Generation of higher-order squeezing in multiphoton micromaser

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Abstract

The generation of steady state higher-order squeezing in the sense of Hong and Mandel [Phys. Rev. Lett. 54, 323(1985); Phys. Rev. A32, 974(1985)] and also of Hillery [Phys. Rev. A36, 3796(1987)] in a multiphoton micromaser is studied. The results show that the cotangent state which is generated by the coherent trapping scheme in a multiphoton micromaser can exhibit not only second-order squeezing but also fourth-order and squared field amplitude squeezings. The influence of the cavity loss on the squeezings is investigated.

1 Introduction

The one-atom micromaser which has been developed in recent years[1-3] is an unique device in experimentally studying the interaction of a single atom with the quantized electromagnetic field in a cavity. It has theoretically or experimentally been shown that the field with nonclassical properties such as sub-poissonian photon distribution[4-5] and second quadrature squeezing[6-8] can be generated in a one-photon micromaser. The key physical process in the micromaser is the interaction of a two-level atom with a single-mode quantized electromagnetic field inside the cavity. As usual, the Jaynes- Cummings model[9] is employed to describe the process and the corresponding Hamiltonian is written as

$$\hat{H} = \hbar\omega_0 \hat{S}_z + \hbar\omega \hat{a}^+ \hat{a} + \hbar g(\hat{a}\hat{S}_- + \hat{a}^+ \hat{S}_+) \tag{1}$$

where $\hat{S}_z = \frac{1}{2}(|e><e| - |g><g|)$, $\hat{S}_- = |e><g|$ and $\hat{S}_+ = |g><e|$. In the above, $|g>$ and $|e>$ denote the lower and upper states of the atom; $\pm\frac{1}{2}\hbar\omega_0$ are energies of the atomic levels; $\hat{a}^+$ and

$\hat{a}$ creation and annihilation operators of photons with frequency ω; $\hbar g$ is the atom-field coupling constant. It is seen in (1) that at a time only one photon is exchanged between the atom and the field while the transition of the atom from one level to another takes place. So, a micromaser which is built on the basis of (1) is called one-photon micromaser. The generalization of (1) is

$$\hat{H} = \hbar\omega_0\hat{S}_z + \hbar\omega\hat{a}^+\hat{a} + \hbar g(\hat{a}^m\hat{S}_- + \hat{a}^{+m}\hat{S}_+) \tag{2}$$

where m is the photon multiple and other symbols have the same meanings as in (1). The difference of (2) from (1) is that m photons are allowed to be emitted or absorbed in the transition of the atom. A number of theoretical analyses have shown that time-dependent squeezing effects can be generated in the interaction described by (2)[10-13]. A micromaser which is built on the basis of (2) is called multiphoton micromaser($m \geq 2$). The purpose of this paper is to investigate higher order squeezing properties of cotangent state produced by the coherent trapping approach[14] in a multiphoton micromaser.

2 Higher-order squeezing properties of cotangent state

Suppose that at time t the state vector of the atom-field system is

$$|\Psi(t)> = \sum_{n=N_d}^{N_u} S_n|n> \otimes(\alpha|e> + \beta|g>) \tag{3}$$

in which the field is in the superposition $\sum_{n=N_d}^{N_u} S_n|n>$ and the atom in a coherent state $\alpha|e> +\beta|g>$. While the atom is flying inside the cavity, the state vector of the atom-field coupling system is evolving in time according to the time-dependent Schrödinger equation with (2). When the atom exits out of the cavity at time $t' = t + \tau$ the atom-field coupling system gets into the state

$$\begin{aligned} |\Psi(t+\tau)> &= \exp[-i(\omega_0\hat{S}_z + \omega\hat{a}^+\hat{a})\tau] \\ &\times \{\sum_{n=N_d}^{N_u} S_n[\alpha\cos(\sqrt{(n+m)!/n!}g\tau)|n> \\ &- i\beta\sin(\sqrt{n!/(n-m)!}g\tau)|n-m>]|e> \\ &+ \sum_{n=N_d}^{N_u} S_n[\beta\cos(\sqrt{n!/(n-m)!}g\tau)|n> \\ &- i\alpha\sin(\sqrt{(n+m)!/n!}g\tau)|n+m>]|g>\}. \end{aligned} \tag{4}$$

If requiring that except the exponential factor $\exp[-i(\omega_0\hat{S}_z+\omega\hat{a}^+\hat{a})\tau]$ induced by $\hbar\omega_0\hat{S}_z+\hbar\omega\hat{a}^+\hat{a}$ during the period τ the whole system completely returns to the initial state (3) after the atom left out of the cavity, i.e., $\exp[i(\omega_0\hat{S}_z+\omega\hat{a}^+\hat{a})\tau]|\Psi(t+\tau)>=|\Psi(t)>$, and making the interaction time τ fulfill the conditions

$$\sqrt{N_d!/(N_d-m)!}g\tau=q\pi, \quad q=0,2,4,\cdots, \tag{5}$$

$$\sqrt{(N_u+m)!/N_u!}g\tau=p\pi, \quad p=1,3,5,\cdots, \tag{6}$$

we can write (4) as

$$|\Psi(t+\tau)>=\exp[-i(\omega_0\hat{S}_z+\omega\hat{a}^+\hat{a})\tau]\sum_{n=N_d}^{N_u}S_n|n>\otimes(\beta|g>-\alpha|e>) \tag{7}$$

where S_n are determined by the recurrence relation

$$S_n=-i\frac{\alpha}{\beta}\cot(\frac{1}{2}\sqrt{n!/(n-m)!}g\tau)S_{n-m} \tag{8}$$

with $n=N_d+m, N_d+2m,\cdots,N_u$. In (7), except the phase factor $\exp(-i\omega\tau\hat{a}^+\hat{a})$, the field returns to the initial state $\sum_{n=N_d}^{N_u}S_n|n>$ and the magnitudes of the atomic level occupation probability amplitudes for the lower and upper states are same as in the initial state $\alpha|e>+\beta|g>$ but the relative phase of α to β changes π. Therefore, We can conclude that if the field is pumped into the state $\sum_{n=N_d}^{N_u}S_n|n>$ from an initial state it will no longer be affected by the succeeding atoms which are initially in the coherent state $\alpha|e>+\beta|g>$. In this sense, we say that the state $\sum_{n=N_d}^{N_u}S_n|n>$ is steady. Since the relation (8) is the cotangent function the corresponding state of the field is named the cotangent state[14].

To investigate squeezing properties of the cotangent state, we introduce the two slowly varying quadrature components of the field amplitude

$$\hat{d}_1=\frac{1}{2}(\hat{a}e^{i\omega t}+\hat{a}^+e^{-i\omega t}); \quad \hat{d}_2=\frac{1}{2i}(\hat{a}e^{i\omega t}-\hat{a}^+e^{-i\omega t}). \tag{9}$$

In an arbitrary state of the field, the Nth-order moment of fluctuation of the field in $\hat{d}_i(i=1,2)$ is

$$\begin{aligned}<(\Delta\hat{d}_i)^N>=<:(\Delta\hat{d}_i)^N:>+\frac{N!}{(N-2)!}\frac{1}{8}<:(\Delta\hat{d}_i)^{N-2}:>\\+\frac{N!}{2!(N-4)!}\frac{1}{8^2}<:(\Delta\hat{d}_i)^{N-4}:>+\cdots+\frac{(N-1)!!}{2^N}\end{aligned} \tag{10}$$

where $\Delta\hat{d}_i = \hat{d}_i - < \hat{d}_i >$ and $<: (\Delta\hat{d}_i)^n :>$ $(n = N, N-2, \cdots, 2)$ is the nth-order moment of the $\hat{d}_i$'s mean squared fluctuation in the normal order. For a coherent state of the field, one has $< (\Delta\hat{d}_i)^N > = (N-1)!!/2^N$. When the Nth-order moment of the mean squared fluctuation of $\hat{d}_i$ in a state is smaller than in a coherent state, that is, $< (\Delta\hat{d}_i)^N > < (N-1)!!/2^N$, we say that the state is a Nth-order squeezed state in the $\hat{d}_i$'s component. Hong and Mandel[15] have shown that the Nth-order squeezed state with an even N is nonclassical.

We may also define the two quadrature operators of the square of the field amplitude

$$\hat{Y}_1 = \frac{1}{2}(\hat{a}^2 e^{2i\omega t} + \hat{a}^{+2} e^{-2i\omega t}); \quad \hat{Y}_2 = \frac{1}{2i}(\hat{a}^2 e^{2i\omega t} - \hat{a}^{+2} e^{-2i\omega t}). \tag{11}$$

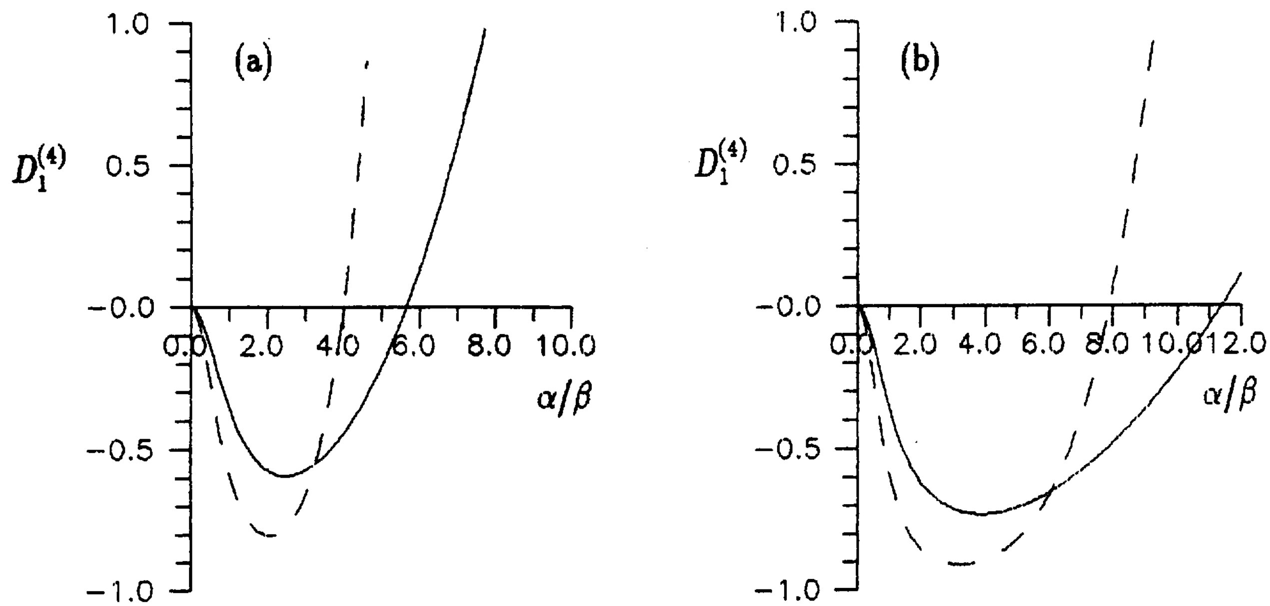
It is easily shown that $\hat{Y}_1$ and $\hat{Y}_2$ fulfill the commutator $[\hat{Y}_1, \hat{Y}_2] = i(2\hat{N}+1)$, where $\hat{N} = \hat{a}^+\hat{a}$. The uncertainty relation for their variances is $< (\Delta\hat{Y}_1)^2 >< (\Delta\hat{Y}_2)^2 > \geq < \hat{N} + \frac{1}{2} >^2$. For a coherent state of the field, the equality holds and we have $< (\Delta\hat{Y}_1)^2 > = < (\Delta\hat{Y}_2)^2 > = < \hat{N} + \frac{1}{2} >$. If either $< (\Delta\hat{Y}_1)^2 >$ or $< (\Delta\hat{Y}_2)^2 >$ is less than $< \hat{N} + \frac{1}{2} >$ in a state of the field, the state is called a squared amplitude squeezed state. Hillery[16] has shown that this squeezed state is also nonclassical.

For convenience, we will in the following discussions employ the two quantities

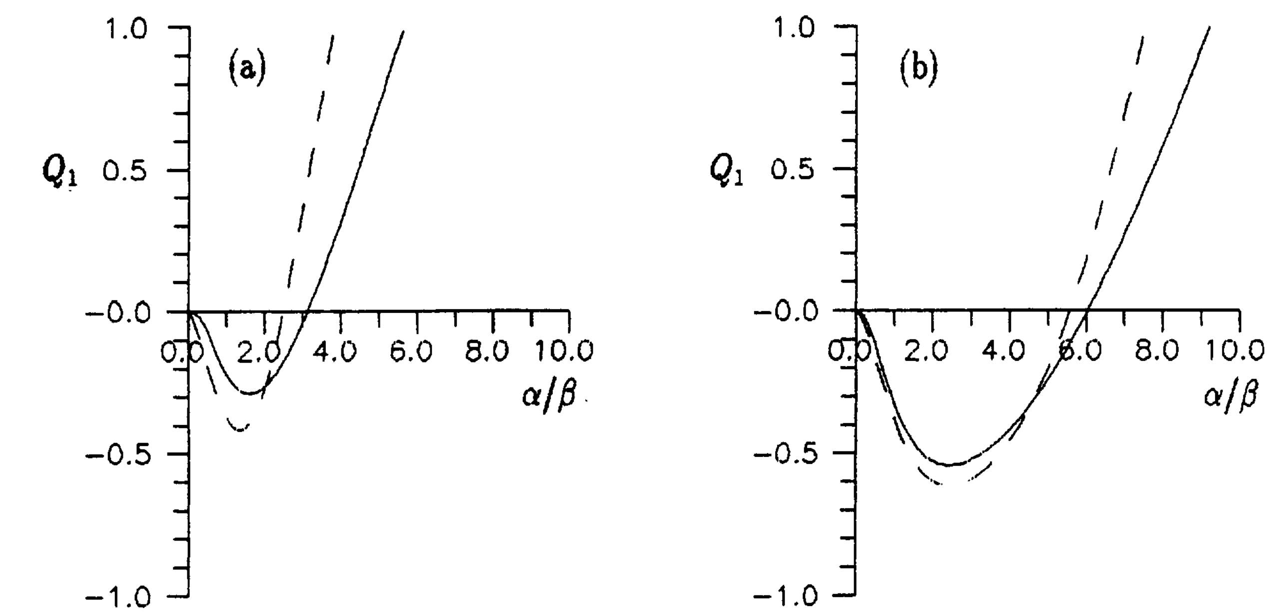
$$D_i^{(N)} = \frac{< (\Delta\hat{d}_i)^N >}{2^{-N}(N-1)!!} - 1, \quad ; \quad Q_i = \frac{< (\Delta\hat{Y}_i)^2 >}{< \hat{N} + \frac{1}{2} >} - 1, \quad (i = 1, 2) \tag{12}$$

to measure the squeezing degree. When $D_i^{(N)} < 0$ or $Q_i < 0$ the squeezings appear according to the above definition for squeezing.

Squeezing properties of the cotangent state with various photon multiples m have numerically been investigated. In our calculations the relative phase of the upper level probability amplitude α to the lower level one β is chosen $\pi/2$. We have found that the pronounced second- and fourth-order squeezings appear only for $m = 1$. In Figs. 1, $D_1^{(2)}$ and $D_1^{(4)}$ of the cotangent state with $m = 1$ are depicted against the ratio of α to β. It is observed in these figures that the strongest squeezing effect can be reached for a given N_u by a proper choice of α/β. For example, $D_1^{(4)} = -0.91$ can be acquired for $N_u = 40$ with $\alpha/\beta = 3.2$. This corresponds to the initial state of the atoms in which the occupation probabilities for the upper and lower levels are 0.91 and 0.09, respectively. We also notice that the fourth-order squeezing appears only in the regions of the second one. It means that the present fourth-order squeezing is not intrinsic[10] and is induced from the second one. In Figs. 2, Q_1 of the cotangent state with $m = 1$ and $m = 2$ versus α/β is shown. It is observed in these figures that for a given N_u there exists the value of α/β for the optimal squeezing. For example, Q_1 can reach -0.54 and -0.62 for $m = 1$ and $m = 2$, respectively, when $\alpha/\beta = 2.5$ and



Figs. 1: $D_1^{(2)}$(solid line) and $D_1^{(4)}$ (dashed line) versus α/β with $N_d = 0$ and $m = 1$.
(a)$N_u = 10$;(b)$N_u = 40$.



Figs. 2: Q_1 versus α/β with $N_d = 0$, $m = 1$(solid line) and $m = 2$(dashed line).
(a)$N_u = 10$;(b)$N_u = 40$.

$N_u = 40$. It means that these squeezing degrees can be acquired when the atoms are initially in the coherent state with the level occupation probabilities $\alpha^2 = 0.86$ and $\beta^2 = 0.14$. We also notice that the optimal squeezing in the two photon case is always stronger than in the one photon case for a given N_u. In our calculations we find that the squared amplitude squeezing disappears when $m > 2$. It means that this squeezing can be realized only in one-and two-photon micromasers by the coherent trapping approach.

3 Dynamic process of generation of steady state squeezing

We now turn our attention to dynamically generating the squeezing effects discussed above. Suppose that each of the atoms entering the cavity is initially in the coherent state $\alpha|e> +\beta|g>$, and the flight time of the atoms inside the cavity is τ. If at time t_i the density matrix of the field is $\hat{\rho}^{(f)}(t_i)$, at time $t_i + \tau$ the atom will leave out of the cavity and density matrix elements of the field can be written as

$$\begin{aligned}
&\hat{\rho}^{(f)}_{n'n}(t_i+\tau) = \exp[i(n-n')\omega\tau] \\
&\times\{[|\alpha|^2\cos(\sqrt{(n'+m)!/n'!}g\tau)\cos(\sqrt{(n+m)!/n!}g\tau) \\
&+|\beta|^2\cos(\sqrt{n'!/(n'-m)!}g\tau)\cos(\sqrt{n!/(n-m)!}g\tau)]\hat{\rho}^{(f)}_{n'n}(t_i) \\
&+|\beta|^2\sin(\sqrt{(n'+m)!/n'!}g\tau)\sin(\sqrt{(n+m)!/n!}g\tau)\hat{\rho}^{(f)}_{n'+mn+m}(t_i) \\
&+|\alpha|^2\sin(\sqrt{n'!/(n'-m)!}g\tau)\sin(\sqrt{n!/(n-m)!}g\tau)\hat{\rho}^{(f)}_{n'-mn-m}(t_i) \\
&+i\alpha\beta^*\cos(\sqrt{(n'+m)!/n'!}g\tau)\sin(\sqrt{(n+m)!/n!}g\tau)\hat{\rho}^{(f)}_{n'n+m}(t_i) \\
&-i\alpha^*\beta\sin(\sqrt{(n'+m)!/n'!}g\tau)\cos(\sqrt{(n+m)!/n!}g\tau)\hat{\rho}^{(f)}_{n'+mn}(t_i) \\
&+i\alpha^*\beta\cos(\sqrt{n'!/(n'-m)!}g\tau)\sin(\sqrt{n!/(n-m)!}g\tau)\hat{\rho}^{(f)}_{n'n-m}(t_i) \\
&-i\alpha\beta^*\sin(\sqrt{n'!/(n'-m)!}g\tau)\cos(\sqrt{n!/(n-m)!}g\tau)\hat{\rho}^{(f)}_{n'-mn}(t_i)\}. \qquad (13)
\end{aligned}$$

If the next atom enters the cavity at time t_{i+1}, there will be no atom inside the cavity within the time interval $t_i + \tau \le t \le t_{i+1}$. We suppose that during that interval the field relaxes at the rate γ to the thermal reservoir with the mean photon number n_b. This process is described by the master equation[5]

$$\begin{aligned}
&\frac{d\hat{\rho}^{(f)}(t)}{dt} = \frac{\gamma}{2}(n_b+1)(2\hat{a}\hat{\rho}^{(f)}(t)\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho}^{(f)}(t) - \hat{\rho}^{(f)}(t)\hat{a}^+\hat{a}) \\
&+\frac{\gamma}{2}n_b(2\hat{a}^+\hat{\rho}^{(f)}(t)\hat{a} - \hat{a}\hat{a}^+\hat{\rho}^{(f)}(t) - \hat{\rho}^{(f)}(t)\hat{a}\hat{a}^+), \quad t_i+\tau \le t \le t_{i+1}. \qquad (14)
\end{aligned}$$

On the basis of (13)-(14), we can study the dynamic evolution of the field while the atoms one by one pass through the cavity.

In the present calculations, we choose that the field is initially in the vacuum, and make the relative phase of α to β being $\pi/2$ and τ satisfying the conditions (5)-(6) with $q = 0$ and $p = 1$. Meanwhile, we take $\gamma = 5s^{-1}$, $g = 10kHz$ which are consistent with the parameters used in the current micromaser[4]. If the injection of the atoms is regular, i.e., the time distances between the adjacent atoms are same. In this case, the relaxation time of the field to the reservior is equal to $1/R - \tau$ where R is the atomic flux. In Figs.3-6, for the single photon case, the evolution of $D_1^{(4)}$ and Q_1 against the number of the atoms which have left out of the cavity is shown with various values of the atomic flux R. In these figures, the dashed line represents the result with $\gamma = 0$. According to the conditions for the present calcualtions, the steady state with $\gamma = 0$ must be the cotangent state. It is observed that when R is small the field has not the second- and fourth-order squeezing properties since the steady state resuts from the balance between the gain and the loss. As R increases, the gain brought by the atoms will overpass the cavity loss the steady state will arise from the coherent trapping because of the condition (6). Then the steady state exhibits the squeezing behaviour as shown in the figures. We also notice that when R is adequate large $D_1^{(4)}$ and Q_1 of the steady state are very close to the values of the cotangent state with the same parameters.

In Figs.7 and 8, for the two-photon case, the evolution of Q_1 against the number of the atoms is depicted. It is observed that the evolution behaviour is similar to shown in Figs. 5 and 6, and the squeezing becomes deeper than in the one photon case with the same value of R.

4 Conclusion

We have shown that the cotangent state produced by the coherent trapping scheme in a one-photon micromaser can exhibit steady state fourth-order as well as squared amplitude squeezings. The last squeezing can also appear in the cotangent state produced in a degenerate two-photon micromaser. The cotangent state of the field with these squeezing effects can be reached from the cavity vacuum by the atomic coherent pumping. The influence of the cavity loss on the squeezing effects has been investigated. The results show that when the flux of the atoms entering the cavity is moderately large the squeezings are not essentially affected by the cavity loss.

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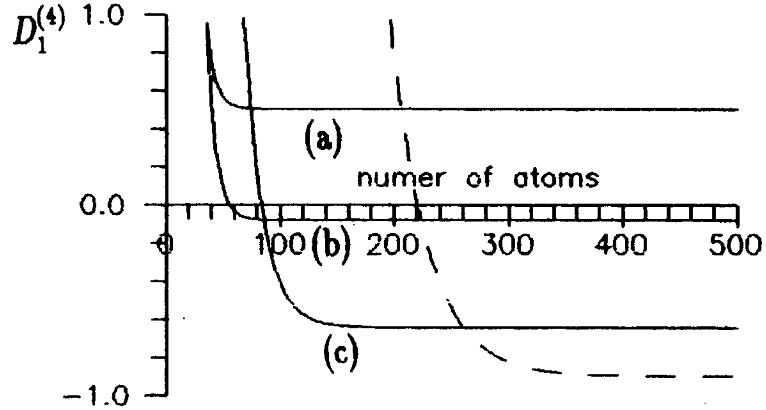

Fig.3: $D_1^{(4)}$ versus the number of atoms for one photon case with $N_u = 30$ and $\alpha/\beta = 2.9$. The values of the atomic flux for the lines are (a) $22s^{-1}$, (b) $30s^{-1}$ and (c) $50s^{-1}$.

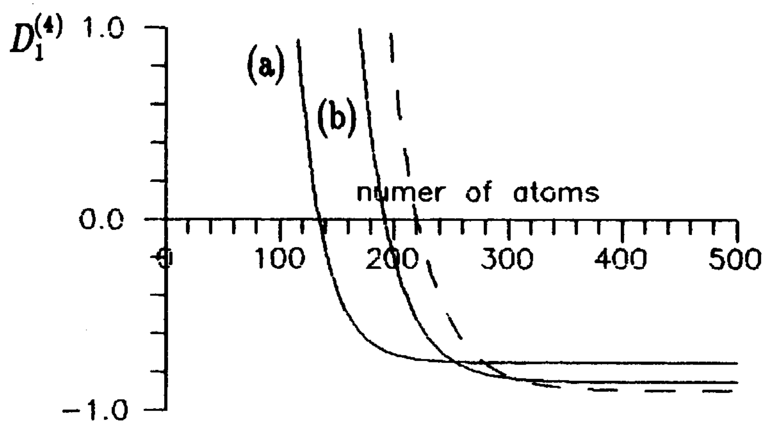

Fig.4: Same as Fig.3 but (a) $100s^{-1}$ and (b) $500s^{-1}$.

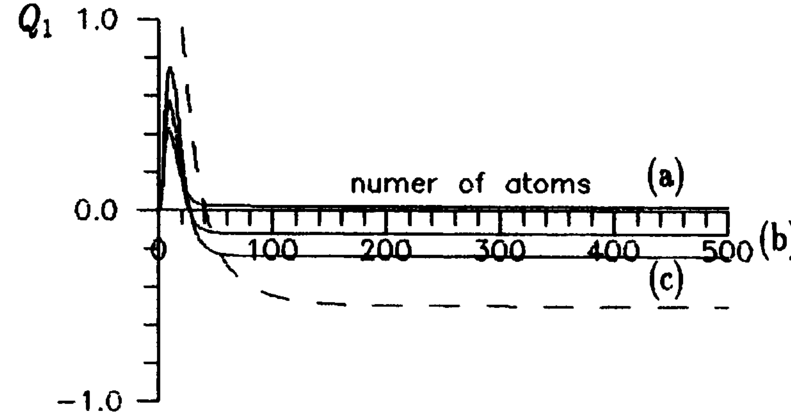

Fig.5: Q_1 versus the number of atoms for one photon case with $N_u = 30$ and $\alpha/\beta = 2.3$. (a)$20s^{-1}$;(b)$30s^{-1}$; (c)$50s^{-1}$.

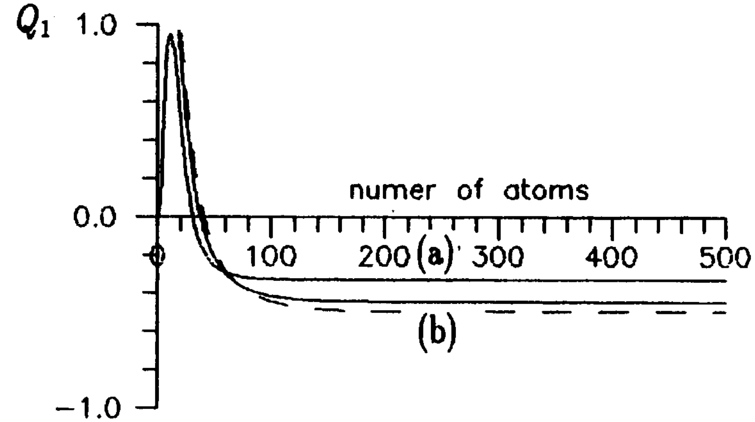

Fig.6: Same as Fig.5 but (a) $100s^{-1}$ and (b) $500s^{-1}$.

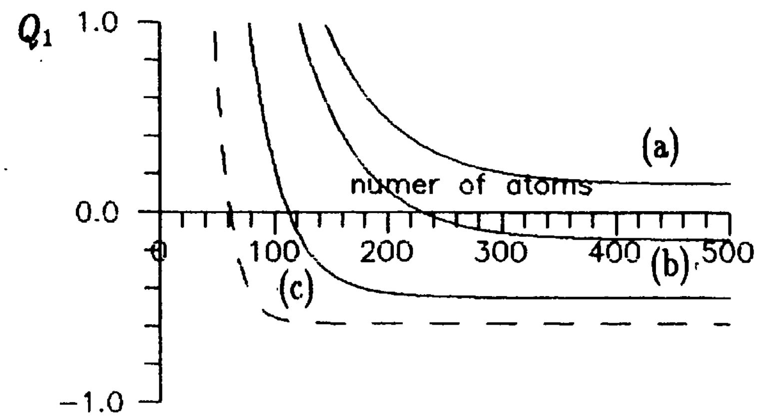

Fig.7: Q_1 versus the number of atoms for tow-photon case with $N_a = 30$ and $\alpha/\beta = 2.3$. (a)$33s^{-1}$;(b)$35s^{-1}$; (c)$50s^{-1}$.

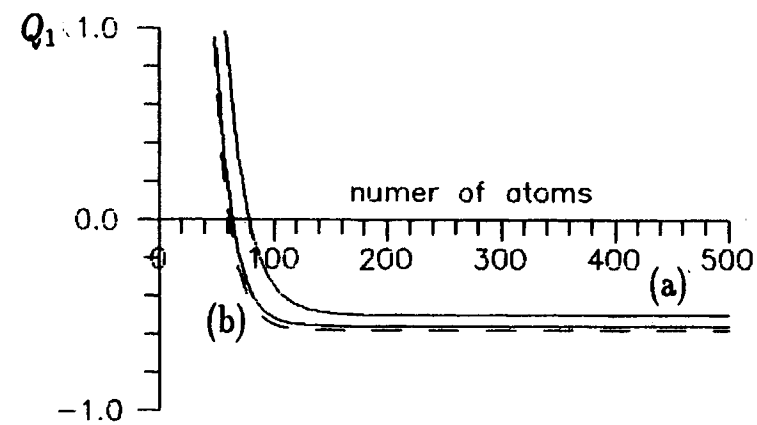

Fig.8: Same as Fig.7 but (a)$100s^{-1}$ and (b) $500s^{-1}$.

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