m}^{1/2} \mid z, k, k-m >_q, (m = 1, 2, \cdots, k), \tag{5}$$

for the k states of (1), it is easy to prove that the relations hold:

$$_q < z, k, 0 \mid a_q^+ a_q \mid z, k, 0 >_q = \mid z \mid^2 A_{k-1}/A_0, \tag{6}$$

$$_q < z, k, m \mid a_q^+ a_q \mid z, k, m >_q = \mid z \mid^2 A_{m-1}/A_m, (m = 1, 2, \cdots, k-1), \tag{7}$$

$$_q < z, k, 0 \mid a_q^{+2} a_q^2 \mid z, k, 0 >_q = \mid z \mid^4 A_{k-2}/A_0, \tag{8}$$

$$_q < z, k, 1 \mid a_q^{+2} a_q^2 \mid z, k, 1 >_q = \mid z \mid^4 A_{k-1}/A_1, \tag{9}$$

$$_q < z, k, m \mid a_q^{+2} a_q^2 \mid z, k, m >_q = \mid z \mid^4 A_{m-2}/A_m, (m = 2, 3, \cdots, k-1). \tag{10}$$

By means of (6)-(10) the q-coherent degrees of the second order of the k states given by (1) can be obtained, respectively, they are

$$g_{q0}^{(2)}(0) = \frac{_q < z, k, 0 \mid a_q^{+2} a_q^2 \mid z, k, 0 >_q}{_q < z, k, 0 \mid a_q^+ a_q \mid z, k, 0 >_q^2} = \frac{A_0 A_{k-2}}{A_{k-1}^2}, \tag{11}$$

$$g_{q1}^{(2)}(0) = \frac{_q < z, k, 1 \mid a_q^{+2} a_q^2 \mid z, k, 1 >_q}{_q < z, k, 1 \mid a_q^+ a_q \mid z, k, 1 >_q^2} = \frac{A_1 A_{k-1}}{A_0^2} \tag{12}$$

$$g_{qm}^{(2)}(0) = \frac{_q < z, k, m \mid a_q^{+2} a_q^2 \mid z, k, m >_q}{_q < z, k, m \mid a_q^+ a_q \mid z, k, m >_q^2} = \frac{A_{m-2} A_m}{A_{m-1}^2}, (m = 2, 3, \cdots, k-1). \tag{13}$$

Substituting (2) into (11), we obtain

$$g_{q0}^{(2)}(0) = \frac{\sum_{m=0}^{\infty} \left[\sum_{n=0}^{m} \frac{1}{[kn]_q![km-kn+k-2]_q!}\right] x^{km}}{x^k \sum_{m=0}^{\infty} \left[\sum_{n=0}^{m} \frac{1}{[kn+k-1]_q![km-kn+k-1]_q!}\right] x^{km}} = \frac{f_{q1}(x)}{x^k f_{q2}(x)}, \tag{14}$$

where $x = \mid z \mid^2$. Consider $k \geq 3$, while $[n]_q > [n-1]_q \geq 1$, therefore we have

$$\sum_{n=0}^{m} \frac{1}{[kn]_q![km-kn+k-2]_q!} > \sum_{n=0}^{m} \frac{1}{[kn+k-1]_q![km-kn+k-1]_q!}, \tag{15}$$

and hence $f_{q1}(x) > f_{q2}(x)$, so that $g_{q0}^{(2)} > 1$ when $x < 1$, However, when $x > 1$, there surely exist values of x (e.g., $x^k > f_{q1}(x)/f_{q2}(x)$) for which the relation holds:

$$g_{q0}^{(2)}(0) = \frac{f_{q1}(x)}{x^k f_{q2}(x)} < 1. \tag{16}$$

Therfore, the state $\mid z,k,0 >_q$ may exhibis antibunching effect when $x > 1$. Substituting (2) into (12), we have

$$g^{(2)}_{q1}(0) = \frac{x^k \sum_{m=0}^{\infty}\left[\sum_{n=0}^{m}\frac{1}{[kn+1]_q![km-kn+k-1]_q!}\right]x^{km}}{\sum_{m=0}^{\infty}\left[\sum_{n=0}^{m}\frac{1}{[kn]_q![km-kn]_q!}\right]x^{km}} = \frac{x^k f_{q3}(x)}{f_{q4}(x)}, \qquad (17)$$

Obviously,

$$\sum_{n=0}^{m}\frac{1}{[kn+1]_q![km-kn+k-1]_q!} < \sum_{n=0}^{m}\frac{1}{[kn]_q![km-kn]_q!}, \qquad (18)$$

so that $f_{q3}(x) < f_{q4}(x)$. Therefore $g^{(2)}_{q1}(0) < x^k$,i.e., when $x \leq 1, g^{(2)}_{q1}(0) < 1$. From (2) and (13), we obtain

$$\begin{aligned}
g^{(2)}_{qm}(0) &= \frac{\sum_{n'=0}^{\infty}\left[\sum_{n=0}^{n'}\frac{1}{[kn+m-2]_q![kn'-kn+m]_q!}\right]x^{kn'}}{\sum_{n'=0}^{\infty}\left[\sum_{n=0}^{n'}\frac{1}{[kn+m-1]_q![kn'-kn+m-1]_q!}\right]x^{kn'}} \\
&< \frac{\sum_{n=0}^{\infty}\frac{[n+1]_q}{[m-2]_q![m]_q!}x^{kn}}{\sum_{n=0}^{\infty}\frac{[n+1]_q}{\{[kn+m-1]_q!\}^2}} < \frac{\frac{1}{[m-2]_q![m]_q!}\sum_{n=0}^{\infty}[n+1]_q x^{kn}}{\frac{1}{\{[m-1]_q!\}^2}} = \frac{[m-1]_q\sum_{n=0}^{\infty}[n+1]_q x^{kn}}{[m]_q} (19)
\end{aligned}$$

Obviously,

$$\lim_{x\to 0}\sum_{n=0}^{\infty}[n+1]_q x^{kn} = [1]_q = 1, \qquad (20)$$

¿From Eq.(19),we obtain

$$\lim_{x\to 0} g^{(2)}_{qm}(0) < \lim_{x\to 0}\frac{[m-1]_q\sum_{n=0}^{\infty}[n+1]_q x^{kn}}{[m]_q} = \frac{[m-1]_q}{[m]_q} < 1. \qquad (21)$$

Therefore, the states $\mid z,k,m >_q$ $(m = 2,3,\cdots,k-1)$ exhibit antibunching effect when $x \to 0$. We sum up the above results and obtain that all of the k states given by (1) show the antibunching effect, i.e., they are all nonclassical states.It had been proved[11] that the k states is the complete, therefore, they form a nonclassical cmplete representation. For example, in this picture,the q-coherent state $\mid z >_q$ may be expressed as:

$$\mid z >_q = exp(-\mid z \mid^2 /2)\sum_{i=0}^{k-1}A_i^{1/2}(\mid z \mid^2,k)\mid z,k,i >_q \qquad (22)$$

It is interesting to note that when $q \to 1$, the eigenstates of the operator a_q^k become the states considered in our paper [14]. Therefore, this letter is a generalization of our paper[14] in the condition q-deformed.

References

[1] L.C.Biednharn,J.phys.A.**22**,L873(1989).

[2] C.P.Sun and H.C.Fu,J.phys.A.**22**,L983(1989)

[3] R.Floreanini,V.P.Spiridon et al.,Phys.lett.B.**242**, 383(1991).

[4] G.L.Alvarez,C.Gomez et al.,Nucl.Phys.B. **330**,347(1991).

[5] J.R.Klauder and B.S.Skagerstam,Coherent states (Singapore, World Scientific,1989)

[6] P.P.Kulish and E.V.Damaskinsky,J.Phys.A. **23**,L415(1990).

[7] A.J.Bracken et al.,J.Phys.A.**24**,1379(1991).

[8] A.Solomon and J.Katriel,J.Phys.A.**24**,L1209(1990).

[9] L.M.Kuang and F.B.Wang,Phys.Lett.A.**173**,221(1993); **169**,225(1992).

[10] L.M.Kuang, F.B.Wang et al., Acta Opt.Sin.**13**,1008(1993)(in Chinese)

[11] L.M.Kuang,F.B.Wang and G.J.Zeng,Phys.Lett.A.**176**,1(1993).

[12] Wang Jisuo and Wang Chuankui,Acta Opt.Sin.**14**,1043(1994),(in Chinese).

[13] D.F.Walls,Nature.**306**,141(1983)

[14] Jinzuo Sun,Jisuo Wang and Chuankui Wang,Phys.Rev.A.**44**,3369(1991).