QUANTUM STATE ENGINEERING VIA COHERENT-STATE SUPERPOSITIONS

J.Janszky, P.Adam, S.Szabo, P.Domokos
Research Laboratory for Crystal Physics
P.O. Box 132, H-1502 Budapest, Hungary

Abstract

The quantum interference between the two parts of the optical Schrödinger-cat state makes possible to construct a wide class of quantum states via discrete superpositions of coherent states. Even a small number of coherent states can approximate the given quantum states at a high accuracy when the distance between the coherent states is optimized, e. g. nearly perfect Fock state $|n\rangle$ can be constructed by discrete superpositions of $n+1$ coherent states lying in the vicinity of the vacuum state.

1 Introduction

Recently, much attention has been paid to the problem of generating quantum states of an electromagnetic field mode. In micromaser experiments various schemes have been proposed that allow us to create states with controllable number-state distribution [1, 2]. There are theoretical results presenting that certain quantum states can be arbitrarily well approximated by discrete superpositions of coherent states [3, 4]. The significance of applying a coherent-state expansion instead of the number-state one is to open new prospects in "quantum state engineering". Nonlinear interaction of the field, being initially in a coherent state, with a Kerr-like medium [5] or in degenerate parametric oscillator [6] leads to superpositions of finite number of coherent states. Back-action evading and quantum nondemolition measurements can also yield such superposition states [7, 8]. An atomic interference method has been developed, which can result in arbitrary superposition of coherent states on a circle in phase space [9]. Based on these promising schemes, implementation of experiments capable to produce required superpositions of coherent states can be anticipated.

In this paper we shall discuss the possibility to construct quantum states using coherent states superpositions. We find a simple set of superposition states which coincides with the Fock basis for any practical purpose.

2 Schrödinger-cat states

The superpositions of coherent states [10, 11],

$$| \alpha, \phi\rangle = c_\phi(| \alpha\rangle + e^{i\phi} | -\alpha\rangle), \tag{1}$$

referred to as Schrödinger-cat states, when the constituent coherent states are macroscopically distinguishable, have attracted much interest.

The two most typical superposition states are the even or "male" ($\phi = 0$) and odd or "female" ($\phi = \pi$) cat states. The case with small difference between the constituent states, by analogy, could be called Schrödinger-kitten states.

Although the coherent states are the most classical of all pure states of light, their simple superposition described by Eq. (1) shows remarkable nonclassical features as a consequence of the quantum interference [12, 13, 14].

The Wigner function of a Schrödinger-cat state with real $\alpha = x$

$$W(\beta) = \frac{{c_+}^2}{\pi}\left[\exp(-2|\beta - x|^2) + \exp(-2|\beta + x|^2) + 2\exp(-2|\beta|^2)\cos\left(4x\mathrm{Im}\beta + \phi\right)\right] \tag{2}$$

leads us to better understanding the origin of the quantum interference. The first two terms in the Wigner function of Eq. (2) correspond to the Gaussian bells of the constituent coherent states while the third term describes an interference fringe pattern between the bells. We note that although two coherent states with strongly different arguments are almost orthogonal to each other, the maximal amplitude of the interference fringe remains two times larger than the amplitudes of the constituent coherent states, independently from the distance between them.

The wavelength of the fringe decreases with the increase of the distance between the coherent states, the phase of the fringe depends on the relative phase ϕ in Eq. (1) between the composite part of the cat state (Fig. 1).

The picture becomes more complicated if we superpose more than 2 coherent states. In this case multiple fringes can constructively or destructively interfere with each other and also with the original coherent state bells to produce different nonclassical states as we will show in the next Section.

3 State engineering

Let us consider a pure state given as a superposition of coherent states along the real axis in phase space [12, 3, 15]

$$\mid \psi\rangle = \int F(x) \mid x\rangle dx \ . \tag{3}$$

Let us consider the following discrete superposition of coherent states along the real axis of the phase space

$$\mid \psi_N\rangle = \sum_{k=1}^{N} F_k \mid x_k\rangle \ . \tag{4}$$

Here the coherent states $\mid x_k\rangle$ are chosen to be equally distributed at distances d along the real axis around the coherent state $\mid x_0\rangle$ that belongs to the center of the corresponding one-dimensional distribution function $F(x)$ (Eq. 3), i.e.

$$x_k = x_0 + \left(k - \frac{N+1}{2}\right) d \ , \quad k = 1, \ldots N \ . \tag{5}$$

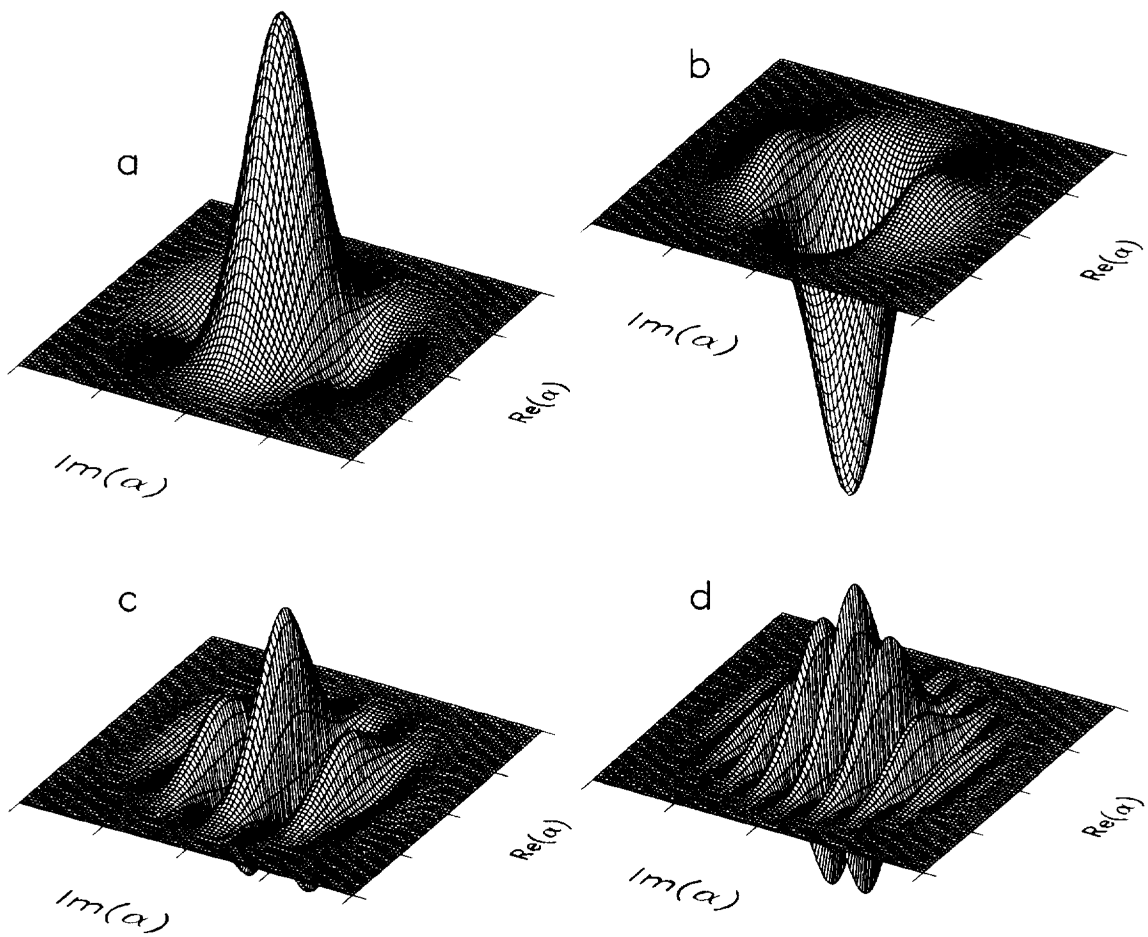

FIG. 1. The interference parts (fringes) of the Wigner functions of Schrödinger-cat states consisting of two coherent states put along the real axis of phase space. The phase difference between the coherent states changes the phase of the fringes, leading e. g. to the so-called male or female cat states (Fig.1a, $\phi = 0$ and Fig. 1b, $\phi = \pi$ respectively). Increasing the distance x of the coherent state from the origin of the phase space decreases the wave length of the fringes (Fig. 1a, $x = 0.6$; Fig. 1c, $x = 2$; Fig. 1d, $x = 4$).

The coefficients F_k are derived from the one-dimensional continuous distribution (Eq. 3)

$$F_k = cF(x_k) \; , \tag{6}$$

where c is a normalization constant.

As an example we consider displaced squeezed number states $\mid n, \zeta, Z\rangle$. Their interesting nonclassical properties were widely discussed in the literature [16]. The one-dimensional coherent-state representation of squeezed displaced number state along the real axis of phase space has the form [17]

$$F(x) = \tilde{c}_n H_n\left(\frac{x-Z}{\sqrt{2uv}}\right)\exp\left(-\frac{u-v}{2v}x^2 + \frac{(uZ - vZ^*)}{v}x\right) \; , \mathrm{Re}\left(\frac{u-v}{2v}\right) > 0 \; . \tag{7}$$

The parameters u and v are connected to the complex squeezing parameter ζ in the usual way

$$u = \cosh r \; , \quad v = e^{i\theta}\sinh r \; . \tag{8}$$

We note that states $\mid 0, \zeta, Z\rangle$ are the well-known squeezed coherent states.

In Fig. 2. we show how a squeezed Fock state builds up as we use more and more coherent states in the superposition. Here $n = 1$, the squeezing parameter $r = 0.5$. The sampling distance d for each N was optimized, minimizing the mismatch between the desired and the approximating states. In Fig. 2a even at $N = 3$ coherent states the resulting state began to resemble the desired state. Fig. 2b shows state made of 4 coherent states. The emerging target state can be clearly seen. As we added more coherent states ($N = 5$ and $N = 6$ for Figs. 2c and 2d respectively) the approximation became more and more perfect. In fact, the picture of the Wigner function of the superposition of 6 coherent state is indistinguishable from that of the squeezed 1-photon state.

Another possibility for state construction is if we begin the discretization described in this section with a one-dimensional representation of the state on a circle in phase-space [3, 18].

The discrete superposition of $n+1$ coherent states (a generalization of the female cat state) situated symmetrically on a circle with radius r in phase space

$$\mid n, r\rangle = c(r)\frac{\sqrt{n!}e^{\frac{r^2}{2}}}{(n+1)r^n}\sum_{k=0}^{n} e^{\frac{2\pi i}{n+1}k} \mid re^{\frac{2\pi i}{n+1}k}\rangle \; , \tag{9}$$

for small enough radius r leads to the n-photon Fock state $\mid n\rangle$ [19].

There are several experimental schemes which are appropriate to generate superposition states composed of coherent states lying on a circle in phase space. Making an initial coherent field interact with a sequence of two-level atoms detuned from the cavity resonance leads to such superpositions [8]. In the special case of their scheme, when the "phase-shift per photon" accumulated by the atomic dipoles crossing the cavity is a rational multiple of π, a symmetrical superposition of finite number of coherent states on a circle emerges. Required discrete superpositions on a circle, including the elements of the basis set given in Eq.(9), can be prepared in a single-atom interference method in a designed apparatus [9]. Superposition on a circle with small radius, that is essential in our case, can be generated in both of the above mentioned experimental schemes by starting with a field initially in coherent state with a small amplitude. The progress in quantum optics seems to enable us in the near future to create experimentally these superposition states.

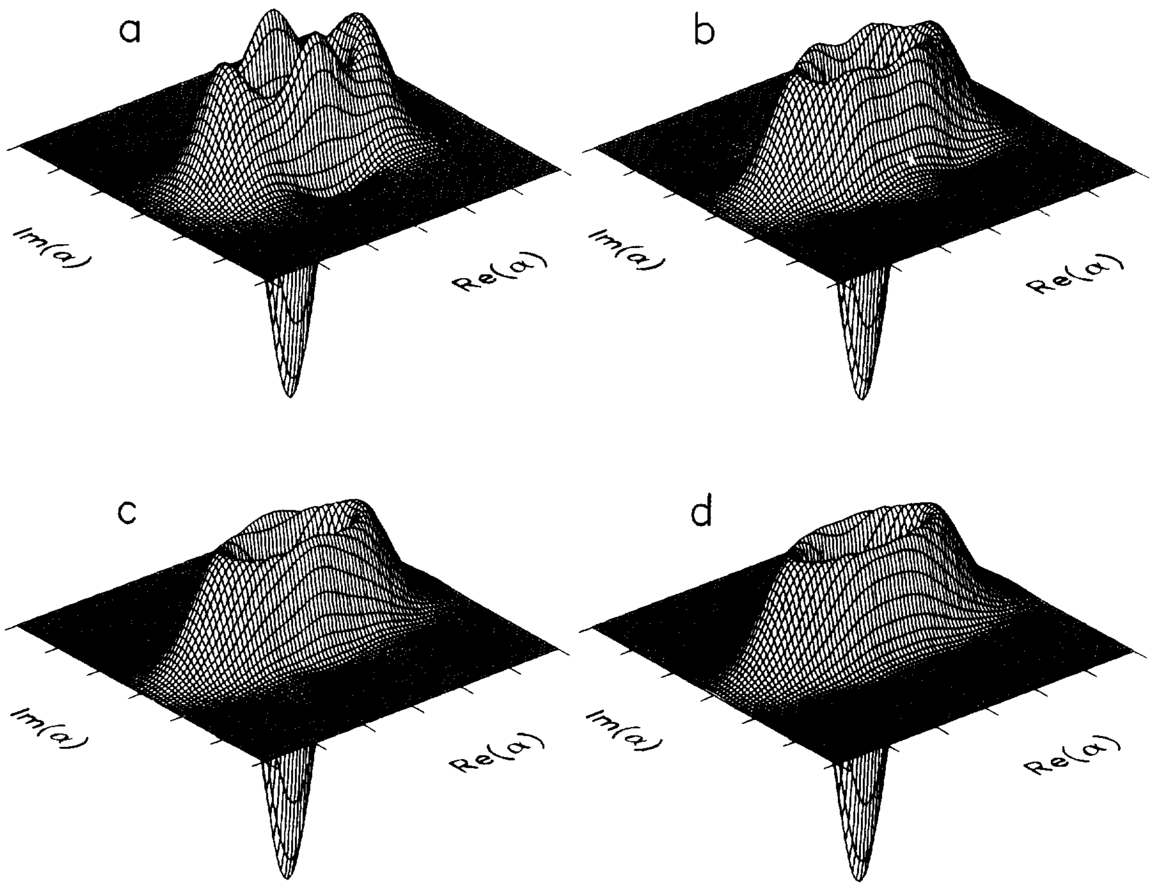

FIG. 2. Wigner functions of the coherent-state superpositions along the real axis approximating the squeezed number state $\mid n = 1, r = 0.5\rangle$. The numbers N of the constituent coherent states are equal to 3 (a), 4 (b), 5 (c), 6 (d) and the optimized distances d_{opt} of adjacent coherent states are equal to 1.27 (a), 1.13 (b), 1.03 (c) and 0.96 (d). Superposition of 6 coherent states gives surprisingly good approximation, while even that of 3 coherent states has features resembling the desired squeezed 1-photon state.

Acknowledgments

This work was supported by the National Research Scientific Fund (OTKA) of Hungary, under Contracts No. T14083, F17380.

References

[1] K. Vogel, V. M. Akulin, W. P. Schleich, Phys. Rev. Lett. **71**, 1816 (1993).

[2] B. M. Garraway, B. Sherman, H. Moya-Cessa, P. L. Knight, G. Kurizki, Phys. Rev. A **49**, 535 (1994).

[3] J. Janszky, P. Domokos, P. Adam, Phys. Rev. A **48**, 2213 (1993).

[4] P. Domokos, J. Janszky, P. Adam, T. Larsen, Quantum Opt. **6**, 187 (1994).

[5] B. Yurke, D. Stoler, Phys. Rev. Lett. **57**, 13 (1986).

[6] M. Wolinsky, H. J. Carmichael, Phys. Rev. Lett. **60**, 1836 (1988).

[7] B. Yurke, W. Schleich, D. F. Walls, Phys. Rev. A **42**, 1703 (1990).

[8] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, N. Zagury, Phys. Rev. A **45**, 5193 (1992).

[9] P. Domokos, J. Janszky, P. Adam, Phys. Rev. A **50**, 3340 (1994).

[10] V. V. Dodonov, I. A. Malkin, and V. I. Man'ko, Physica **72**, 597 (1974).

[11] M. Hillery, Phys. Rev. A **36**, 3796 (1987).

[12] J. Janszky, A.V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990).

[13] V. Buzek, P. L. Knight, Opt. Commun. **81**, 331 (1991).

[14] W. Schleich, M. Pernigo, Fam Le Kien Phys. Rev. A, **44**, 2172 (1991).

[15] P. Adam, I. Foldesi, J. Janszky, Phys. Rev. A **49**, 1281 (1994).

[16] M. S. Kim, F. A. M. Oliviera, P. L. Knight, Phys. Rev. A **40**, 2494 (1989).

[17] I. Foldesi, P. Adam, J. Janszky, Phys. Lett. A, *in print* (1995).

[18] P. Domokos, J. Janszky, P. Adam, Phys. Rev. A **50**, 4293 (1994).

[19] J. Janszky, P. Domokos, S. Szabó, P. Adam, Phys. Rev. A **51**, 4191 (1995).