

# NONCLASSICAL PROPERTIES OF Q-DEFORMED SUPERPOSITION LIGHT FIELD STATE

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## Abstract

In this paper, the squeezing effect, the bunching effect and the anti-bunching effect of the superposition light field state which involving  $q$ -deformation vacuum state  $|0\rangle_q$  and  $q$ -Glauber coherent state  $|z\rangle_q$  are studied, the controllable  $q$ -parameter of the squeezing effect, the bunching effect and the anti-bunching effect of  $q$ -deformed superposition light field state are obtained.

### 1 Introduction

In recent years people have made progress in the research of some concrete physical problem using quantum groups  $SU_q(2)$ , Quantum algebra has been realized by using  $q$ -oscillator and the parametrized Fock state  $|n\rangle_q$  was obtained too. From this  $q$ -Glauber coherent state  $|z\rangle_q$  was introduced. Hao Sanyu<sup>[1]</sup> showed that the coherent degree can be controlled by  $q$ -deformation parameter. Zhu Chongxu<sup>[1]</sup> showed that some quantum statistical properties of  $q$ -even-odd coherent state can be controlled by  $q$ -parameter.

We studied the squeezing effect of  $q$ -deformed superposition light field which involving  $q$ -deformation vacuum state  $|0\rangle_q$  and  $q$ -Glauber Coherent state  $|z\rangle_q$ . The results showed that the squeezing effect, the bunching effect and the anti-bunching effect can be controlled by  $q$ -parameter.

### 2 Nonclassical properties of $q$ -deformed superposition Light field state.

The  $q$ -deformed superposition Light field state is

$$|\psi\rangle_q = \alpha|0\rangle_q + \beta|z\rangle_q \quad (1)$$

where

$$|z\rangle_q = e^{-\frac{1}{2}|z|^2} \sum_{n=0}^{\infty} \frac{Z^n}{\sqrt{[N]!}} |n\rangle_q \quad (2)$$

$$Z = Re^{i\theta}, \alpha = r_1 e^{i\theta_1}, \beta = r_2 e^{i\theta_2} \quad (3)$$

$$[X] = \frac{q^{\frac{X}{2}} - q^{-\frac{X}{2}}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}} (q \neq 1) \quad e_q^x = \sum_{n=0}^{\infty} \frac{x^n}{[N]!}, [X]! = [X] \cdot [X-1] \cdots [1] \quad (4)$$

The normalization condition is

$$\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2e_q^{-\frac{1}{2}|z|^2}\cos(\theta_1 - \theta_2) = 1 \quad (5)$$

## 2.1 The squeezing effect of q-deformed superposition light field state

The two orthogonal components of q-deformed electromagnetic field are defined as

$$Y_1 = \frac{1}{2}(a_q^+ + a_q), Y_2 = \frac{1}{2i}(a_q^+ - a_q) \quad (6)$$

where  $a_q$  is q-annihilation operator and  $a_q^+$  is q-creation operator. Because of  $[Y_1, Y_2] = \frac{i}{2}[a_q, a_q^+]$ , so we have the uncertainty relation.

$$\langle (\Delta Y_1)^2 \rangle \langle (\Delta Y_2)^2 \rangle \geq \frac{1}{4} |\langle [Y_1, Y_2] \rangle|^2 \quad (7)$$

If the squeezing exists, then we have

$$F_i = \langle (\Delta Y_i)^2 \rangle - \frac{1}{4} < 0 \quad (i = 1, 2) \quad (8)$$

For q-deformed superposition light field state, we have

$$\begin{aligned} \langle \psi | a_q a_q^+ | \psi \rangle &= (\alpha^* \langle 0 | + \beta^* \langle Z |) a_q a_q^+ (\alpha | 0 \rangle + \beta | Z \rangle) \\ &= |\alpha|^2 + \beta^* \alpha e_q^{-\frac{1}{2}|z|^2} + \alpha^* \beta e_q^{-\frac{1}{2}|z|^2} + |\beta|^2 e_q^{-|z|^2} \sum_{n=0}^{\infty} \frac{|Z|^{2n}}{[n]_1!} [n+1] \end{aligned} \quad (9)$$

$$\langle \psi | a_q^+ a_q | \psi \rangle = (\alpha^* \langle 0 | + \beta^* \langle Z |) a_q^+ a_q (\alpha | 0 \rangle + \beta | Z \rangle) = |\beta|^2 |Z|^2 \quad (10)$$

$$\begin{aligned} \langle \psi | a_q | \psi \rangle &= (\alpha^* \langle 0 | + \beta^* \langle Z |) a_q (\alpha | 0 \rangle + \beta | Z \rangle) \\ &= Z \alpha^* \beta e_q^{-\frac{1}{2}|z|^2} + |\beta|^2 Z \end{aligned} \quad (11)$$

$$\begin{aligned} \langle \psi | a_q^+ | \psi \rangle &= (\alpha^* \langle 0 | + \beta^* \langle Z |) a_q^+ (\alpha | 0 \rangle + \beta | Z \rangle) \\ &= \beta^* \alpha Z^* e_q^{-\frac{1}{2}|z|^2} + |\beta|^2 Z^* \end{aligned} \quad (12)$$

$$\begin{aligned} \langle \psi | a_q^{+2} | \psi \rangle &= (\alpha^* \langle 0 | + \beta^* \langle Z |) a_q^{+2} (\alpha | 0 \rangle + \beta | Z \rangle) \\ &= \beta^* \alpha Z^{*2} e_q^{-\frac{1}{2}|z|^2} + |\beta|^2 Z^{*2} \end{aligned} \quad (13)$$

$$\begin{aligned} \langle \psi | a_q^2 | \psi \rangle &= (\alpha^* \langle 0 | + \beta^* \langle Z |) a_q^2 (\alpha | 0 \rangle + \beta | Z \rangle) \\ &= \beta \alpha^* Z^2 e_q^{-\frac{1}{2}|z|^2} + |\beta|^2 Z^2 \end{aligned} \quad (14)$$

From (8) - (14), we can have

$$\begin{aligned} F_1 &= \frac{1}{4} \{ \langle a_q^{+2} \rangle + \langle a_q^2 \rangle + \langle a_q a_q^+ \rangle + \langle a_q^+ a_q \rangle - (\langle a_q^+ \rangle + \langle a_q \rangle)^2 \} - \frac{1}{4} \\ &= \frac{1}{4} [ e_q^{-\frac{1}{2}|z|^2} \tau_1 \tau_2 k^2 2 \cos(\theta_2 + 2\varphi - \theta_1) + \tau_2^2 R^2 \cdot 2 \cos 2\varphi + e_q^{-\frac{1}{2}|z|^2} \tau_1 \tau_2 2 \cos(\theta_2 - \theta_1) \\ &+ \tau_2 q^{-|z|^2} \sum_{n=0}^{\infty} \frac{R^{2n}}{[n]_1!} [n+1] + \tau_2^2 R^2 + \tau_1^2 - (R \tau_1 \tau_2 e_q^{-\frac{1}{2}|z|^2} 2 \cos(\theta_2 + \varphi - \theta_1) + R \tau_2^2 2 \cos \varphi)^2 - 1 ] \end{aligned} \quad (15)$$

$$\begin{aligned} F_2 &= \frac{1}{4} [ -\langle a_q^{+2} \rangle - \langle a_q^2 \rangle + \langle a_q a_q^+ \rangle + \langle a_q^+ a_q \rangle + (\langle a_q^+ \rangle - \langle a_q \rangle)^2 ] - \frac{1}{4} \\ &= \frac{1}{4} [ -e_q^{-\frac{1}{2}|z|^2} \tau_1 \tau_2 R^2 2 \cos(\theta_2 - 2\varphi - \theta_1) - \tau_2^2 R^2 2 \cos 2\varphi + e_q^{-\frac{1}{2}|z|^2} \tau_1 \tau_2 2 \cos(\theta_1 - \theta_2) \\ &+ \tau_2^2 e_q^{-|z|^2} \sum_{n=0}^{\infty} \frac{R^{2n}}{[n]_1!} [n+1] + \tau_2^2 R^2 + \tau_1^2 + (R \tau_1 \tau_2 e_q^{-\frac{1}{2}|z|^2} \sin(\theta_2 + \varphi - \theta_1) - R \tau_2^2 2 \sin \varphi)^2 - 1 ] \end{aligned} \quad (16)$$

It is clear  $F_1$  and  $F_2$  are periodic function of  $\varphi$ . Numerical value calculating showed that  $Y_1$  and  $Y_2$  may be more than zero and less than zero accompanying the variation of  $q$ . This result shows that the generally squeezing may exist and can be controlled by  $q$ .

2.2 The bunching effect, the anti-bunching effect of q-deformed superposition light field state  
 For q-deformed superposition light field state, we have

$$\langle \psi | a_q^{+2} a_q^2 | \psi \rangle = |\beta|^2 |Z|^4 \quad (17)$$

$$g_q^{(2)}(0) = \frac{\langle \psi | a_q^{+2} a_q^2 | \psi \rangle}{(\langle \psi | a_q^+ a_q | \psi \rangle)^2} = \frac{|\beta|^2 |Z|^4}{|\beta|^4 |Z|^4} = \frac{1}{|\beta|^2} = \frac{1}{r_2^2} \quad (18)$$

When  $\cos(\theta_1 - \theta_2) \geq 0$ , from (5) we have

$$r_1^2 + r_2^2 \leq 1 \quad (19)$$

From (19), we get  $r^2 < 1$ , so that

$$g_q^{(2)}(0) = \frac{1}{r^2} > 1 \quad (20)$$

(18) shows that the bunching effect exists.

When  $\cos(\theta_1 - \theta_2) < 0$ , we have  $r_1^2 + r_2^2 > 1$ , so that  $r_2^2$  may be more than 1 and we have

$$g_q^{(2)}(0) = \frac{1}{r^2} < 1 \quad (22)$$

(22) Shows that the anti-bunching effect exist.

### 3. Conclusion

The results of this paper shows that the squeezing effect, the bunching effect and the anti-bunching effect of q-deformed superposition light field state may exist and can be controlled by q-parameter.

### Reference

- [1] Hao Sanyu, ACTA PHYSICA SINICA, 42, 1057 (1993)
- [2] Zhu Chongxu et al, ACTA PHYSICA SINICA, 43, 1262(1994).

