SOLVABLE QUANTUM MACROSCOPIC MOTIONS and DECOHERENCE MECHANISMS in QUANTUM MECHANICS on NONSTANDARD SPACE

Tsunehiro KOBAYASHI
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan

Abstract

Quantum macroscopic motions are investigated in the scheme consisting of N-number of harmonic oscillators in terms of ultra-power representations of nonstandard analysis. Decoherence is derived from the large internal degrees of freedom of macroscopic matters.

1. Introduction

How to describe motions of macroscopic matters in quantum mechanics is not only a very interesting problem but also a very important problem to develop the present situation of theoretical physics. Before going into the details we shall start from the question "What are macroscopic matters?". One may characterize them in terms of the following three properties:
(1) The number of constituents N is very large and cannot be precisely counted in measurements.
(2) Every measurement of energy E of macroscopic matters is accompanied by experimental margin of uncertainty ΔE and an enormous number of different quantum states are contained within the energy uncertainty.
(3) Macroscopic matters are usually classical objects. This means that the density matrices describing their quantum states have no interference terms (decoherence mechanism exists.).

The first character indicates that we have no way to measure the precise number of the constituents in realistic measurement processes. Furthermore we may say that the precise determination of the quantum states for all the constituents are impossible. This property has a close connection with the second character. In usual measurements the energies of macroscopic matters are not quantum mechanical order ($O(\hbar)$) which disappears in the limit of $\hbar \to 0$ ($lim_{\hbar\to 0}\ O(\hbar)$). It means that every measurement of the energy of macroscopic objects may contain some uncertainty ΔE which is in the order $O(\hbar)$. How to introduce these features in quantum mechanics is the main theme of this paper.

An interesting possibility is to describe the macroscopic matters on the Hilbert spaces extended by nonstandard analysis,[1] where infinity (∞) like $N \to \infty$ and infinitesimal (≈ 0) like $\hbar \to 0$ are treated rigorously. It should also be pointed out that quantum states of N constituents which may be described by the direct product of the quantum states of the constituents such that $\Psi_N(r_1,..,r_N) = \prod_{i=1}^{N} \phi_{E_i}(r_i)$ become ultra-products in the limit $N \to \infty$. Then we can represent the macroscopic states in terms of ultra-power representation of nonstandard analysis by introducing some equivalence relation based on the ultra-filter on the ultra-products.

From the discussions of quantum mechanics on nonstandard spaces[2,3,4] we know that
(I) there exist new eigenfunctions called as "ultra-eigenfunctions" which are not described by the

superposition of eigenfunctions on usual quantum mechanics on real number space ($\mathcal{R}$), and (II) in the limit $\hbar \rightarrow 0$ we can introduce infinitesimal energy uncertainties ΔE which are in the order $O(\hbar)$. It is important that the introduction of such energy uncertainties is expressed by the monad (infinitesimal neighborhoods) of real numbers on nonstandard spaces.

Now we may expect that we can describe macroscopic states in terms of new eigenfunctions (ultra-eigenfunctions) containing the energy uncertainty $\Delta E \sim O(\hbar)$. In this paper I shall present a solvable model to realize the above consideration.

2. Model

Let us investigate a system consisting of N-harmonic oscillators which are bounded around a fixed point X_0. The Hamiltonian is given by $H_N = \frac{1}{2m}\sum_{i=1}^{N} p_i^2 + \frac{k}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}(x_i - x_j)^2 + \frac{K}{2}\sum_{i=1}^{N}(x_i - X_0)^2$, where m the mass of the constituents, k and K the oscillator constants, p_i and x_i, respectively, stand for the momentum and position operators of i-th constituent. This Hamiltonian describes the bounded N-oscillator system moving in the harmonic oscillator potential of which center is at X_0. Our interest is focused on the relative motion between the fixed point X_0 and the center of mass(CM) of the N-oscillator system, because the motion will become the observable as the motions of the macroscopic system in the macroscopic limit $N \rightarrow \infty$. The Hamiltonian is separable in terms of the following choice of coordinates; $R^N = X_G^N - X_0$, $\rho_n^N = [nx_{n+1} - (\sum_{i=1}^{n} x_i)]/\sqrt{n(n+1)}$, for $n = 1, 2, ..., N-1$, where $X_G^N = \frac{1}{N}\sum_{i=1}^{N} x_i$ is the CM(center of mass) coordinate of the N-oscillator system. We can rewrite the Hamiltonian as

$$H^N = \frac{1}{2mN}(P^N)^2 + \frac{NK}{2}(R^N)^2 + \sum_{n=1}^{N-1} H_n, \tag{1}$$

where $H_n = \frac{1}{2m}p_n^2 + \frac{1}{2}(k + K)\rho_n^2$. The eigenfunctions for (1) are obtained as follows; $H^N\Psi^N = (E_R + \sum_{n=1}^{N-1}\epsilon_n)\Psi^N$, where

$$\Psi^N(R^N, [\rho_n]) = \Phi_R(R^N)\prod_{n=1}^{N-1}\phi_{l_n}(\rho_n) \tag{2}$$

with $[\rho_n] \equiv [\rho_1, \rho_2, \cdots, \rho_{N-1}]$, which satisfy $H_R\Phi_R = E_R\Phi_R$ and $H_n\phi_{l_n} = \epsilon_n\phi_{l_n}$ with $E_R = (n_R + 1/2)\omega_R\hbar$ ($\omega_R = \sqrt{K_N/M_N} = \sqrt{K/m}$) and $\epsilon_n = (l_n + 1/2)\omega\hbar$ ($\omega = \sqrt{(k+K)/m}$). Note that the eigenvalues of 'H^N and $\sum_n H_n$ are, respectively, given by $E^N = E_R + \epsilon_L^N$ and $\epsilon_L^N \equiv \sum_{n=1}^{N-1}\epsilon_n = (L + \frac{1}{2}(N-1))\omega\hbar$ with $L = \sum_{n=1}^{N-1} l_n$ and all the energies are of the order of $O(\hbar)$, i.e. $E^N \sim E_R \sim \epsilon_L^N \sim O(\hbar)$. We see that E_R and ϵ_L^N are not enough to specify the state given in (2) uniquely. That is, there are many different states having a fixed value of ϵ_L^N, of which multiplicity is evaluated as $W(N, L) = \frac{(L+N-2)!}{L!(N-2)!}.Ingeneralweshouldwriteeigenfunctionsspecifiedby\mathrm{E}_N$ and ϵ_L^N in terms of the suprepositons of those different states such that

$$\Psi_L^N(E^N; R^N, [\rho_n]) = \Phi_{E_R}(R^N)\sum_{l_1=0}^{L}\cdots\sum_{l_{N-1}=0}^{L}\delta_{\sum_{n=1}^{N-1} l_n, L}\, a([l_n])\prod_{n=1}^{N-1}\phi_{l_n}(\rho_n) \tag{3}$$

with $[l_n] \equiv [l_1, l_2, \cdots, l_{N-1}]$, where $a([l_n])$ are the coefficients satisfying the constraint required from the normalization $\sum_{l_1=0}^{L}\cdots\sum_{l_{N-1}=0}^{L}\delta_{\sum_{n=1}^{N-1} l_n, L}\, |a([l_n])|^2 = 1$.

3. Oscillator system in nonstandard spaces

Now let us study the limit represented by $N \to \infty$. The state given in (2) becomes an infinite direct-product

$$\Psi_{E_R,L}(R;[\rho_n]) = \Phi_{E_R}(R) \prod_{n=1} \phi_{l_n}(\rho_n). \tag{4}$$

The Hamiltonian (1) is modified as $\hat{H} = H_R + \sum_{n=1} \hat{H}_n$, where in order to evade the divergence arising from the sum of zero point oscillations $\sum_{n=1}^{N-1} \frac{1}{2}\hbar\omega$ in the limit of $N \to \infty$ $\hat{H}_n$ is taken as $\hat{H}_n = H_n - \frac{1}{2}\hbar\omega$. We have

$$\hat{H}\Psi_{E_R,L}(R,[\rho_n]) = (E_R + \epsilon_L)\Psi_{E_R,L}(R,[\rho_n]), \tag{5}$$

where $H_R\Phi_{E_R}(R) = E_R\Phi_{E_R}(R)$, and $\epsilon_L = L\hbar\omega$ with $L = \sum_{n=1} l_n$. Note that, since $l_n \in \mathcal{N}$ for $\forall n \in \mathcal{N}$, then $L \in \mathcal{N}$.

Following the expression of (3), we can write the most general wave-functions for the macroscopic object characterized by the CM(center of mass) energy eigenvalue E_R as $\Psi(E_R,[C_L([l_n])];R,[\rho_n]) = \Phi_{E_R}(R)\phi([C_L([l_n])];[\rho_n])$, where

$$\phi([C_L([l_n])];[\rho_n]) = \sum_{L=0}^{L_f} \sum_{l_1=0}^{L} \sum_{l_2=0}^{L} \cdots \delta_{\sum_{n=1} l_n, L}\, C_L([l_n]) \prod_{n=1} \phi_{l_n}(\rho_n), \tag{6}$$

L_f is an arbitrary natural number ($L_f \in \mathcal{N}$) and the normalization condition is given by $\sum_{L=0}^{L_f} \sum_{l_1=0}^{L} \sum_{l_2=0}^{L} \cdots \delta_{\sum_{n=1} l_n, L} |C_L([l_n])|^2 = 1$. The expectation value of the total energy is obtained as $< E >= E_R + \Delta E([C_L])$, where $\Delta E([C_L]) \equiv \sum_{L=0}^{L_f} \sum_{l_1=0}^{L} \sum_{l_2=0}^{L} \cdots \delta_{\sum_{n=1} l_n, L} |C_L([l_n])|^2\, \epsilon_L$.

Now let us consider measurements of the CM energy. When we try to observe it by using a photon as a probe, we have to measure it through the interaction of the photon with the constituents. This means that we cannot measure the CM energy directly and then we have to take account of the internal motions of the macroscopic object. In realistic measurement processes for macroscopic objects, which will be carried out by using a photon flux composed of many photon, we should consider that direct observable is the total energy rather than the CM energy. In those measurements the total of the internal energy $\Delta E([C_L]) \equiv < E > - < E_R >$ may be understood to be the errors for the CM energy. Note that the errors should not be confused with those arising from inefficiencies of detectors. We may conclude that we have always to take account of the existence of these errors in the observed CM energies when we discuss the CM motions of macroscopic objects which are studied in the classical mechanics.

In nonstandard analysis the error must be infinitesimal. Then as we take into account that the center of mass energy E_R and its angular frequency ω_R are observables represented by real numbers, the possibility allowed here is only the following choice; "$n_R \in {}^*\mathcal{N} - \mathcal{N},\ L \in \mathcal{N},\ \hbar \approx 0$." ¿From now on we define the macroscopic limit st_{macro} by taking

$$N^{-1} \approx 0 \quad and \quad \hbar \approx 0. \tag{7}$$

Let us investigate the equivalence relation introduced on the ultra-products. We can explicitly write this equivalence relation for the macroscopic objects by using the ultra-filter in nonstandard analysis as follows; $\phi_L([\rho]) = \prod_{n=1} \phi_{l_n}(\rho)$ with $\epsilon_L = \sum_n l_n\hbar\omega$ and $\phi_{L'}([\rho_{n'}]) = \prod_{n'=1} \phi_{l'_{n'}}(\rho_{n'})$ with

$\epsilon_{L'} = \sum_{n'} l'_{n'}\hbar\omega$ are equivalent ($\longleftrightarrow_{macro}$), if and only if the number of $n \in \mathcal{N}$ satisfying $l_n \neq 0$ and that of $n' \in \mathcal{N}$ satisfying $l_{n'} \neq 0$ are finite numbers. That is, it is represented as

$$\phi_L \longleftrightarrow_{macro} \phi_{L'}, \ \ if\ and\ only\ if\ the\ sets\ of\ numbers\ defined\ by$$

$$(n \in \mathcal{N}; l_n \neq 0)\ and\ (n' \in \mathcal{N}; l_{n'} \neq 0)\ are\ finite\ sets\ of\ \mathcal{N}.$$

The physical space for the macroscopic motions is represented by $\mathcal{S}_{macro}({}^*\mathcal{H}) = {}^*\mathcal{H}/ \longleftrightarrow_{macro}$.

Let us start from the most general expression of ultra-eigenfunctions satisfying

$$\hat{H}\Psi_c(R,[\rho_n]) \approx_{macro} E\Psi_c(R,[\rho_n]), \tag{8}$$

where $st_{macro}(E) \in \mathcal{R}$ and $\neq 0$, that is, it is observable in the classical limit. In the above equation Ψ_c is factorized with respect to the CM motion and the internal ones as $\Psi_c = \Phi_c(R)\phi([\rho_n])$. Since E have the freedom of the order of $O(\hbar)$, the general expression for the internal motions is given as $\phi([C_L([l_n])];[\rho]) = \sum_{L=0}^{L_f}\sum_{l_1=0}^{L}\cdots\sum_{l_n=0}^{L}\cdots\delta_{\sum_{n=1} l_n, L}\, C_L([l_n])\prod_{n=1}\phi_{l_n}(\rho_n)$, of which energy expectation value is obtained as

$$\Delta E([C_L([l_n])]) \equiv \sum_{L=0}^{L_f}\sum_{l_1=0}^{L}\cdots\sum_{l_n=0}^{L}\cdots\delta_{\sum_{n=1} l_n, L}\, |C_L([l_n])|^2\epsilon_L \sim O(\hbar). \tag{9}$$

We can derive the equation for the CM motions by operating the internal trace operation represented by the partial trace operation for all the internal variables $({}^{\forall}\rho)$, that is,

$$< \phi([C_L([l_n])];[\rho]), H\Psi_c(R,[\rho]) >_{internal} = (H_R + \Delta E)\Phi_c(R) \approx_{macro} H_R\Phi_c(R). \tag{10}$$

As was shown in Ref.s[3,4], it is required for us to solve the equation only in the classical region satisfying $st_{macro}(E-\Delta E-V(R)) \in \mathcal{R}_+$. In order to obtain stationary states represented by $\Phi_c^{E_R}(R) = \hat{N}e^{iW(R)/\hbar}$, where $\hat{N}$ denotes the normalization constant, we can reduce the Schroedinger equation to that for $W(R)$ as $i\frac{\hbar}{2M}\frac{d^2W}{dR^2} \approx_{macro} \frac{1}{2M}(\frac{dW}{dR})^2 + \frac{1}{2}\hat{K}R^2 - (E-\Delta E)$. This equation has already solved in Ref.s 3 and 4 and is given in the classical region as

$$W(R) \approx W_{cl}^{E,\Delta E}(R) + \frac{1}{2}i\hbar\ln(u_{cl}^{E,\Delta E}(R)), \tag{11}$$

where $W_{cl}^{E,\Delta E}(R) = \int^R\sqrt{2M(E-\Delta E-V(R')}dR'$, $u_{cl}^{E,\Delta E}(R) = \sqrt{2M(E-\Delta E-V(R))}$ and $V(R) = \frac{1}{2}\hat{K}R^2$. In the non-classical region we may take $\Psi_c = 0$. (In details for the derivation of $\Phi_c^{E_R}$ and their orthogonality, see Ref.s[3,4].) It should be stressed that $\rho_c = |\Phi_c^{E_R}|^2$ in the classical limit gives the exact distribution for the ensemble of the particles moving in the potential $V(R)$, which is expected from classical mechanics.

4. Decoherence mechanism of ultra-eigenfunctions

As was shown in the last section, the most general expression of the ultra-eigenfunctions has the $[C_L([l_n])]$-dependence. Through the observations of classical quantities written only by the CM(center of mass) variables, we can not fix the coefficients $[C_L([l_n])]$ at all. In other words the CM energy is determined only within the error $\Delta E([C_L([l_n])])$, for which only the constraint $st_{macro}(\Delta E([C_L([l_n])])) = 0$ is required. Therefore, we may introduce integration procedures with respect to the coefficients $[C_L([l_n])]$ in order to take off the apparent dependence on those unobservable parameters in the density matrices. It should be stressed that this integration stands for the average over undetermined energy uncertainties ΔE and then it has well-defined physical meaning and its introduction is not ad hoc. Let us study this situation in the density matrix for the following superposed state of two ultra-eigenfunctions with different energies, $st_{macro}(E-E')\neq 0$, $\Psi^{E,E';\Delta E}=c_E\Psi_c^{E,\Delta E}+c_{E'}\Psi_c^{E',\Delta E}$, where $|c_E|^2+|c_{E'}|^2=1$. The density matrix is given by

$$\rho_c^{E,E';\Delta E}=|c_E|^2\rho_c^{E,E;\Delta E}+|c_{E'}|^2\rho_c^{E',E';\Delta E}+(c_E c_{E''}^{*}\Psi_c^{E,\Delta E}\Psi_c^{E',\Delta E\dagger}+h.c.). \tag{12}$$

In order to obtain the density matrix for the CM motions which is independent of the coefficients, we introduce the integrations with respect to the undetermined complex coefficients $[C_L([l_n])]$. The number of the coefficients is counted as $\hat{W}\equiv\sum_{L=0}^{L_f}W(N,L)$, where $W(N,L)$ is the number of the different combinations for $[l_n]$. The multiplicity $\hat{W}$ is same as that of the equivalent internal wave-functions $\phi_L([l_n])$. Then we can rewrite the internal state $\phi([C_L([l_n])])$ as $\phi(I;[\rho_n])=\sum_{I=1}^{\hat{W}}C_I\phi_I([\rho_n])$, where $[C_I]\equiv[C_1,C_2,\cdots,C_{\hat{W}}]$ are the new coefficients and $\phi_I([\rho])$ stands for the internal wave-function corresponding to the number I. Of course, they satisfy the relation $<\phi_I,\phi_{I'}>=\delta_{I,I'}$. The energy expectation value is rewritten by $\Delta E([C_I])=\sum_{I=1}^{\hat{W}}|C_I|^2\epsilon_I$. Using these coefficients, we can write the integrations with respect to $[C_I]$ as follows;

$$\hat{\rho}_c^{E,E'}\equiv\prod_{I=1}^{\hat{W}}\int d^2C_I\mathcal{G}(C_I)\rho_c^{E.E';\Delta E}, \tag{13}$$

where $\int d^2C_I$ stands for the integrals with respect to the real and imaginary parts of C_I and $\mathcal{G}(C_I)$ is the metric function for C_I satisfying the condition $st_{macro}(\prod_{I=1}^{\hat{W}}\int d^2\mathcal{G}(C_I)\sum_{I'=1}^{\hat{W}}|C_{I'}|^2)=1$ so as to derive the normalization condition $st_{macro}\,(Tr(\hat{\rho}_c^{E,E'}))=1$. Since the metric should not depend on the phases of C_I, we take as $\mathcal{G}(C_I)=\frac{1}{2\pi}\mathcal{G}(|C_I|)>0$.

In the density matrix the integrations are written down as follows;

$$\prod_I\int d^2C_L\mathcal{G}(C_I)\sum_{I'}\sum_{I''}C_{I'}C_{I''}^{*}\frac{\hat{N}^E\hat{N}^{E'*}}{\sqrt{u_{cl}^{E,\Delta E}(R)u_{cl}^{E',\Delta E'}(R)}}\times e^{i\frac{1}{\hbar}(W_{cl}^{E,\Delta E}(R)-W_{cl}^{E',\Delta E'}(R))}\phi_{I'}([\rho])\phi_{I''}([\rho])^{*}, \tag{14}$$

where $\Delta E=\sum_{I'}|C_{I'}|^2\epsilon_{I'}$ and $\Delta E'=\sum_{I''}|C_{I''}|^2\epsilon_{I''}$. The diagonal term with $E=E'$ is written as

$$\prod_I\frac{1}{2\pi}\int d^2C_I\mathcal{G}(|C_I|)[\sum_{I'}|C_{I'}|^2|\phi_{I'}|^2+\sum_{I'}\sum_{I''\neq I'}C_{I'}C_{I''}^{*}\phi_{I'}\phi_{I''}^{*}]\frac{|\hat{N}_E|^2}{\sqrt{u_{cl}^{E,\Delta E}(R)u_{cl}^{E,\Delta E'}(R)}}. \tag{15}$$

The second term becomes zero because of the integrations with respect to the phases of $C_I=|C_I|e^{i\theta_I}$ from zero to 2π. Then we can evaluate the diagonal term as

$$\hat{\rho}_c^{E,E}(R,[\rho])=\prod_I\int d|C_I|\mathcal{G}(|C_I|)[\sum_{I'}|C_{I'}|^2|\phi_{I'}|^2]\frac{N_E^2}{u_c^{E,\Delta E}(R)}. \tag{16}$$

Now let us estimate the interference terms. Taking into account that differences with the order $O(\hbar)$ having no contribution in the st_{macro}- operation are allowed in the expression of $\Phi_c^E(R)$ and also the order of the error ΔE is $O(\hbar)$, we can use the following equivalent expression for $W_{cl}^{E,\Delta E}(R)$

in the classical region $0 < st_{macro}(E-V(R)) \in \mathcal{R}_+$; $W_{cl}^{E,\Delta E}(R) \approx_{macro} \int^R \sqrt{2M(E-V(R'))}dR' - \frac{1}{2}\Delta E(|C_I|)\int^R \sqrt{\frac{2M}{E-V(R')}}dR'$. Then we can write the off-diagonal term as

$$\hat{\rho}_c^{E,E'}(R,[\rho]) \approx_{macro} \prod_I \int d^2C_I \mathcal{G}(C_I) \sum_{I'}\sum_{I''} C_{I'}C_{I''}^* \frac{\hat{N}^E \hat{N}^{E'*}}{\sqrt{u_{cl}^E(R)u_{cl}^{E'}(R)}}$$

$$\times e^{-\frac{i}{2}\sum_{I'} I'|C_{I'}|^2\omega(f(E;R)-f(E';R))} e^{i\frac{1}{\hbar}(W_{cl}^E(R)-W_{cl}^{E'}(R))} \phi_{I'}([\rho])\phi_{I''}([\rho])^*, \tag{17}$$

where $u_{cl}^E(R) = \sqrt{2M(E-V(R))}$, $W_{cl}^E(R) = \int^R \sqrt{2M(E-V(R'))}$, $f(E;R) = \int^R \sqrt{\frac{2M}{E-V(R')}}dR'$ and $u_{cl}^{E,\Delta E}(R) \approx_{macro} u_{cl}^E(R)$ are used. As same as the second term of the diagonal elements, the terms with $I' \neq I''$ disappear by the integrations over the phases of C_Is. The remaining terms with $I' = I''$ include the following integrals with respect to $|C_I|$s;

$$\prod_{I=1}^{\hat{W}} \int d|C_I|\mathcal{G}(|C_I|)e^{iIa_I|C_I|^2} \sum_{I'} |C_{I'}|^2, \tag{18}$$

where $a_I = -\frac{1}{2}(f(E;R) - f(E';R))\omega \not\approx 0$ in the classical region. The normalization can be rewritten as $\sum_{I'=1}^{\hat{W}}(<|C_{I'}|^2> \prod_{I\neq I'}^{\hat{W}} <1>) = (<1>^{\hat{W}-1})(\sum_{I'=1}^{\hat{W}} <|C_{I'}|^2>) = 1$, where $<A_I> \equiv \int d|C_I|\mathcal{G}(|C_I|)A_I$. Taking into account that this equation must be satisfied for arbitrary number of $\hat{W} \in \mathcal{N}$, it is reasonable to impose the following relations $\sum_{I'=1}^{\hat{W}} <|C_{I'}|^2> = 1$ and $<1> = 1$. We obtain the relations $q_I \equiv |<e^{iIa_I|C_I|^2}>| < <1> = 1$ and $|\sum_{I'=1}^{\hat{W}} <|C_{I'}|^2 e^{iI'a_{I'}|C_{I'}|^2}>| < \sum_{I'=1}^{\hat{W}} <|C_{I'}|^2> = 1$ because of $\mathcal{G}(|C_I|) > 0$ and $\forall a_I \neq 0$ and $\not\approx 0$ for $^\forall I \in \mathcal{N}$ in the classical region. We estimate the integrations as

$$|\sum_{I'=1}^{\hat{W}}(<|C_{I'}|^2 e^{iI'a_{I'}|C_{I'}|^2}> \prod_{I\neq I'}^{\hat{W}} <e^{iIa_I|C_I|^2}>)| < (q_{max})^{\hat{W}-1} \approx_{macro} 0, \tag{19}$$

where q_{max} denotes the maximum number among q_Is and the last equality is derived from the fact that $q_{max} < 1$ and $\hat{W}$ goes to infinity in the macroscopic limit. ¿From the above result we know that the magnitudes of the off-diagonal terms in $\hat{\rho}_c^{E,E'}$ are infinitesimal and the contributions from the off-diagonal terms are always infinitesimal in the evaluation of expectation values for all operators ($\mathcal{O}$) which are written only in terms of the CM variables. (In details, see Ref.5.)

5. Remarks

There is no space enough to explain the coherent states reproducing the classical trajectories of the CM motions.We may conclude that the quatum states of macroscopic objects are well described in terms of the ultra-eigenfunctions of quantum mechanics on nonstandard spaces.[5]

References

[1] A. Robinson, Non Standard Analysis (North-Holland, Amsterdam, 1970).
[2] M. O. Farrukh, J. Math. Phys. 16 (1975)177.
[3] T. Kobayashi, Proceedings of the Symposium on Symmetries in Science VII, eds. B. Gruber et al. (Plenum, New York, 1993)287 and Soryushiron-Kenkyu 89(1994)211 (in Japanese).
[4] T. Kobayashi, IL Nuovo Cim. 110B(1995)61.
[5] T. Kobayashi, preprint of Unversity of Tsukuba, UTHEP-279 (1994).