CONTINUOUS FEEDBACK
AND MACROSCOPIC COHERENCE

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Abstract

We show that a model, recently introduced for quantum nondemolition measurements of a quantum observable, can be adapted to obtain a measurement scheme which is able to slow down the destruction of macroscopic coherence due to the measurement apparatus.

1 Introduction

One of the most important limitations in the observation of quantum coherence at macroscopic level is the possibility of generating at least to macroscopic quantum states which show the quantum coheh rence. Since the seminal work of Yurke and Stoler [1] it becomes clear that a Kerr medium could be used to generate such states at optical level. They showed, indeed, that the unitary evolution of an initial coherent state, interacting with a Kerr medium with a well defined length, will produce a superposition of coherent states. For instance an initial states $|\alpha\rangle$ will generate the superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle) \tag{1}$$

after an interaction time $t_0 = \pi/(2\Omega)$ where Ω is the strenght of Kerr nonlinearity. At well defined shorter times three or more coherent states could also be generated [1]. This, of course, requires the precise knowledge of the length of the medium (or interaction time). It is also well known, and was shown in great details by Daniel and Milburn [2], that as soon as one takes into account the loss in the Kerr medium the generation of those states is suddenly inhibited. Thus, the best should be to have a Kerr medium with high nonlinearity to loss ratio. Recently [3], quadrature squeezed light was observed in semiconductors at frequencies less than half of band gap, where large ratios of nonlinearity to loss can be obtained [4]. Then, semiconductors could be the best media to generate the superposition of states because of the large ratio of the nonlinear phase shift to the optical losses which in the reported experiment [3] was extimated greater than 100. Furthermore, it has been recently shown [5] that a quasi-superposition of macroscopic states, with interference fringes still present, could be generated in a Kerr medium with the above ratio of 10,

when one uses a squeezed bath to model the loss. In this context it was also shown that a squeezed bath could be realized by a suitable feedback [6]. Moreover, it was also shown [7] that by using a time modulation of the Kerr nonlinearity one could obtain the coherent superposition without the precise knowledge of the length of the medium (or interaction time) by only adjusting the phase of the time modulation. However, even though we could assume that such a macroscopic superposition (or quasi-superposition) has been generated, one should have some experimental apparatus suitable to observe the interference pattern. Yurke and Stoler [1] pointed out that any unavoidable dissipation, introduced by the measurement process, will suddenly destroy the interference fringes which are the signature of the coherent superposition. Kennedy and Walls [8], following a suggestion of Mecozzi and Tombesi [9], showed that a phase-sensitive experimental apparatus, like the one modeled by a squeezed bath, might preserve the macroscopic coherence. In the present paper we will show that such an experimental device could be phisically realized by using an appropriate quantum nondemolition (QND) model, introduced by Alsing, Milburn and Walls [10], when one takes into account the detunings of the coupled modes with respect to the cavity characteristic frequencies.

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2   The Model

We consider a cavity supporting two different modes, with annihilation operators a and b. The two modes are coupled by a nonlinear crystal, so that (in the interaction picture)

$$H_{int} = \hbar\chi X_\xi Y_\varphi \,, \tag{2}$$

where $X_\xi = \left(ae^{i\xi} + a^\dagger e^{-i\xi}\right)/2$ and $Y_\varphi = \left(be^{i\varphi} + b^\dagger e^{-i\varphi}\right)/2$. This interaction could be achieved by, for example, a crystal with a $\chi^{(2)}$ nonlinearity in which two processes driven by classical fields, amplification at the frequency $\omega_s = \omega_a + \omega_b$, and frequency conversion at the frequency $\omega_d = \omega_a - \omega_b$, have equal strengths [10]. Because of the QND condition, when the "meter" mode b is heavily damped at rate k_b, one can monitor the quadrature X_ξ of the signal mode a just by performing a homodyne measurement of a quadrature Y_δ of the mode b. In fact, when $k_b \gg k_a$ (damping rate of the a mode) the homodyne photocurrent $I(t)$ can be directly expressed in terms of the "instantaneous" mean value $\langle X_\xi(t)\rangle_c$, conditioned on the result of the measurement [11, 12], as

$$I(t) = \eta\chi \left[2\sin(\delta - \varphi)\langle X_\xi(t)\rangle_c + \sqrt{\frac{2k_b}{\eta\chi^2}}\xi(t)\right] \,, \tag{3}$$

where η is the efficiency of the homodyne detection and $\xi(t)$ is a Gaussian white noise with $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$.

The QND-mediated feedback model of [6, 11] is obtained by taking part of the output homodyne photocurrent $I(t)$ and feeding it back to the cavity so to add a driving term $H_{fb}(t) = \hbar g I(t)X_\theta$ to the a mode Hamiltonian. The constant g represents the gain of the feedback process and $X_\theta = \left(ae^{i\theta} + a^\dagger e^{-i\theta}\right)/2$. If one adiabatically eliminates the meter mode b and applies the Markovian feedback theory recently developed by Wiseman and Milburn [13], the dynamics of the a

mode can be exactly determined, and in [11] we have shown that in the unstable regime the decoherence time of an optical Schrödinger cat can be appreciably increased, so to facilitate its detection.

In the present paper we reconsider this model and we eliminate the electro-optical feedback loop. We simply detune the two modes in the cavity, so that their uncoupled evolution is no more driven by the standard vacuum bath term alone, but by

$$\mathcal{L}_a\rho = k_a \left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right) - i\left[\delta_a a^\dagger a, \rho\right] \tag{4}$$

and an analogous expression holds for the b mode. The effect of the two nonzero detunings δ_a and δ_b can be intuitively described in terms of an "internal feedback" mechanism, because the detunings mix the two quadratures X_ξ and Y_φ with their respective $\pi/2$ out of phase quadrature, so that any variation of X_ξ is "fed back" to the X_ξ dynamics itself by the joint action of the detunings and the nonlinear coupling. Provided that the adiabatic condition $k_b \gg k_a$ is satisfied, the homodyne measurement of the quadrature Y_δ allows monitoring the a mode quadrature X_ξ also in the presence of nonzero detunings. In fact, when $\delta_b \neq 0$, Eq. (3) generalizes to

$$I(t) = \eta\chi \left\{ \left[\frac{2k_b^2}{k_b^2 + \delta_b^2} \sin(\delta - \varphi) \right. \right. \tag{5}$$
$$\left. \left. - \frac{2k_b\delta_b}{k_b^2 + \delta_b^2} \cos(\delta - \varphi) \right] \langle X_\xi(t)\rangle_c + \sqrt{\frac{2k_b}{\eta\chi^2}}\xi(t) \right\} ,$$

so that from the homodyne photocurrent it is still possible to reconstruct the marginal probability distribution of the quadrature X_ξ, which is the quantity usually considered for revealing the interference fringes associated to an optical Schrödinger cat. We have therefore the model defined by the following master equation for the density matrix D of the two modes

$$\dot{D} = \mathcal{L}_a D + \mathcal{L}_b D - \frac{i}{\hbar}\left[H_{int}, D\right] , \tag{6}$$

where the superoperator $\mathcal{L}_i$ ($i = a, b$) is given by (4). We shall now see that all the interesting results obtained for the feedback model of [11] (the preservation of macroscopic quantum coherence in particular) can also be obtained with this simpler model.

Eq. (6) can be exactly solved, because the Wigner function of the two modes evolves according to the Fokker-Planck equation for a four-dimensional Ornstein-Uhlenbeck process [14]. Anyway, the analytical expressions in the general case are very cumbersome and therefore we shall explicitly discuss only the adiabatic limit $k_b \gg k_a$, where the meter mode b can be adiabatically eliminated, and which, as we have seen above, is the most interesting case for our purposes. After the adiabatic elimination of the b mode, one gets the following master equation for the a mode reduced density matrix ρ

$$\dot{\rho} = \mathcal{L}_a\rho - \frac{\Gamma}{2}\left[X_\xi, [X_\xi, \rho]\right] + iF\left[X_\xi, \{X_\xi, \rho\}\right] , \tag{7}$$

where $\Gamma = \chi^2 k_b / 2(k_b^2 + \delta_b^2)$, $F = \chi^2\delta_b / 4(k_b^2 + \delta_b^2)$.

3 Macroscopic Coherence

We will now focus on the detection of optical Schrödinger cats rather than on their generation, and therefore we shall assume that at $t = 0$ a superposition of coherent states of the a mode has been already prepared, i.e., we consider an initial condition $\rho(0) = \sum_{\alpha,\beta} N_{\alpha,\beta} |\alpha\rangle \langle \beta|$. The exact time evolution from this initial state can be obtained with the same method of [11] and it is better expressed in terms of the normally ordered characteristic function $\chi(\lambda, \lambda^*; t) = \mathrm{Tr}\left\{\rho(t) \exp(\lambda a^\dagger) \exp(-\lambda^* a)\right\}$

$$\chi(\lambda, \lambda^*; t) = \sum_{\alpha,\beta} N_{\alpha,\beta} \langle \beta|\alpha\rangle \exp\left\{ B^*(t)\lambda - A(t)\lambda^* \right. \tag{8}$$

$$\left. -\nu(t)|\lambda|^2 + \frac{\mu(t)}{2}\lambda^{*\,2} + \frac{\mu(t)^*}{2}\lambda^2 \right\},$$

where

$$
\begin{aligned}
A(t) &= \left[\frac{\alpha}{2}\frac{2\Delta - iF + 2i\delta_a}{2\Delta} - \beta^*\frac{iFe^{-2i\xi}}{4\Delta}\right] e^{-(k_a+\Delta)t} \\
&+ \left[\frac{\alpha}{2}\frac{2\Delta + iF - 2i\delta_a}{2\Delta} + \beta^*\frac{iFe^{-2i\xi}}{4\Delta}\right] e^{-(k_a-\Delta)t}
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
B^*(t) &= \left[\frac{\beta^*}{2}\frac{2\Delta + iF - 2i\delta_a}{2\Delta} + \alpha\frac{iFe^{2i\xi}}{4\Delta}\right] e^{-(k_a+\Delta)t} \\
&+ \left[\frac{\beta^*}{2}\frac{2\Delta - iF + 2i\delta_a}{2\Delta} - \alpha\frac{iFe^{2i\xi}}{4\Delta}\right] e^{-(k_a-\Delta)t}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\nu(t) &= \frac{F}{16}\left(\frac{\Gamma\delta_a}{\Delta^2} - \frac{2F}{\Delta}\right)\left(\frac{1 - e^{-2(k_a+\Delta)t}}{2(k_a + \Delta)}\right) \\
&\quad - \frac{\Gamma\delta_a(2\delta_a - F)}{8\Delta^2}\left(\frac{1 - e^{-2k_a t}}{2k_a}\right) \\
&\quad + \frac{F}{16}\left(\frac{\Gamma\delta_a}{\Delta^2} + \frac{2F}{\Delta}\right)\left(\frac{1 - e^{-2(k_a-\Delta)t}}{2(k_a - \Delta)}\right)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\mu(t) &= \frac{F^2 e^{-2i\xi}}{16(2\delta_a - F + 2i\Delta)}\left(\frac{\Gamma\delta_a}{\Delta^2} - \frac{2F}{\Delta}\right)\frac{1 - e^{-2(k_a+\Delta)t}}{2(k_a + \Delta)} \\
&\quad - \frac{F\Gamma e^{-2i\xi}\delta_a}{8\Delta^2}\frac{1 - e^{-2k_a t}}{2k_a} \\
&\quad + \frac{F^2 e^{-2i\xi}}{16(2\delta_a - F - 2i\Delta)}\left(\frac{\Gamma\delta_a}{\Delta^2} + \frac{2F}{\Delta}\right)\frac{1 - e^{-2(k_a-\Delta)t}}{2(k_a - \Delta)}
\end{aligned}
\tag{12}
$$

$$\Delta = \sqrt{\delta_a F - \delta_a^2}\,. \tag{13}$$

We see that the system is stable and reaches a steady state if and only if

$$\Delta < k_a \quad \text{i.e.} \quad \chi^2 \delta_a \delta_b < 4 \left(k_b^2 + \delta_b^2 \right) \left(k_a^2 + \delta_a^2 \right) . \tag{14}$$

In the stable case, the stationary state is described by a Gaussian density operator of the form

$$\rho_{st} = Z^{-1} \exp \left\{ -na^\dagger a - \frac{m}{2} a^{\dagger 2} - \frac{m^*}{2} a^2 \right\} , \tag{15}$$

where Z is a normalization constant and the equilibrium parameters m and n can be written as

$$n = \frac{\nu_\infty + 1/2}{\sqrt{(\nu_\infty + 1/2)^2 - |\mu_\infty|^2}} \tag{16}$$

$$\times \log \left\{ \frac{\left[\sqrt{(\nu_\infty + 1/2)^2 - |\mu_\infty|^2} + 1/2 \right]^2}{\nu_\infty (\nu_\infty + 1) - |\mu_\infty|^2} \right\}$$

$$m = \frac{\mu_\infty}{\nu_\infty + 1/2} n , \tag{17}$$

where the asymptotic values ν_∞ and μ_∞ are easily obtained from (11) and (12). An interesting aspect of this stationary state is that it can show arbitrary quadrature squeezing. For example, the stationary variance of the quadrature X_ξ is given by

$$\left\langle X_\xi^2 \right\rangle = \frac{1}{2} \left[\frac{1}{2} + \frac{\delta_a \left(\Gamma \delta_a + 2F k_a \right)}{8 k_a \left(k_a^2 - \Delta^2 \right)} \right] \tag{18}$$

and one has squeezing when $\delta_a \delta_b < 0$ and $k_b / k_a < |\delta_b / \delta_a|$. It is easily seen that when $\delta_a = 0$ no squeezing is possible, while for $\delta_a \neq 0$ but $\delta_b = 0$ extra noise is added to the system. The possibility to obtain squeezing with this model is thus only due to the existence of detunings, which give a sort of implicit feedback.

4   Interference Fringes

Let us now focus on the detection of the interference fringes associated to a linear superposition of coherent states. These fringes can generally be seen from the marginal probability distribution of the quadrature X_ξ, $P(x_\xi) = \langle x_\xi | \rho(t) | x_\xi \rangle$, where $|x_\xi\rangle$ is the eigenstate of X_ξ with eigenvalue x_ξ. As we have seen above, this probability distribution can be reconstructed from the homodyne measurement of the meter mode b and its general expression can be easily obtained from the characteristic function (8) [8, 11]

$$P(x_\xi, t) = \sum_{\alpha, \beta} N_{\alpha, \beta} \frac{\langle \beta | \alpha \rangle}{\sqrt{\pi \sigma_x^2(t)}} \exp \left\{ -\frac{(x_\xi - \delta_{\alpha, \beta}(t))^2}{\sigma_x^2(t)} \right\} , \tag{19}$$

where

$$\sigma_x^2(t) = \frac{1}{2} + \nu(t) + \text{Re}\left\{\mu(t)e^{2i\xi}\right\} , \tag{20}$$

$$\delta_{\alpha,\beta}(t) = \frac{A(t)e^{i\xi} + B^*(t)e^{-i\xi}}{2} . \tag{21}$$

As a special case we consider the initial superposition trated by Yurke and Stoler [1], produced by the unitary evolution of a coherent state in a Kerr medium

$$\rho(0) \tag{22}$$
$$= \frac{1}{2}\left(e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle\right)\left(e^{i\pi/4}\langle\alpha| + e^{-i\pi/4}\langle-\alpha|\right) .$$

With this choice (19) simplifies to

$$P(x_\xi, t) = \frac{1}{2}\left\{p_+^2(x_\xi, t) + p_-^2(x_\xi, t)\right.$$
$$\left. + 2p_+(x_\xi, t)p_-(x_\xi, t)\sin\left[\Omega(x_\xi, t)\right]|\langle\alpha|-\alpha\rangle|^{\eta(t)}\right\} . \tag{23}$$

The first two terms $p_\pm^2(x_\xi, t)$ describe the two Gaussian peaks corresponding to the two coherent states $|\pm\alpha\rangle$ of the initial superposition and they are explicitely given by

$$p_\pm^2(x_\xi, t) = \frac{1}{\sqrt{\pi\sigma_x^2(t)}}\exp\left\{-\frac{(x_\xi \mp \delta_{++}(t))^2}{\sigma_x^2(t)}\right\} , \tag{24}$$

where

$$\delta_{++}(t) = \text{Re}\left\{\alpha e^{i\xi}r(t)\right\} \tag{25}$$

$$r(t) = e^{-k_a t}\left(\cosh\Delta t - i\frac{\delta_a}{\Delta}\sinh\Delta t\right) . \tag{26}$$

The third term in (23) describes the quantum interference between the two coherent states, where the function

$$\Omega(x_\xi, t) = \frac{2x_\xi}{\sigma_x^2(t)}\text{Im}\left\{\alpha e^{i\xi}r(t)\right\} \tag{27}$$

gives the probability oscillations associated with the interference fringes and the factor $|\langle\alpha|-\alpha\rangle|^{\eta(t)} = \exp\left\{-2|\alpha|^2\eta(t)\right\}$ describes the suppression of quantum coherence due to dissipation. It is clear that this suppression is practically immediate for macroscopically distinguishable states (i.e., large $|\alpha|$), unless $\eta(t) \simeq 0$. It is therefore important to analyze the behavior of this decoherence function $\eta(t)$, which is equal to

$$\eta(t) = 1 - \frac{|r(t)|^2}{2\sigma_x^2(t)} . \tag{28}$$

To be more specific, if we want to determine the conditions under which the detection of macroscopic quantum coherence is facilitated, we have to compare $\eta(t)$ with the corresponding decoherence function of a standard vacuum bath, which is given by [8]

$$\eta_{vac}(t) = 1 - e^{-2k_a t} . \tag{29}$$

This function shows that in the standard case, after a time $t \simeq 1/(2k_a)$, it is $\eta_{vac}(t) \simeq 1$ and therefore the quantum interference is quickly washed out. On the contrary, in the present model it is possible that $\eta(t)$ assumes much smaller values, so to significantly slow down the destruction of the interference pattern.

5 Conclusions

Differently from a very large part of the literature on optical Schrödinger cats, we have focused on their detection rather than their generation because, as realized since the paper by Yurke and Stoler [1], to detect a linear superposition of macroscopically distinguishable states is more difficult than to create it. To the best of our knowledge, only the paper by Brune *et al.* [15] affords a detailed discussion of both aspects. our opinion, Brune *large number of atoms reconstruction of the probability distribution revealing the contrary shows how to prepare a fully optical detection scheme based on a very simple model. offer a promising way to both measurements and detect a linear superposition of coherent states.*

Acknowledgments

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