

# Theoretical Analysis about Quantum Noise Squeezing of Optical Fields from an Intracavity Frequency-Doubled Laser

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## Abstract

The dependence of the quantum fluctuation of the output fundamental and second-harmonic waves upon cavity configuration has been numerically calculated for the intracavity frequency-doubled laser. The results might provide a direct reference for the design of squeezing system through the second-harmonic-generation.

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## 1 Introduction

The SHG is a highly efficient process for producing the squeezed state light. The generation of squeezed light by SHG in a passive cavity has been studied intensively in theory and experiment<sup>[1-3]</sup>. Some authors have discussed the nonclassical properties of the output fields from an intracavity frequency-doubled laser. Most of them consider an idea laser system<sup>[4-5]</sup>.

For the experimental physicists, it is interesting to analyze the nonclassical properties of the output optical fields from a realistic intracavity frequency-doubled laser system. In this paper, the intensity fluctuations spectra of the fundamental and the SH wave in output fields have been calculated. The dependences of the intensity fluctuations on the configuration of the laser cavity and the losses in the cavity have been discussed. These results will provide a direct reference for the design of squeezer with SHG.

## 2 Fundamental and SH fluctuations spectra

The system we consider here is a single-ended resonator that contains a laser medium and a  $\chi^{(2)}$ -nonlinear crystal. The laser is pumped by a coherent laser source and the fundamental frequency mode ( $\omega_1$ ) and SH frequency mode ( $\omega_2 = 2\omega_1$ ) are coupled by a  $\chi^{(2)}$ -nonlinearity. Based on Lax-Louisell laser theory<sup>[6]</sup> and in the rotating frame, the semiclassical equations of motion for this system are given by:

$$\alpha_1 = (-\gamma_1 - i\Delta_1)\alpha_1 + k\alpha_1\alpha_2 + \frac{g\alpha_1}{1 + b|\alpha_1|^2/g} \quad (1)$$

$$\alpha_2 = (-\gamma_2 - i\Delta_2)\alpha_2 - \frac{1}{2}k\alpha_1^2 \quad (2)$$

where  $\alpha_1, \alpha_2$  is the complex amplitude of fundamental and SH wave,  $\Delta_1 = \omega_1 - \omega_L$  and  $\Delta_2 = \omega_2 - 2\omega_L$  are the detuning between the cavity modes and the lasing transition  $\omega_L$ ,  $\gamma_1, \gamma_2$  are the

cavity damping rates,  $g$  is the pump parameter,  $b$  is the saturation parameter of the laser medium,  $k$  is the nonlinear coupling constant.

$$k = \frac{2\chi^{(2)}}{n^3} \left( \frac{\hbar\omega^3}{V\epsilon_0} \right)^{\frac{1}{2}} \frac{l}{L} \quad (3)$$

The constant  $k$  depends on the nonlinearity of the crystal and the configuration of the cavity.  $V$  is the mode volume,  $l$  is the nonlinear crystal length,  $L$  is the cavity length.

It is useful to introduce the real parameters  $p_j$  and  $q_j$  to describe the real and imaginary parts of the field  $\alpha_j$  respectively.

$$p_j = \frac{1}{2}(\alpha_j + \alpha_j^*) \quad q_j = \frac{1}{2i}(\alpha_j - \alpha_j^*) \quad (4)$$

In the stationary state, the real variables are the solution of the following equations:

$$\frac{k^2 b}{2\gamma_2 g} p_1^4 + \left( \frac{k^2}{2\gamma_2} + \frac{b\gamma_1}{g} \right) p_1^2 + (\gamma_1 - g) = 0 \quad (5)$$

$$p_2 = -\frac{k}{2\gamma_2} p_1^2 \quad (6)$$

whereas the imaginary parts are taken as zero  $q_1 = q_2 = 0$ .

When the pump parameter  $g$  approaches the critical value<sup>[5]</sup>

$$g_c = \frac{1}{4}(2\gamma_1 + \gamma_2) \left( 1 + \left( 1 + \frac{8b\gamma_2^2}{k^2(2\gamma_1 + \gamma_2)} \right)^{\frac{1}{2}} \right) \quad (7)$$

the phase variables  $q_j$  become unstable and the system presents self-sustained oscillation. We are interested in the regime below the threshold of the instabilities, in which the equations of motion (1) and (2) can be linearized around the stationary state given by equations (5) and (6).

At the case of resonance ( $\Delta_j = 0$ ), we obtain expressions for the outgoing amplitude squeezing spectra of the fundamental and SH wave at the analytic frequency  $\Omega$ . When the phase angles equal to zero the optimum squeezing can be obtained. Setting zero as the shot-noise level, we have:

$$S_1(\Omega) = -\frac{1 - R_1}{1 - R_1 + L_1} \frac{8\gamma_{R_1} \left( \frac{k}{2} |\bar{p}_2| - \frac{g}{(1+b|\bar{p}_1|^2/g)^2} \right) (\gamma_2^2 + \Omega^2)}{D} \quad (8)$$

$$S_2(\Omega) = -\frac{1 - R_2}{1 - R_2 + L_2} \frac{8\gamma_{R_2} \left( \frac{k}{2} |\bar{p}_2| - \frac{g}{(1+b|\bar{p}_1|^2/g)^2} \right) k^2 |\bar{p}_1|^2}{D} \quad (9)$$

where

$$D = \left( k^2 |\bar{p}_1|^2 + \frac{2b\gamma_2 |\bar{p}_1|^2}{(1+b|\bar{p}_1|^2/g)^2} - \Omega^2 \right)^2 + \Omega^2 \left( \gamma_2 + \frac{2b|\bar{p}_1|^2}{1+b|\bar{p}_1|^2/g} \right)^2 \quad (10)$$

$R_j$  is the reflectivity of the output coupler at the frequency  $\omega_j$ .  $L_j$  is the rest losses per roundtrip in the resonator that include absorption, scattering and residual transmission through mirrors other than the output coupler.  $\gamma_{R_j}$  is the cavity damping rate which only depends on the output coupler loss ( $1 - R_j$ ),  $\gamma_j$  is total cavity damping rate which depends on total losses ( $1 - R_j + L_j$ ). From the equations, it can be seen that the squeezing increase when the rest losses are decreased.

### 3 Numerical calculation and discussions

Following numerical calculation was processed according to our realistic experimental setup and parameters. The experimental system is shown in Fig.1

Fig.1

A Nd:YAG laser medium and a nonlinear crystal KTP are contained in a semimonolithic laser cavity. One side of Nd:YAG crystal was coated as the input coupler ( $M_1$ ). The length of Nd:YAG and KTP both are 5mm. The input coupler is high reflectivity for both fundamental and SH waves and the output coupler ( $M_2$ ) is high reflectivity for the fundamental wave. Former works [4-5] have indicated that the squeezing increases with pump parameter. Considering  $g < g_c$ , we chose the pump parameter  $g = 10^9 s^{-1}$ , that corresponds to the pump power of 2W in our system, to discuss the dependence of the squeezing on the configuration of the cavity and the reflectivity of the output coupler for SH wave ( $R_2$ ). The saturation parameter  $b$  of laser crystal is  $0.2s^{-1}$ , the rest losses of fundamental wave is 0.5% and the rest losses of SH wave is 1%.

Fig.2

Fig.2 shows that the squeezing degree of the SH wave at zero analytic frequency as a function of the cavity length and  $R_2$ . Here the curvature radius of output coupler is designated as 30mm. It can be seen that for the designated curvature radius we can find an optimum  $R_2$  ( $R_2 = 88\%$ ) and an optimum cavity length ( $L = 25mm$ ) to get the maximum squeezing ( $S_2(0) = -0.21$ ). For a certain  $R_2$  there is a correspondent optimum cavity length which is a near half-concentric configuration.

Fig.3

In Fig.3 the curvature radius of output coupler is taken as 100mm. In this case  $R_2 = 92\%$   $L = 46mm$  should be an optimum option which is a near half-confocal cavity other than above near half-concentric.

Fig.4

Fig.4 is the squeezing spectra of the fundamental wave (1) and the SH wave (2) as a function of analytic frequency  $\Omega$  at the above-mentioned optimum configurations of the cavity. For Fig.4(a)  $\rho = 30mm$ ,  $L = 25mm$  and  $R_2 = 88\%$ ; for Fig.4(b)  $\rho = 100mm$ ,  $L = 46mm$  and  $R_2 = 92\%$ . In this designed system the squeezing of the fundamental wave is much less than SH wave. The squeezing bandwidth in Fig.4(a) is larger than Fig.4(b), so that in the experiment the length of laser cavity should be chosen as short as possible to obtain higher intracavity density of power, larger squeezing bandwidth and more compact configuration.

## 4 Conclusion

We have calculated the dependence of quantum noise squeezing upon the reflectivity of output coupler and the length of cavity in the intracavity- doubled laser. The results might provide some references for designing squeezer with intracavity SHG.

## References

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## Figure Caption

Fig.1 The laser configuration

Fig.2 The dependence of squeezing at  $\Omega = 0$  upon the reflectivity  $R_2$  and cavity length with  $\rho = 30mm$

Fig.3 The dependence of squeezing at  $\Omega = 0$  upon the reflectivity  $R_2$  and cavity length with  $\rho = 100mm$

Fig.4 The squeezing spectra for the fundamental wave (1) and SH wave (2). (a)  $\rho = 30mm$ , (b)  $\rho = 100mm$

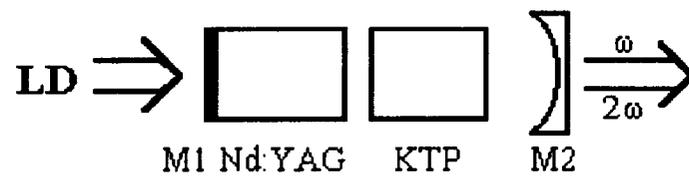


Fig. 1 The laser configuration

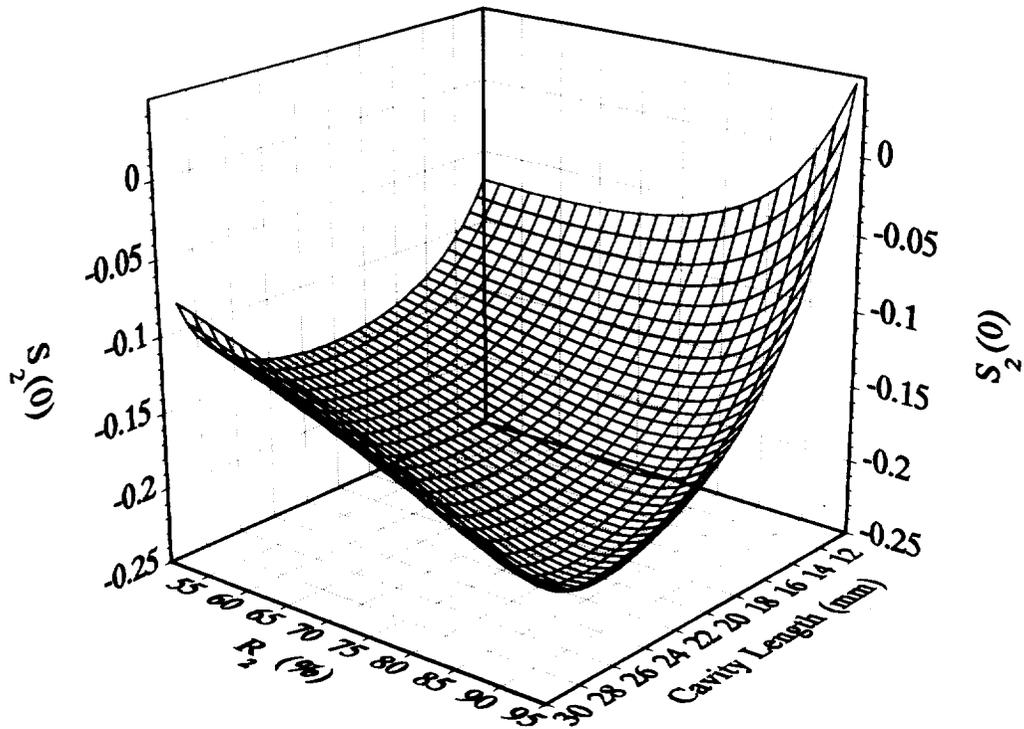


Fig.2 The dependence of squeezing at  $\Omega=0$  upon the reflectivity  $R_2$  and cavity length with  $\rho=30\text{mm}$

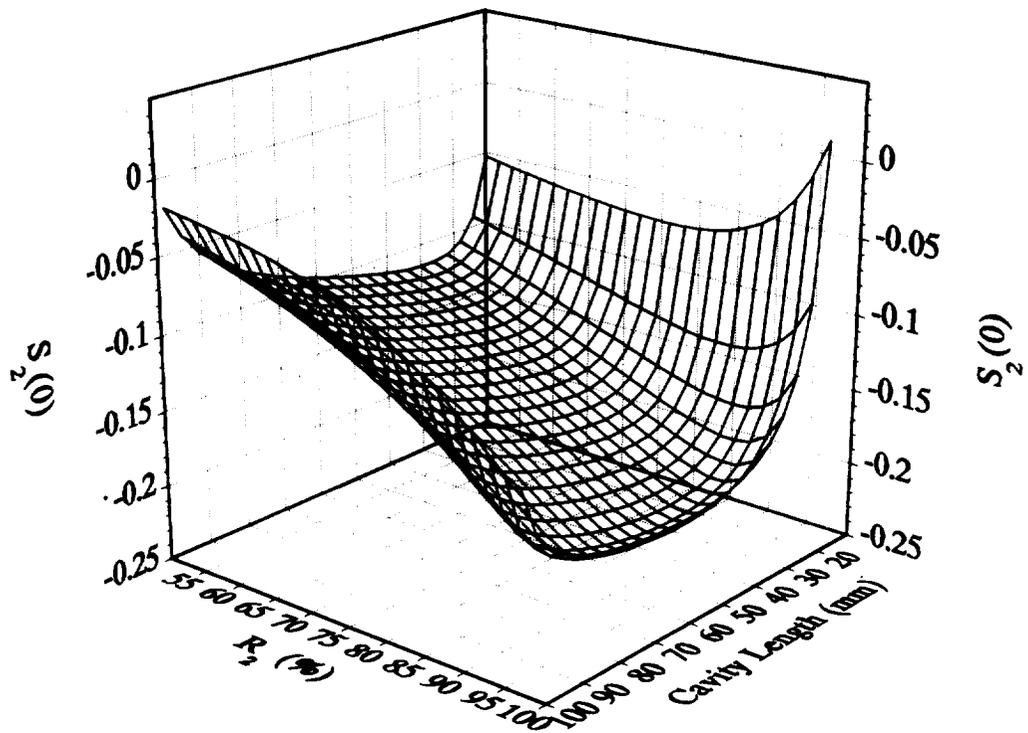


Fig.3 The dependence of squeezing at  $\Omega=0$  upon the reflectivity  $R_2$  and cavity length with  $\rho=100$ mm

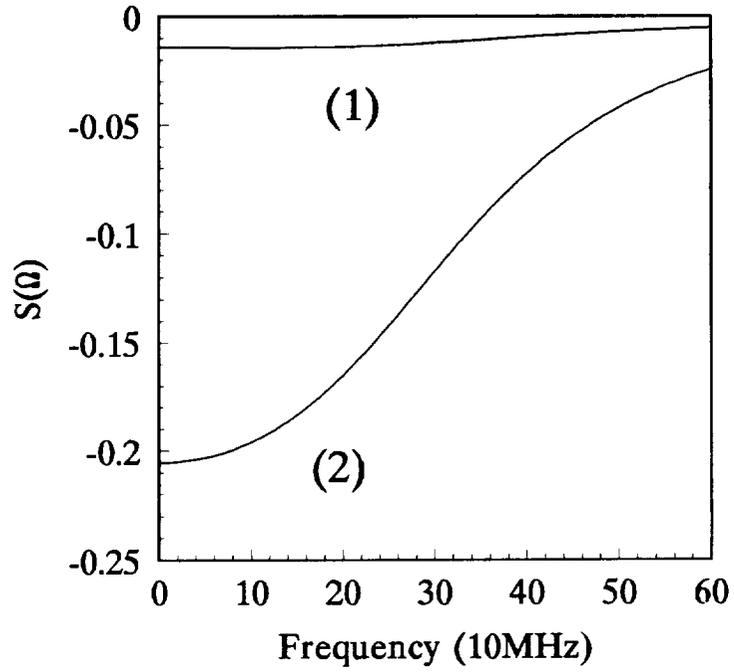


Fig.4(a) The squeezing spectra for the fundamental wave (1) and SH wave (2) ( $\rho=30\text{mm}$ )

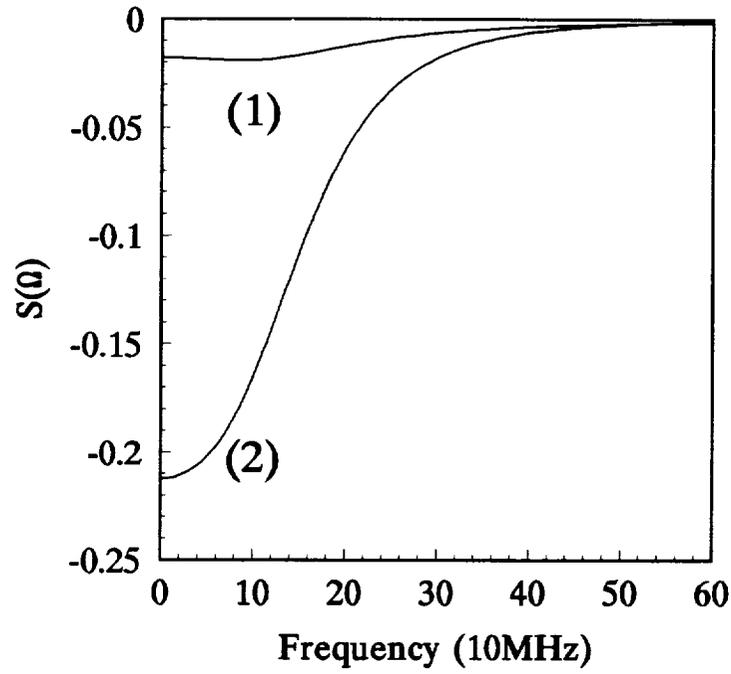


Fig.4(b) The squeezing spectra for the fundamental wave (1) and SH wave (2) ( $\rho=100\text{mm}$ )