The contradiction between the measurement theory of quantum mechanics and the theory that the velocity of any particle can not be larger than the velocity of light

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Abstract

By the measurement theory of quantum mechanics and the method of Fourier transform, we proved that the wave function $\psi(x,y,z,t)=\frac{8}{(2\pi\sqrt{2L})^3}\Phi(L,t,x)\Phi(L,t,y)\Phi(L,t,z)$. According to the theory that the velocity of any particle can not be larger than the velocity of light and the Born interpretation, when $|\delta|>(ct+L)$, $\Phi(L,t,\delta)=0$. But according to the calculation, we proved that for some δ, even if $|\delta|>(ct+L)$, $\Phi(L,t,\delta)\neq 0$.

By the measurement theory of quantum mechanics, if someone measures the coordinate of a particle, it will make the particle to the eigenstate of the coordinate. The eigen function (with eigen value zero) of the coordinate of a particle can be assumed as follows:

$$\alpha(x,y,z)=\frac{1}{(\sqrt{2L})^3} \quad \text{when } -L<x<L,\ -L<y<L,\ -L<z<L$$

$$\alpha(x,y,z)=0 \quad \text{otherwise} \tag{1}$$

L is an infinitesimally positive real number. Now assume someone measures the coordinate of a particle at the place $\vec{r}=0$ ($\vec{r}$ represents the coordinate), suppose this particle is measured at the time $t=0$, thus this particle is made to the eigenstate (with eigen value zero) of the coordinate, the wave function $\psi(\vec{r},t)$ of this particle will satisfy the following condition

$$\psi(x,y,z,0)=\alpha(x,y,z) \tag{2}$$

By the Fourier transform and the Schrödinger wave equation, it is not difficult to see

$$\psi(\vec{r},t)=\frac{1}{(2\pi)^{3/2}}\int\varphi(\vec{k})\exp[i(\vec{k}\cdot\vec{r}-\frac{\hbar k^2t}{2\mu})]d^3\vec{k} \tag{3}$$

Where $\varphi(\vec{k})=\frac{1}{(2\pi)^{3/2}}\int\psi(\vec{r},0)e^{-i\vec{k}\cdot\vec{r}}d^3\vec{r}$

$$=\frac{1}{(2\pi)^{3/2}}\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L}\frac{1}{(\sqrt{2L})^3}\exp[-ik_xx-ik_yy-ik_zz]dx\,dy\,dz$$

$$=\frac{1}{(2\pi)^{3/2}}\frac{1}{(\sqrt{2L})^3}\frac{2}{k_x}\sin(k_xL)\frac{2}{k_y}\sin(k_yL)\frac{2}{k_z}\sin(k_zL) \tag{4}$$

Thus $\psi(x,y,z,t)=\frac{1}{(2\pi)^{3/2}}\int\varphi(\vec{k})\exp[i(\vec{k}\cdot\vec{r}-\frac{\hbar k^2t}{2\mu})]d^3\vec{k}$

$$=\frac{1}{(2\pi)^3}\frac{8}{(\sqrt{2L})^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{k_x}\sin(k_xL)\frac{1}{k_y}\sin(k_yL)\frac{1}{k_z}\sin(k_zL)$$

$$\exp\{i[k_xx-\frac{\hbar k_x^2t}{2\mu}+k_yy-\frac{\hbar k_y^2t}{2\mu}+k_zz-\frac{\hbar k_z^2t}{2\mu}]\}dk_x\,dk_y\,dk_z$$

$$=\frac{1}{(2\pi)^3}\frac{8}{(\sqrt{2L})^3}\Phi(L,t,x)\Phi(L,t,y)\Phi(L,t,z) \tag{5}$$

Where

$$\Phi(L,t,x)=\int_{-\infty}^{+\infty}\frac{\sin(k_xL)}{k_x}\exp[i(k_xx-\frac{\hbar k_x^2t}{2\mu})]dk_x \tag{6}$$

$$\Phi(L,t,y)=\int_{-\infty}^{+\infty}\frac{\sin(k_yL)}{k_y}\exp[i(k_yy-\frac{\hbar k_y^2t}{2\mu})]dk_y \tag{7}$$

$$\Phi(L,t,z)=\int_{-\infty}^{+\infty}\frac{\sin(k_zL)}{k_z}\exp[i(k_zz-\frac{\hbar k_z^2t}{2\mu})]dk_z \tag{8}$$

Thus $\Phi(0,t,x)=0$ (9)

$$\frac{\partial\Phi(L,t,x)}{\partial L}=\int_{-\infty}^{+\infty}\cos k_xL\,\exp[i(k_xx-\frac{\hbar k_x^2t}{2\mu})]dk_x$$

$$=\int_{-\infty}^{+\infty}\frac{\exp(ik_xL)+\exp(-ik_xL)}{2}\exp[i(k_xx-\frac{\hbar k_x^2t}{2\mu})]dk_x$$

$$=\frac{1}{2}\{\int_{-\infty}^{+\infty}\exp[ik_x(x+L)]\exp[-i\frac{\hbar k_x^2t}{2\mu}]dk_x$$

$$+\int_{-\infty}^{+\infty}\exp[ik_x(x-L)]\exp[-i\frac{\hbar k_x^2 t}{2\mu}]dk_x\}$$

$$=\frac{1}{2}\{\int_{-\infty}^{+\infty}\exp\{-i[\frac{\hbar k_x^2 t}{2\mu}-k_x(x+L)]\}dk_x$$

$$+\int_{-\infty}^{+\infty}\exp\{-i[\frac{\hbar k_x^2 t}{2\mu}-k_x(x-L)]\}dk_x\}$$

$$=\frac{1}{2}\{\int_{-\infty}^{+\infty}\exp\{-i[(\sqrt{\frac{\hbar t}{2\mu}}k_x-\frac{x+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2-(\frac{x+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\}dk_x$$

$$+\int_{-\infty}^{+\infty}\exp\{-i[(\sqrt{\frac{\hbar t}{2\mu}}k_x-\frac{x-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2-(\frac{x-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\}dk_x\}$$

$$=\frac{1}{2}\{\frac{1}{\sqrt{\frac{\hbar t}{2\mu}}}[\sqrt{\frac{\pi}{2}}-i\sqrt{\frac{\pi}{2}}]\exp[i(\frac{x+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]+\frac{1}{\sqrt{\frac{\hbar t}{2\mu}}}[\sqrt{\frac{\pi}{2}}-i\sqrt{\frac{\pi}{2}}]\exp[i(\frac{x-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\} \quad (10)$$

By (9),(10)

$$\Phi(L,t,x)=\int_0^L\frac{\partial\Phi(L',t,x)}{\partial L'}dL'+\Phi(0,t,x)=\int_0^L\frac{\partial\Phi(L',t,x)}{\partial L'}dL'$$

$$=\int_0^L\frac{1}{2\sqrt{\frac{\hbar t}{2\mu}}}[\sqrt{\frac{\pi}{2}}-i\sqrt{\frac{\pi}{2}}]\{\exp[i(\frac{x+L'}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]+\exp[i(\frac{x-L'}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\}dL' \quad (11)$$

$$\frac{\partial\Phi(L,t,x)}{\partial x}=\int_0^L\frac{1}{2\sqrt{\frac{\hbar t}{2\mu}}}\sqrt{\frac{\pi}{2}}(1-i)\{\frac{\partial}{\partial x}\exp[i(\frac{x+L'}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]+\frac{\partial}{\partial x}\exp[i(\frac{x-L'}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\}dL'$$

$$=\int_0^L\frac{1}{2\sqrt{\frac{\hbar t}{2\mu}}}\sqrt{\frac{\pi}{2}}(1-i)\{\frac{\partial}{\partial L'}\exp[i(\frac{x+L'}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]-\frac{\partial}{\partial L'}\exp[i(\frac{x-L'}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\}dL'$$

$$=\frac{1}{2\sqrt{\frac{\hbar t}{2\mu}}}\sqrt{\frac{\pi}{2}}(1-i)\{\exp[i(\frac{x+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]-\exp[i(\frac{x}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]$$

$$-\exp[i(\frac{x-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]+\exp[i(\frac{x}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\}$$

$$=\frac{1}{2\sqrt{\frac{\hbar t}{2\mu}}}\sqrt{\frac{\pi}{2}}(1-i)\{\exp[i(\frac{x+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]-\exp[i(\frac{x-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\} \quad (12)$$

According to the theory that the velocity of any particle can not be larger than the velocity of light c and the Born interpretation, for any $t(t>0)$, when $|x|>(ct+L)$, $|y|>(ct+L)$, $|z|>(ct+L)$, $\psi(x,y,z,t)$ will be zero. This means if $|\delta|>(ct+L)$, $\psi(\delta,\delta,\delta,t)$ will be zero. Then by (5), when $|\delta|>(ct+L)$, $\Phi(L,t,\delta)$ will be zero. Therefore when $|\delta|>(ct+L)$, $\frac{\partial\Phi(L,t,\delta)}{\partial\delta}$ will be zero.

According to (12).

$$\frac{\partial\Phi(L,t,\delta)}{\partial\delta}=\frac{1}{2\sqrt{\frac{\hbar t}{2\mu}}}\sqrt{\frac{\pi}{2}}(1-i)\{\exp[i(\frac{\delta+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]-\exp[i(\frac{\delta-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2]\} \tag{13}$$

Now assume $\frac{\partial\Phi(L,t,\delta)}{\partial\delta}=0$ (14)

Then it is not difficult to see

$$(\frac{\delta+L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2=(\frac{\delta-L}{2\sqrt{\frac{\hbar t}{2\mu}}})^2+2n\pi \tag{15}$$

Where n is an integer

$$\text{Thus } \frac{\delta L}{\frac{\hbar t}{2\mu}}=2n\pi \tag{16}$$

$$\text{or } \delta=\frac{\hbar t n\pi}{\mu L} \tag{17}$$

This means when $\frac{\delta\mu L}{\hbar t\pi}$ is not an integer, even if $|\delta|>(ct+L)$, $\frac{\partial\Phi(L,t,\delta)}{\partial\delta}$ will not be zero. This result contradicts that when the velocity of any particle can not be larger than the velocity of light and the Born interpretation is valid, if $|\delta|>(ct+L)$, $\frac{\partial\Phi(L,t,\delta)}{\partial\delta}$ will be zero. This contradiction means that there is a contradiction between the measurement theory of quantum mechanics and the theory that the velocity of any particle can not be larger than the velocity of light.

Reference

Eugen Merzbacher, Quantum Mechanics, John Wiley & Sons, New York, 1970, p. 26