Q(α) Function and Squeezing Effect

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Abstract

The relation of squeezing and Q (α) function is discussed in this paper. By means of Q function, the squeezing of field with gaussian Q(α) function or negative P(α)function is also discussed in detail.

1 Introduction

In quantum optics, P(α), Q(α)and W(α)are coammon quasiprobability distribution function [1], but only Q(α) perserve good function (positive and nonregular). Recently, by means of Fokker—Plank equation for Q function, M. S. Kim et. al discussed the fouth—order squeezing[2], In this paper, we consider the relation between Q function and squeezing, and study the squeezing of field with gaussian Q function or negative P(α)function.

for any field density operator ρ, the Q function is definded as

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle \tag{1}$$

it satisfies the normalization condition

$$\int d\alpha^2 Q(\alpha) = 1 \tag{2}$$

For antinormally ordered operator $f(a, a^+) = f^{(a)}(a, a^+)$, one can get following equation

$$\langle f(a, a^+)\rangle = \int d^2\alpha Q(\alpha) f^{(a)}(\alpha, \alpha^*) = 1 \tag{3}$$

where α and $\alpha^+$are annihilation and creation operators respeotively Ddfining parameter

$$S = \langle :(a + a^+)^2:\rangle - \langle a + a^+\rangle^2 \tag{4}$$

For squeezing, S should be negative

Now, we suppose that Q function can be expanded as following form

$$Q(a) = \frac{1}{\pi}e^{-\beta|\alpha|^2}\sum C_{m,n}\alpha^m\alpha^{*n}, (C_{m,n} = C^*_{n,m}) \tag{5}$$

Using mathmatical identity [3]

$$\int \frac{d^2\alpha}{\pi} e^{-\beta|\alpha|^2+\rho\alpha+\tau\alpha^*} = \frac{1}{\beta} e^{\rho\tau/\beta}, (\beta > 0) \tag{6}$$

one can have

$$\int \frac{d^2\alpha}{\pi} \alpha^m \alpha^{*n} e^{-\beta|\alpha|^2} = \frac{n!\delta_{mn}}{\beta^{m+1}} \tag{7}$$

and the normalization condition is

$$\sum_m C_{m,m} m!/\beta^{m+1} = 1 \tag{8}$$

By means of equations(3)and (7),we have

$$\langle a + a^+ \rangle = \sum_m \frac{2(m+1)!ReC_{m,m+1}}{\beta^{m+2}} \tag{9}$$

$$\langle a^2 + a^{+2} \rangle = \sum_m \frac{2(m+2)!ReC_{m,m+2}}{\beta^{m+3}} \tag{10}$$

$$\langle a^+ a \rangle = \sum_m \frac{(m+1)-\beta}{\beta^{m+2}} m! C_{m,m} \tag{11}$$

and

$$S = \sum_m \frac{2(m+2)!ReC_{m,m+2}}{\beta^{m+3}} + 2\sum_m \frac{m+1-\beta}{\beta^{m+2}} m! C_{m,m} - [\sum_m \frac{2(m+1)!ReC_{m,m+1}}{\beta^{m+2}}]^2 \tag{12}$$

If the field exists squeezing,then

$$\sum_m [\frac{(m+2)!ReC_{m,m+2}}{\beta^{m+3}} + \frac{(m+1-\beta)m!C_{m,m}}{\beta^{m+2}}] < [\sum_m \frac{2(m+1)!ReC_{m,m+1}}{\beta^{m+2}}]^2 \tag{13}$$

2 Squddzing of field with gaussian Q function

We introduce the gaussian Q function as

$$Q(\alpha) = \sqrt{t^2 - 4|A|^2} exp[-t(\alpha^* - \omega^*)(\alpha - \omega) + A^*(\alpha^* - \omega^*)^2 + A(\alpha - \omega)^2] \tag{14}$$

where $t > 2|A|$. Using integration formula[3]

$$\int \frac{d^2z}{\pi} e^{-\mu|z|^2 + fz^2 + gz^{*2} + \tau z + \sigma z^*} = \frac{1}{\sqrt{\mu^2 - 4fg}} e^{\frac{\mu\tau\sigma + \tau^2 g + \sigma^2 f}{\mu^2 - 4fg}} \tag{15}$$

and equation (3), one can show

$$\langle a + a^+ \rangle = \omega + \omega^* \tag{16}$$

$$\langle a^2 + a^{+2} \rangle = \omega^{*2} + \omega^2 + \frac{2(A + A^*)}{t^2 - 4|A|^2} \tag{17}$$

$$\langle a^+ a \rangle = |\omega|^2 + \frac{t}{t^2 - 4|A|^2} - 1 \tag{18}$$

and easyly obtain

$$S = \frac{2(A + A^* + 4|A|^2 + t - t^2)}{t^2 - 4|A|^2} \tag{19}$$

Thus the condition for the existence of squeezing is

$$A + A^* + 4|A|^2 < t^2 - t \tag{20}$$

If $A=0$, squeezing means $t>1$, if $t<1$ and $A=0$, no squeezing exists in the field. It is worth to point out that the field with $A=0$ and $t>1$ has not been found uptill now.

3 Squeezing of field with negative P(α) function

The relation of $P(\alpha)$ and $Q(\alpha)$ is

$$Q(\alpha) = \int \frac{d^2\beta}{\pi} e^{-|\beta - \alpha|^2} P(\beta) \tag{21}$$

for nonclassical field, its $P(\alpha)$ function has two situations [4]: i) $P(\alpha)$ is negative, ii) $P(\alpha)$ is more singular than $\delta-$ function. We consider the nonclassical field with negative $P(\alpha)$ function[5]

$$\rho = \int d^2\alpha P(\alpha) |\alpha\rangle\langle\alpha| \tag{22}$$

Suppose $P(\alpha)$ as

$$P(\alpha) = \frac{1}{\pi} e^{-t|\alpha|^2} \sum_{i,j} P_{i,j} \alpha^i \alpha^{*j} \tag{23}$$

Using equations (6) and (21), we obtain

$$Q(\alpha) = \frac{1}{\pi} \sum_{i,j} P_{i,j} e^{\frac{-t}{1+t}|\alpha|^2} \sum_{l=o}^{min(i,j)} \frac{i!j!\alpha^{i-l}\alpha^{*j-l}}{l!(i-l)!(j-l)!(l+t)^{i+j-l+1}} \tag{24}$$

comparing with equation(5),one can have

$$\beta = \frac{t}{1+t} \tag{25}$$

$$C_{m\ n} = \sum_{l} P_{m+l,n+l} \frac{(m+l)!(n+l)!}{l!m!n!(1+t)^{m+n+l+1}} \tag{26}$$

Obviously,the field with negative P function can exihibites squeezing for some situation,but,if $P(\alpha)$ is only the function of $|\alpha|$,i.e, $P(\alpha)$ is sphere symmetry in phase space,then

$$P_{i,j} = 0 \qquad (i \neq j) \tag{27}$$

$$C_{m,n} = 0 \quad (m \neq n) \tag{28}$$

Form equation (12),one can get

$$S > 0 \tag{29}$$

In conclusion ,it is clearly that no suqueezing exists in the field with negative P(α) function which is sphere symmetry in phase space.

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