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A PROPOSAL FOR TESTING LOCAL REALISM WITHOUT USING ASSUMPTIONS RELATED TO HIDDEN VARIABLE STATES

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Abstract

A feasible experiment is discussed which allows us to prove a Bell's theorem for two particles without using an inequality. The experiment could be used to test local realism against quantum mechanics without the introduction of additional assumptions related to hidden variables states. Only assumptions based on direct experimental observation are needed.

The experiment I wish to discuss is represented in Fig.1. It is a variant of Franson's two-photon correlation experiment [1]. However, variants of other experiments could also be considered [2-4]. A source (S) emits pairs of photons (γ_1 and γ_2). The photons are emitted simultaneously [5], but there is uncertainty about the time of emission. H_1 and H_2 are 50%:50% beam splitters. As in an experiment recently discussed [6], H_1', H_2', H_3, and H_4 are not 50%:50% beam splitters, and have real amplitude transmissivities T_1, T_2, T_3, and T_4, and real amplitude reflectivities R_1, R_2, R_3, and R_4. M_1, M_1', M_2, M_2' and M_2'' are mirrors, and ϕ_1, ϕ_2, and ϕ_3 are phase shifters. L_2-S_2=L_1-S_1=$c\Delta T$ is much greater than the coherence lengths of the packets associated with γ_1 and γ_2. This implies that $\Delta\omega_1\Delta T \gg 1$ and $\Delta\omega_2\Delta T \gg 1$, where $\Delta\omega_1$ and $\Delta\omega_2$ are the uncertainties in the angular frequencies of γ_1 and γ_2. However, $\Delta(\omega_1+\omega_2)\Delta T \ll 1$. As is well known [1], in this case the situation in which both photons follow the long paths is indistinguishable from the situation in which both photons follow the short paths. In the present proposal a balanced Mach-Zehnder interferometer for photons γ_2, constituted by H_3, H_2', M_2'', and H_4 has been introduced.

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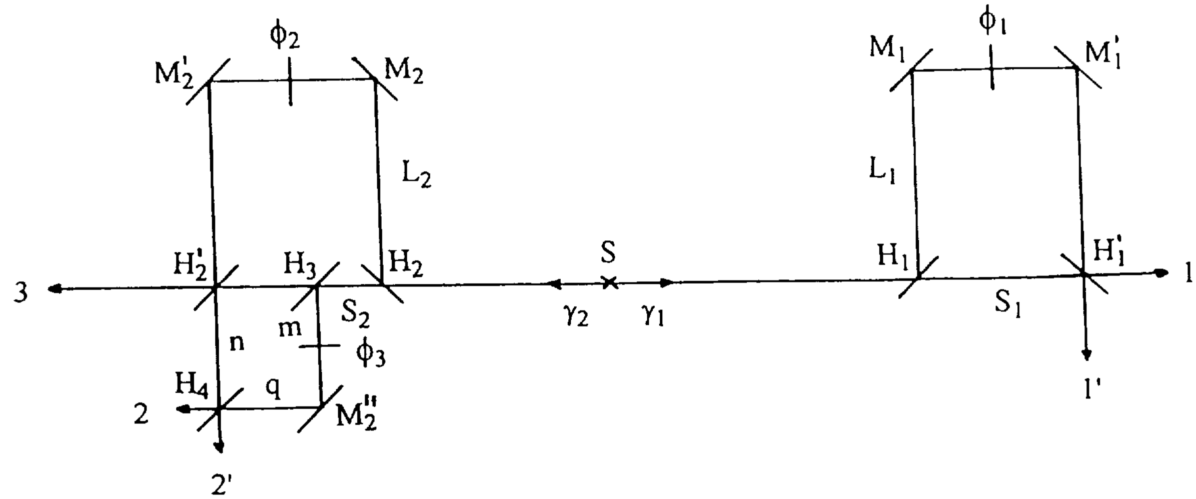

FIG.1. Experiment proposed.

I will consider four different situations: (**A**) H_1' and H_4 are removed; (**B**) H_1' is in place and H_4 is removed; (**C**) H_1' is removed and H_4 is in place; (**D**) H_1' and H_4 are in place. The detections relevant to our discussion are only the coincident detections occuring at sites 1 and 2, 1 and 2', 1' and 2, and 1' and 2'. Naturally, the probability of coincident detections occuring at sites 1' and 2 in situation **A**, $P_A^c(1',2)=0$, since in situation **A** $\gamma_1(\gamma_2)$ has to follow the long(short) path to be detected at 1'(2).

The probability amplitude of coincident detections occuring at sites 1 and 2' in situation **B** is [6]

$$A_B^c(1,2') = \sum_{\omega_1\omega_2} c_{\omega_1\omega_2}\,(\alpha_1\alpha_2+\beta_1\beta_2)\ , \tag{1}$$

where $c_{\omega_1\omega_2}$ is the probability amplitude of having a photon γ_1 with a frequency ω_1 and a photon γ_2 with frequency ω_2, $\alpha_1 = 2^{-1/2}\exp(i\omega_1 t_S)T_1$ is the probability amplitude of having a photon $\gamma_1(\omega_1)$ following the short path, where t_S is the time spent by light to follow the short path, $\alpha_2=2^{-1/2}T_3 iR_2\exp[i\omega_2(t_S+t')]$ is the probability amplitude of having a photon $\gamma_2(\omega_2)$ following the short path, where t' is the time spent by light from H_2' to H_4, $\beta_1=i2^{-1/2}\exp(i\phi_1)\exp(i\omega_1 t_L)iR_1$ is the probability amplitude of having a photon $\gamma_1(\omega_1)$ following the long path, where t_L is the time spent by light to follow the long path, and $\beta_2=i2^{-1/2}\exp(i\phi_2)T_2\exp[i\omega_2(t_L+t')]$ is the probability amplitude of having a photon $\gamma_2(\omega_2)$ following the long path. Using (1) and the condition $\Delta(\omega_1+\omega_2)\Delta T \ll 1$ we obtain

$$A_B^c(1,2')=(i/2)\sum_{\omega_1\omega_2} A_{\omega_1\omega_2}(T_1T_3R_2\text{-}BR_1T_2)\ , \tag{2}$$

where $A_{\omega_1\omega_2}\equiv c_{\omega_1\omega_2}\exp[i\omega_1 t_S+i\omega_2(t_S+t')]$ and $B\equiv\exp[i(\phi_1+\phi_2)+i(\omega_{10}+\omega_{20})\Delta T]$, where ω_{10} and ω_{20} are the central frequencies of γ_1 and γ_2. Choosing $T_1T_3R_2=R_1T_2$ and using the condition

$$\sum_{\omega_1\omega_2} |c_{\omega_1\omega_2}|^2=1\ , \tag{3}$$

we obtain

$$P_B^c(1,2')=(1/2)(T_1T_3R_2)^2(1\text{-ReB})\ . \tag{4}$$

In an ideal situation we can have $[P_B^c(1,2')]_{min}=0$ (ReB=1) and $[P_B^c(1,2')]_{max}=(T_1T_3R_2)^2$ (ReB=-1). This follows from quantum mechanical nonlocality. But in a real situation this is not so. Let us then assume that ReB=1-ε (ReB=-1+ε) in the minimum (maximum) case. Then we can introduce the visibility V_B given by

$$V_B=\frac{[P_B^c(1,2')]_{max}\text{-}[P_B^c(1,2')]_{min}}{[P_B^c(1,2')]_{max}+[P_B^c(1,2')]_{min}}=1\text{-}\varepsilon\ . \tag{5}$$

Thus,

$$[P_B^c(1,2')]_{min}=(1/2)(T_1T_3R_2)^2(1\text{-}V_B)\ . \tag{6}$$

Using a similar reasoning, we obtain

$$A_C^c(1,2')=\sum_{\omega_1\omega_2} c_{\omega_1\omega_2}\delta(\rho_1+\rho_2)\ . \tag{7}$$

where $\delta=2^{-1/2}\exp(i\omega_1 t_S)$, $\rho_1=2^{-1/2}T_3 iR_2\exp[i\omega_2(t_S+t')]T_4$, and $\rho_2=2^{-1/2}iR_3\exp(i\phi_3)$ $\exp[i\omega_2(t_S+t')]iR_4$, which leads to

$$A_C^c=(1/2)\sum_{\omega_1\omega_2} A_{\omega_1\omega_2}(iT_3R_2T_4\text{-}CR_3R_4)\ , \tag{8}$$

where $C\equiv\exp(i\phi_3)$. Thus, choosing $T_3R_2T_4=R_3R_4$ and using (3) we obtain

$$P_C^c(1,2')=(1/2)(T_3R_2T_4)^2(1\text{-ImC})\ . \tag{9}$$

As in (5) and (6), we can introduce the visibility V_C to obtain

$$[P_C^c(1,2')]_{min}=(1/2)(T_3R_2T_4)(1-V_C) \ . \tag{10}$$

In an ideal situation we can have $V_C=1$ and $[P_C^c(1,2')]_{min}=0$. This follows from the wave like properties of light.

It is also easy to see that

$$A_D^c(1,2')=\sum_{\omega_1\omega_2} c_{\omega_1\omega_2}[\lambda_1(\sigma_1+\sigma_2)+\lambda_2\sigma_3] \ , \tag{11}$$

where $\lambda_1=2^{-1/2}\exp(i\omega_1 t_S)T_1$, $\sigma_1=2^{-1/2}T_3 iR_2\exp[i\omega_2(t_S+t')]T_4$, $\sigma_2=2^{-1/2}iR_3\exp(i\phi_3)$ $\exp[i\omega_2(t_S+t')]iR_4$, $\lambda_2=i2^{-1/2}\exp(i\phi_1)\exp(i\omega_1 t_L)iR_1$, and $\sigma_3=i2^{-1/2}\exp(i\phi_2)T_2$ $\exp[i\omega_2(t_L+t')]T_4$, which leads to

$$A_D^c=(i/2)T_1T_3R_2T_4\sum_{\omega_1\omega_2} A_{\omega_1\omega_2}(1-B+iC) \ . \tag{12}$$

Then, choosing ϕ_1 and ϕ_2 such that $P_B^c(1,2')=[P_B^c(1,2')]_{min}$, and ϕ_3 such that $P_C^c(1,2')=[P_C^c(1,2')]_{min}$, we obtain

$$P_D^c(1,2')=(1/2)(T_1T_3R_2T_4)^2[(3/2)-V_B-V_C+V_BV_C-(1-V_B)^{1/2}(1-V_C)^{1/2}] \ . \tag{13}$$

To prove a Bell's theorem for two particles without using an inequality we can consider the ideal situation: $V_B=V_C=1$. I will assign the value $i(1)$ for detections that occur at sites 1 and 2' (1' and 2). Thus, assuming there can be hidden variables states (**HVS**) of the photon pair which mimic quantum mechanics, we can only have: (**A**) $a_R^c(\lambda)b_R^c(\lambda)=i,-1$; (**B**) $a_P^c(\lambda)b_R^c(\lambda)=i,1$, from (6); and (**C**) $a_R^c(\lambda)b_P^c(\lambda)=i,1$, from (10). $a_P^c(\lambda)$ ($b_R^c(\lambda)$) represents the result of a measurement performed at 1,1' (2,2') when H_1' (H_4) is in place (removed), and so on, the superscript c refers to coincident detections, and λ represents the **HVS** of the photon pair [7]. Assuming locality, that is, that $a_R^c(\lambda)$ is the same in **A** and **C**, for example, we see that $a_P^c(\lambda)=i\xrightarrow{B}b_R^c(\lambda)=1\xrightarrow{A}a_R^c(\lambda)=i\xrightarrow{C}b_P^c(\lambda)=1$. That is, $P_D^c(1,2')=0$ (local realism), in disagreement with $P_D^c(1,2')=(1/4)(T_1T_3R_2T_4)^2$ (quantum mechanics), from (13).

Introducing some assumptions which are based on direct experimental observation the above argument can be extended to the case of a real (i.e., non-ideal) experiment. Let us initially consider situation **C** and select only those events in which detection at 2' occurs. In this case, whenever a coincident detection at 1 occurs we know that γ_1 and γ_2 have followed the short paths. I will assume that: (A1) if H_1' had been in place (sit.**C**$\longrightarrow$sit.**D**) the number of photons *following the short path* that would be coincidentally detected at 1 could not be greater than the number of photons coincidentally detected at 1 when H_1' is removed (I will return to this point). Therefore, the number of coincident detections at 1 and 2' in sit.**D** *which correspond to the possibility in which* γ_1 *and* γ_2 *follow the short paths* cannot be greater than $N_C^c(1,2')$, the number of coincident detections at 1 and 2' in sit.**C**.

Let us now consider situation **B** and select only those events in which detection at 1 occurs. In this case, only the coincident detections at 1 and 2' can correspond to the possibility in which γ_1 and γ_2 follow the long paths. According to (A1), if H_4 had been in place (sit.**B**$\longrightarrow$sit.**D**) the number of photons *following path n* that would be coincidentally detected at 2' could not be greater than the number of photons coincidentally detected at 2' when H_4 is removed. Therefore, the number of coincident detections at 1 and 2' in sit.**D** *which correspond to the possibility in which* γ_1 *and* γ_2 *follow the long paths* cannot be greater than $N_B^c(1,2')$, the number of coincident detections at 1 and 2' in sit.**B**. Hence, $N_D^c(1,2') \leq N_C^c(1,2') + N_B^c(1,2')$, or, in terms of probabilities,

$$P_D^c(1,2') \leq P_C^c(1,2') + P_B^c(1,2') \ , \tag{14}$$

since : (A2) coincident detections can only occur when photons of the emitted pair either (a) both follow the long paths, or (b) both follow the short paths.

Let us examine (A1) closer. It was assumed, when changing from situation **C**(**B**) to situation **D**, that the number of detections generated by photons $\gamma_1(\gamma_2)$ following path $S_1(n)$ could not be increased by placing a beam splitter $H_1'(H_4)$ in front of the detectors. Although this may appear to be a nonenhancement assumption [8], this can be directly verified. For example, by blocking path $L_1(q)$ in situation **D**. Now we are not assuming that for *every* **HVS** of a photon the probability of it being detected cannot be enhanced by placing a beam splitter in front of the detector. However, it might still be argued that when $H_1'(H_4)$ is in the position represented in Fig.1, in which case photons from two different directions can impinge on it, its properties are modified, in such a way that photons coming via path $S_1(n)$ become more "detectable" after impinging on $H_1'(H_4)$ and being transmitted, whilst photons coming via path $L_1(q)$ become less "detectable" after impinging on $H_1'(H_4)$ and being reflected [9]. However, this sounds as a much too contrived supposition.

To have a rough estimation of the expected disagreement between the local realistic and the quantum mechanical predictions in a real experiment, we can make $V_B=V_C=V$. Hence, using (6), (10), and (13), we see that in order to have a violation of (14) we must have

$$(T_1T_4)^2[(1/2)-2V+2V^2]/[(T_4^2+T_1^2)(1-V)]>1. \quad (15)$$

Then, making $T_1=T_4=T$, $R_1=R_4=R$, which leads to $T_3=R_2$, $T_2=R_3=[-R+(1+3T^2)^{1/2}]/2T$, we obtain

$$(T^2/4)(1-4V+4V^2)/(1-V)>1 \ . \quad (16)$$

We see that the minimum visibility we must have in order to violate (16) is given by $V>0.87$ $(T=1)$. Apparently our best choice would be $T=1$. However, this corresponds to the situation in which H_1' and H_4 have been removed. In this case the probabilities drop to zero, and we would have to wait an infinite time to get any result. $V=0.90$, $T=1/(1.2)^{1/2}$ $\longrightarrow$ l.h.s.(16) > 1.3. To have an idea of the time necessary to perform an experiment using these data we can calculate the ratio between the probability of having a coincident detection in a Franson's experiment in the case of perfect correlations and the probability given by (13) in the ideal case $(V=1)$. We easily see that we need about eleven times more time to have the same statistics as in a Franson's experiment.

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