# CHIRAL BOSONIZATION OF SUPERCONFORMAL GHOSTS

Shi Deheng　　Shen Yang　　Liu Jinling

*Department of Foundation, The First Aeronautical College of Air Force, Xinyang, Henan 464000, P. R. China*

Xiong Yongjian

*Department of Physics, Xinyang Teacher's College, Xinyang, Henan 464000, P. R. China*



**Abstract**

We explain the difference of the Hilbert space of the superconformal ghosts ($\beta$, $\gamma$) system from that of its bosonized fields $\varphi$ and $\chi$. We calculate the chiral correlation functions of $\varphi$, $\chi$ fields by inserting appropriate projectors.

Recently, many authors have investigated the bosonization ^f superconformal ghosts $\beta$ and $\gamma$[1,2]. Unlike the fermionic ghosts b and c, the bosonization of ($\beta$, $\gamma$) system have some problems.

Locally, ($\beta$,$\gamma$) system is equivalent to two scalar fields $\varphi$ and $\chi$[3]. Although the chiral correlation functions of $\beta$, $\gamma$ fields have been calculated[1,2], the calculation of the chiral correlation functions of $\varphi$, $\chi$ fields will be troublesome. Besides the redundant zero-modes of the bosonized fields, the main reason is that $\varphi$, $\chi$ fields have a large Hilbert space than ($\beta$, $\gamma$) system. In ref. (4), this enlargement was explained as caused by the freedom of choosing the background ghost charge, the so-called picture, and by introducing projectors which specify the picture of each loop, the Hilbert space of $\varphi$, $\chi$ fields are restricted to the degrees of freedom of the ($\beta$, $\gamma$) system. In this paper, we explain this problem from an elementary point of view, and then apply new projector for the calculation of the chiral correlation functions of $\varphi$, $\chi$ fields.

We consider the ($\beta$, $\gamma$) system corresponding to superstring theory, i. e. with conformal dimensions $\frac{3}{2}$, $-\frac{1}{2}$ respectively. Locally, ($\beta$, $\gamma$) system is identified with a scalar field $\varphi$ and a pair of fermions $\zeta$, $\eta$ with conformal weights 0, 1 respectively.

$$\beta=\partial\zeta e^{-i\varphi} \qquad \gamma=\eta e^{i\varphi} \tag{1}$$

The $\varphi$ field is coupled to background charge Q=2, and is described by the action

$$S[\varphi]=\frac{1}{2\pi}\int d^2z(-\partial_z\varphi\,\partial_{\bar z}\varphi-\frac{i}{2}\sqrt{g}\;R\varphi) \tag{2}$$

where, $g_{z\bar z}$ is a Riemann metric and R is the corresponding scalar curvature. We can again bosonizing ($\zeta$, $\eta$) system via another scalar field $\chi$

$$\zeta=e^{i\chi} \qquad \eta=e^{-i\chi} \tag{3}$$

The $\chi$ field is coupled to background charge $Q=-1$ and is described by the action

$$S[\chi]=\frac{1}{2\pi}\int d^2Z(\partial_z\chi\partial_{\bar{z}}\chi+\frac{i}{4}\sqrt{g}R\chi) \tag{4}$$

$\varphi$ and $\chi$ fields are both restricted to taking values on a unit circle $\mathbf{R}/2\pi\mathbf{Z}$ ; this compactification results soliton configurations on Riemann surface $\sum_g$ with genus $g>0$, and insures the necessary holomorphic factorization[5].

The classical soliton sectors can be labeled by the winding numbers for the canonical homology basis $(a_i, b_j)$. The soliton solutions of $\varphi$, $\chi$ fields with winding numbers $(n_i, b_j)$ are given by

$$\begin{aligned}\varphi_{nm}(z)&=\pi(m+\bar{\tau}n)(\mathrm{Im}\tau)^{-1}z+c.c.\\ \chi_{nm}(z)&=i\pi(m+\bar{\tau}n)(\mathrm{Im}\tau)^{-1}z+c.c.\end{aligned} \tag{5}$$

where $\tau$ is the period matrix of $\sum_g$. For simplicity, we have denoted the Jacobi map $\int_{p_0}^{z}\omega$ as $z$. The corresponding action

$$\begin{aligned}S[\varphi_{nm}]&=\frac{\pi}{2}(m+\bar{\tau}n)(\mathrm{Im}\,\tau)^{-1}(m+\tau n)+2S_b\\ S[\chi_{nm}]&=\frac{\pi}{2}(m+\bar{\tau}n)(\mathrm{Im}\tau)^{-1}-S_b\\ S_b&=\pi(m+\bar{\tau}n)(\mathrm{Im}\tau)^{-1}\triangle-c.c.\end{aligned} \tag{6}$$

where, $\triangle$ is Riemann class.

We consider the following correlation functions

$$A_\delta=\int[d\varphi d\zeta d\eta]_\delta e^{-s[\varphi,\zeta,\eta]}\prod_{a=1}^{n+1}\zeta(\chi_a)\prod_{b=1}^{n}\eta(y_b)\prod_{c=1}^{n}e^{iq_c\varphi(z_c)} \tag{7}$$

where $q_c$ are integer satisfying $\sum q_c=2(g-1)$, and $\delta$ is a specific spin structure.

If $\varphi$ and $\chi$ are treated independently, the result will be different from that of the corresponding $\beta$, $\gamma$ fields. We notice that

a) the bosonized fields have redundant zero-modes of $\zeta$ and $\eta$ fields.

and b) the $(\varphi, \chi)$ system has a larger Hilbert space than that of the $(\beta, \gamma)$ system, since $\varphi$ and $\chi$ are not independent globally. Thus we must have appropriate constriants, otherwise, some global configurations will be computed repeatly.

The first aspect can be resolved by inserting operators $\delta(\zeta(\chi))$, $\prod_{i=1}^{g}\delta(\eta(r_i))$ to remove zero-modes of $\zeta$, $\eta$ fields. $\eta$ has zero-modes at $i=1, \cdots, g$, and $\zeta$ has a constant zero-mode, thus $\chi$ is an arbitrary point on $\sum_g$. In order to avoid to compute the similar part of global configurations of $\varphi$, $\chi$ fields, we introduce projector

$$\delta(m_\chi - m_\varphi)\,\delta(n_\chi - n_\varphi) \tag{8}$$

to restrict $\varphi$, $\chi$ on the same soliton sector at the same time.

Now, according to Riemann-Roch theorem, (7) must be modified as follow

$$A_\delta = \int [d\varphi\, d\zeta\, d\eta\,]_\delta\, e^{-s[\varphi,\zeta,\eta]} \prod_{a=1}^{n} \zeta(x_a) \prod_{b=1}^{n} \eta(y_b) \prod_{c=1}^{n} e^{iq_c\varphi(z_c)} \tag{9}$$

Inserting our projector

$$\delta(\zeta(x)) \prod_{i=1}^{g} \delta(\eta(r_i))\, \delta(m_\varphi - m_\chi)\, \delta(n_\chi - n_\varphi)$$

we have

$$A_\delta = \langle \prod_{a=1}^{n} \zeta(x_a) \prod_{b=1}^{n} \eta(y_b) \prod_{c=1}^{n} e^{iq_c\varphi(z_c)} \cdot \delta(\zeta(x)) \prod_{i=1}^{g} \delta(\eta(r_i))\, \delta(m_\chi - m_\varphi)\, \delta(n_\chi - n_\varphi) \rangle_\delta$$

For the $\delta$-functions with fermion arguments, $\delta(\zeta)=\zeta$, $\delta(\eta)=\eta$, and labelling the arbitrary X as $X_{n+1}$, we get

$$A_\delta = \langle \prod_{a=1}^{n+1} e^{i\chi(x_a)} \prod_{b=1}^{n} e^{-i\chi(y_b)} \prod_{c=1}^{n} e^{iq_c\varphi(z_c)} \prod_{i=1}^{g} e^{-i\chi(r_i)}\, \delta(m_\chi - m_\varphi)\, \delta(n_\chi - n_\varphi) \rangle_\delta$$

This result can be written as a soliton sum $\Lambda_{sol,\,\delta}$ multiplied the amplitude of zero soliton sector $A_{00}$

$$A_\delta = \Lambda_{sol,\,\delta} \cdot A_{00} \tag{10}$$

$A_{00}$ is the result of the single-valued part of $\varphi$, $\chi$ fields. It is trivial that[5]

$$A_{00} = \exp\{2\pi \mathrm{Im}(\sum X_a - \sum Y_b - \sum r_i + \sum q_c z_c - \triangle)(\mathrm{Im}\tau)^{-1} \mathrm{Im}(\sum X_a - \sum Y_b$$

$$- \sum r_i + \sum q_c z_c - \triangle))\} \times \left| \frac{\prod_c \sigma(z_c)^{-2q_c}}{\prod_{c_1<c_2} E(z_{c_1}, z_{c_2})^{q_{c_1}q_{c_2}}} \cdot \frac{\prod_b \sigma(y_b) \prod_{r_i} \sigma(r_i)}{\prod_a \sigma(x_a)} \right.$$

$$\left. \times \frac{\prod_{a_1<a_2} E(x_{a_1}, x_{a_2}) \prod_{b_1<b_2} E(y_{b_1}, y_{b_2}) \prod_{r_i<r_j} E(r_i, r_j) \prod_{b<r_i} E(y_b, r_i)}{\prod_{a,\,b} E(x_a, y_b) \prod_{a,\,r_i} E(x_a, r_i)} \right|^2 \tag{11}$$

Using Possion summation formula, we get.

$$\Lambda_{\infty 1,\delta}=(\det\mathrm{Im}\tau)\exp\{-2\pi\mathrm{Im}(\sum x_a-\sum y_b+\sum q_c z_c-\sum r_i-\Delta)(\mathrm{Im}\tau)^{-1}\cdot\mathrm{Im}(\sum x_a-\sum y_b$$

$$+\sum q_c z_c-\sum r_i-\Delta)\}\cdot\left|\sum_{p_\chi}\exp\{\pi i p_\chi\tau p_\chi+2\pi i p_\chi(\sum x_a-\sum y_b-\sum r_i+\Delta)\}\right.$$

$$\cdot\left|\sum_{p_\varphi}\exp\{-\pi i(p_\varphi+\delta')\tau(p_\varphi+\delta')+2\pi i(p_\varphi+\delta')(\sum q_c z_c-2\Delta+\delta'')\}\right|^2 \tag{12}$$

Here, $\delta'$, $\delta''\in(\frac{1}{2}\mathbf{z}/\mathbf{z})^g$, and $\delta=[{\delta' \atop \delta''}]$

From (11) and (12), holomorphic anomaly factors of $A_{00}$ and $\Lambda_{\infty 1,\delta}$ can cancell each other. Thus we can have chiral correlation functions

$$A_\delta^{\mathrm{chiral}}=(\det\mathrm{Im}\tau)^{\frac{1}{2}}\frac{\prod_{b_0=1}^{n}\theta[\delta](-y_{b_0}+\sum x_a-\sum y_b+\sum q_c z_c-2\Delta)}{\prod_{a_0=1}^{n+1}\theta[\delta](-x_{a_0}+\sum x_a-\sum y_b+\sum q_c z_c-2\Delta)}$$

$$\cdot\frac{\prod_{a_1<a_2}E(x_{a_1},x_{a_2})\prod_{b_1<b_2}E(y_{b_1},y_{b_2})}{\prod_{a,b}E(x_a,y_b)\prod_{c_1,c_2}E(z_{c_1},z_{c_2})^{q_{c_1}q_{c_2}}\prod_{c}\sigma(z_c)^{2q_c}} \tag{13}$$

Thus, by inserting appropriate projector to remove the zero-modes of $\zeta$, $\eta$ fields and restrict $\varphi$, $\chi$ on the same soliton sector, we get the correct chiral correlation functions of $\varphi$, $\chi$ fields. As compared with ref. (4), our approach is more comprehensive.

## References


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