ON A NEW DETECTION SCHEME FOR M-ARY ORTHOGONAL COHERENT STATE SIGNAL

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Abstract

We propose a new receiver for M-ary orthogonal coherent-state signal. It is shown that the proposed receiver performs better than a photon counting receiver as to signal detection error probability criterion. It is also shown that the error probability of the proposed receiver is almost minimum for the signal.

1 Introduction

Recent development of technology of the optical communication system brought the error probability of the system almost to the standard quantum limit "SQL", which is the classical error performance limit of an optical communication system. It is well known, however, that ultimate error performance limit of optical communication system is far below the SQL[1,2]. In order to overcome the SQL, quantum phenomena of optical signal, which is one of the most remarkable difference from a communication system using radio frequency carrier, have to be utilized in the detection process. There have been several proposals of detection schemes overcoming the SQL for several signaling schemes[3-12].

Optical M-ary orthogonal signal, especially optical M-ary pulse position modulation (PPM), has a great potential for a very low-energy communication in deep-space data transmission. So far, many authors reported the system performance[14,15]. In these investigations, a photon counting receiver has been employed as a detection scheme. Because, its construction is very simple, and it brings good channel property. The error probability of the receiver for the signal in a coherent-state, however, is much larger than the minimum error probability, which is predicted by the quantum detection theory[13,16]. As far as the author knows, there is no proposal for receiver superior to the photon counting receiver for this signal.

The main purpose of this paper is to propose a new detection scheme for an optical M-ary orthogonal coherent-state signal, which is superior to a photon counting receiver. By comparing its error probability with the minimum error probability, it is shown that it performs quasi-optimally.

2 Proposal of a New Receiver

Pulse position modulation (PPM) signaling is one of the typical orthogonal signals. In a PPM signaling, a symbol of time duration T consists of M time slots of duration $T_s(= T/M)$. Each symbol has only one pulse, and then information is transmitted by the position of the pulse. If PPM signaling is employed for the optical communication system, a laser is pulsed at the transmitter during the slot having the pulse. Therefore, the pulsed slot is in a coherent state, and the other $m-1$ slots are in vacuum states. Then, a symbol S_i (for i=1, 2, ..., M) is expressed by

$$S_i : |\psi_i >= \prod_{j=1}^{M} |\mu_{ij} >_j \qquad \mu_{ij} = \begin{cases} {N_s}^{1/2} & j = i \\ 0 & j \neq i \end{cases} \tag{1}$$

where N_s is an average photon number contained in one optical pulse. At the receiver side, a receiver has to decide the position of the pulsed slot among M slots in the symbol. For this purpose, a photon counting receiver has been employed as a detection scheme. In this case, the pulse position is determined by finding the slot with the maximum photocount among them. If the system is under the quantum noise limit, where no external noise exists, photon number fluctuation of an optical pulse may cause a symbol detection error. Because of the Poissonian statistics of photon number of coherent state, photons are never counted during unpulsed slots. However, no photon may be counted during the pulsed slots. This occurs with the probability of e^{-N_s}. In this case, the detector can not determine the pulsed slot. If one of M symbols is selected randomly, an symbol detection error happens with the probability of

$$Pe^{counting} = \frac{(M-1)}{M} e^{-N_s}. \tag{2}$$

On the other hand, the minimum error probability of the M-ary orthogonal coherent-state signal is given by[13,16]

$$\begin{aligned} Pe^{mini} &= \frac{M-1}{M^2} \left\{ [1 + (M-1)e^{-N_s}]^{1/2} - (1 - e^{-N_s})^{1/2} \right\}^2 \\ &\approx \frac{M-1}{4} \exp[-2N_s] \qquad \text{for } \mathrm{N_s} \gg 1. \end{aligned} \tag{3}$$

By comparing this with the error probability of the photon counting receiver, it is found that the exponent of the former is twice as large as the latter. What causes this difference? As shown in the deviation of Eq.(2), there remained no information about which of M signal has been sent when a photocount of the pulsed slot is zero. That is, the photon counting receiver does not examine whether the incoming signal is $|\psi_i >$ or $|\psi_j >$ $(j \neq i)$, but does whether $|\psi_i >$ or $\prod_{j=1}^{M} |0>_j$. In order to examine whether the incoming signal is $|\psi_i >$ or $|\psi_j >$ $(j \neq i)$, and to make its error probability to approach to the minimum error probability, the information that all the $M-1$ unpulsed slots are in vacuum sates as well as that the pulsed slot is in a coherent state should be used for symbol detection.

For this purpose, we propose a new detection scheme. The block diagram of the proposed receiver is shown in FIG.1. The receiver consists of a local laser, a highly transmissive beam splitter, a photon counter, an optical shutter and its feedback control system. Frequency of the local laser is identical to that of signal field, and its phase is shifted by π [rad.] with respect to the signal of pulsed slot. The intensity of the local field is prepared so that its part reflected by the beam splitter is the same as the transmitted part of the signal. Assuming that the transmission coefficient of the beam splitter is nearly equal to unit, the combination process can be considered as displacement process of coherent component. Let α ($|\alpha|^2 = N_s$) be complex amplitude of the pulsed slot, then the conditional quantum state of the combined field is given in Table I.

TABLE.I Conditional quantum state of combined field.

State of shutter	open	close
Pulsed slot	$\|0>$	$\|\alpha>$
Unpulsed slot	$\|-\alpha>$	$\|0>$

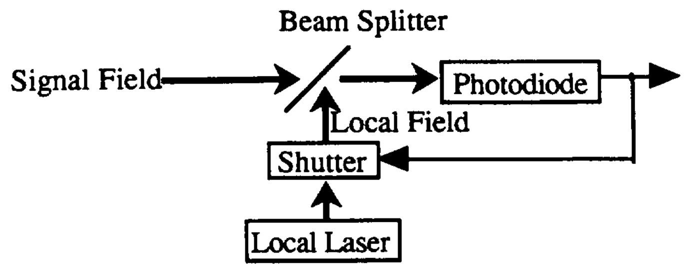


FIG. 1. Block diagram of proposed detection scheme.

Using this construction, the receiver operates in the following way.

1. At the beginning of each symbol, the shutter is *open*.
2. A photon number of combined field is counted during each slot individually.
3. If no photon is counted during a certain, say "ith", slot, the feedback control system switches the shutter into *close* from the next, "$i+1$st", slot till the end of the symbol.

The symbol is decided by the following rules.

1. If the shutter is closed at the ith slot and no other photons are counted after closing the shutter till the end of the symbol, a symbol S_i having an optical pulse at the ith slot, is decided as the transmitted symbol.
2. If some other photons are counted in the ith time-slot after closing the shutter, a symbol S_i is selected.

In the case when S_i is transmitted, if one or more photons are counted during every first i-1 slots, the combined field of the ith slot is in a vacuum state, and then no photons are counted during the slot. Therefore, the shutter is closed from the $i+1$st slot, so that no other photons are detected till the end of the symbol. In this case, S_i is decided as the transmitted symbol by the decision rule 1, and errors never occur. On the other hand, when a symbol S_i is transmitted, if no photon is counted in a certain, say "jth" ($j < i$), time-slot, and the shutter is closed from the j+1st time slot, no photon is counted during from the j+1st to i-1st slots. However, some photon may be counted during the ith slot, whose combined field is in a coherent state $|-\alpha>$. In this case, S_i is also decided correctly as the transmitted symbol by the decision rule 2. If no photons are counted during the ith slot in the previous case, S_j is decided incorrectly, and which causes a symbol detection error. The symbol detection error from S_i to S_j occurs only for $j < i$ with the probability given by

$$P(S_j|S_i) = e^{-2N_s}\left(1 - e^{-N_s}\right)^{(j-1)} \qquad for\ j < i \tag{4}$$

By summing $P(S_j|S_i)$ with respect to j from 1 to i-1, we obtain conditional symbol detection error probability $Pe(S_i)$ as follows:

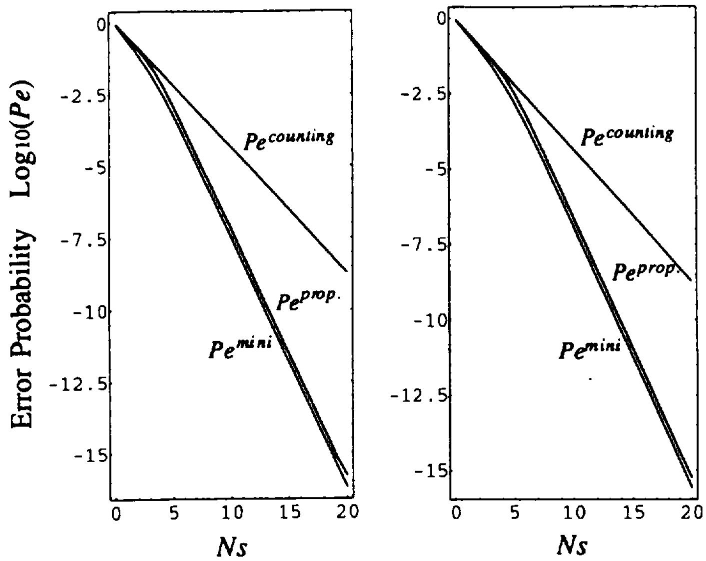
$$\begin{aligned} Pe(S_i) &= \sum_{j=1}^{i-1} P(S_j|S_i) \\ &= e^{-Ns}\{1-(1-e^{-Ns})^{j-1}\} \end{aligned} \tag{5}$$

This is symbol-dependent. Averaging these symbol-dependent error probabilities with respect to *a priori*-probabilities, we obtain average symbol detection error probability. For equally probable signal, an average error probability is given as follows:

$$Pe_{ave.}^{prop.} = \frac{M-1}{M}e^{-Ns} - \frac{1-e^{-Ns}}{\mathrm{M}}\{1-(1-e^{-Ns})^{M-1}\} \tag{6}$$

3 Numerical Results

Symbol detection error probability of the proposed detection scheme is shown as a function of signal energy N_s for symbol lengths M of 64 and 256 in FIGs. 2 (a) and (b), respectively. Those of optimum-quantum receiver, and a photon counting receiver are also shown. It is found in FIG.2 that the proposed scheme is superior to a photon counting receiver on error probability. It is also found that the proposed receiver performs almost optimally. It is easily shown that the error probability of the proposed receiver is approximately only twice as large as the minimum error probability for $N_s \gg 1$. FIG.3 compares the symbol detection error probabilities of the three receivers as a function of block length M for an average photon number N_s of 15. It can be seen from FIG.3 that error probability of the photon counting receiver is almost symbol-length independent, while those of the other two receivers are increasing functions of the length. Though the advantage of the proposed receiver over the photon counting receiver becomes less as the length increases, the proposed receiver is much better than the photon counting receiver for practical use, i.e. $M \leq 1024$. It seems from these results that we can expect the proposed detection scheme to perform ultimately low-energy optical communication.



(a) $M = 64$ (b) $M = 256$

FIG. 2. Symbol detection error probability properties of proposed detection scheme compared with two classical receivers and minimum error probability. (a) is for M=64. (b) is for M=256.

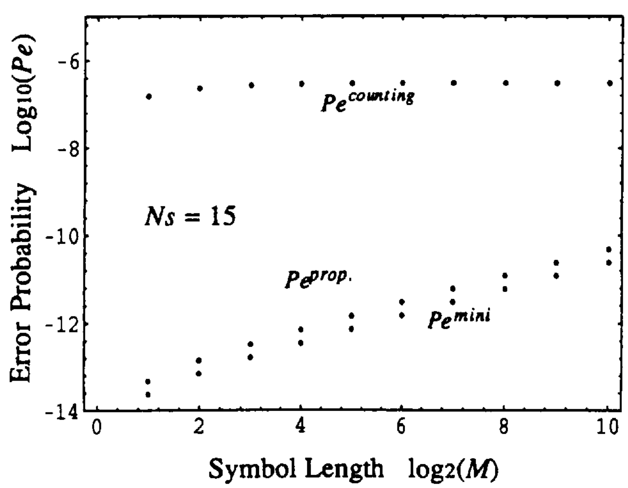


FIG. 3. Error probability dependence on block length M for the case that N_s is 15.

4 Conclusions

In this paper, we proposed a new detection scheme for the M-ary orthogonal coherent-statesignal. The error probability of the scheme was derived. It was shown by comparing its error performance with those of several receivers that the proposed receiver is superior to a photon counting receiver, and it performs almost optimally.

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