Experiments with lasers and frequency doublers

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Abstract

Solid state laser sources, such as diode-pumped Nd:YAG lasers, have given us CW laser light of high power with unprecedented stability and low noise performance. In these lasers most of the technical sources of noise can be eliminated allowing them to be operated close to the theoretical noise limit set by the quantum properties of light. The next step of reducing the noise below the standard limit is known as squeezing. We present experimental progress in generating reliably squeezed light using the process of frequency doubling. We emphasise the long term stability that makes this a truly practical source of squeezed light. Our experimental results match noise spectra calculated with our recently developed models of coupled systems which include the noise generated inside the laser and its interaction with the frequency doubler. We conclude with some observations on evaluating quadrature squeezed states of light.

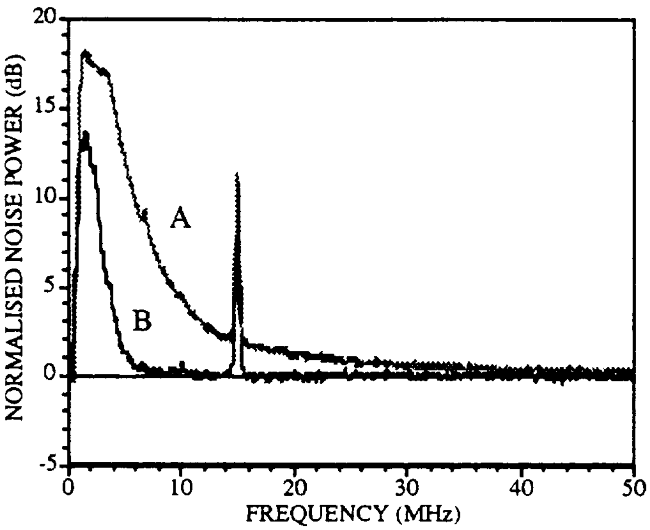
1 Quantum models of coupled systems

Earlier quantum models considered only one system at a time, one resonator or one laser, and predicted the noise properties of such a system in isolation. Using the ideas developed by Gardiner and Carmichael [1] we have developed algorithms which allow us to describe coupled systems. Examples include a laser pumped by another laser, a laser locked to a passive linear or nonlinear resonator (such as a frequency doubler or optical parametric oscillator), or a laser locked to another laser. This new technique is an extremely powerful tool to evaluate the performance of realistic systems, which usually consist of several coupled components, and it was applied to simulate the experiments described in this paper.

2 Removing excess laser noise

It is possible to actively suppress most of the excess technical noise from the laser, including the intrinsic relaxation oscillation, using electro-optic feedback. Such a circuit, with a suitably designed feedback characteristic, will suppress classical fluctuations in the laser light [2] but cannot suppress quantum noise. In fact there is actually a penalty to be paid for the noise suppression: in spectral regions originally free of excess noise, such as well above the relaxation oscillation, the feedback adds classical noise - particularly when the feedback gain is high [3]. Improvements to direct detection feedback can only be made by replacing the beam splitter with a nonlinear optical component, such as a Kerr medium or a frequency doubler [4].

An alternative technique is to passively suppress the noise at higher frequencies by passing the laser through a narrow bandwidth cavity. This arrangement, typically known as a mode cleaner because the cavity improves the spatial properties of the beam, acts as a low pass filter for the laser noise. The impact of the mode cleaner can be see in Figure 1a. Trace A shows the amplitude noise spectrum of the laser used in our experiments, trace B shows the output noise spectrum after a mode cleaner of bandwidth 800 kHz. (The spike is the modulation peak used to lock the mode cleaner). There is a significant improvement, with light reaching the quantum noise limit at 8 MHz, as opposed to beyond 50 MHz.



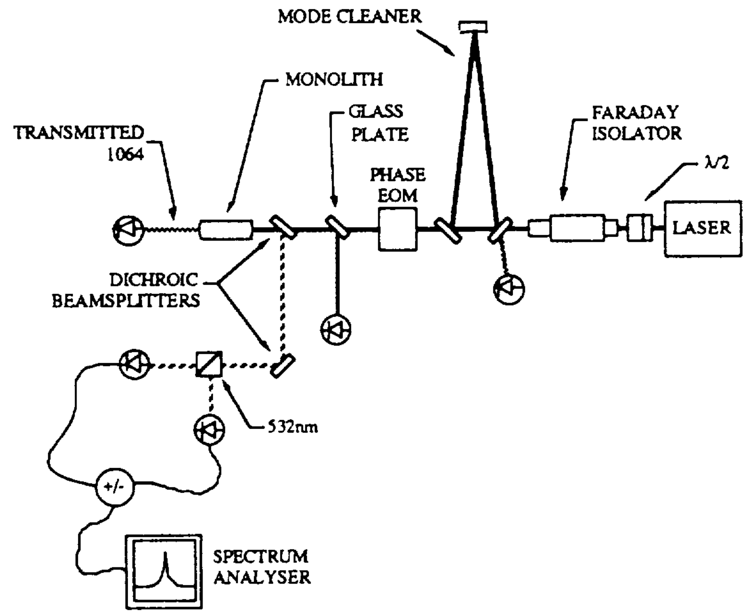


FIG.1a. Intensity noise spectra. A) direct from laser B) after passage through mode cleaner

FIG.1b. Experimental layout for generating squeezed light via frequency doubling

3 Amplitude squeezed light

Having shown that it is possible to remove the technical noise from a practical light source, the question becomes is it possible to produce a practical light source with reduced quantum fluctuations? (In this paper we will concentrate on reduction of amplitude fluctuations.)

This can be done with a diode laser which converts electric current to light with a high quantum efficiency. Currents are a flux of bosons, and thus the Poissonian limit does not apply: standard current regulators generate currents with Poissonian statistics (and thus fluctuations) well below the standard quantum limit. In turn this can be used to drive a laser and generate light with sub Poissonian statistics [5]. However to date, such systems have relatively poor spatial properties and are limited to the red region of the spectrum.

An attractive alternative to diode lasers is to use a nonlinear medium to generate bright, amplitude squeezed light directly. Frequency doubling was one of the first processes which was explored for squeezing [6]. A long sequence of technological improvements was required to improve the reliability of these systems. To date, passive monolithic singly resonant cavities have proved to be by far the most stable systems for noise suppression [7]. In our experiments [8] the doubling material is monolithic MgO:LiNbO$_3$. The end faces are curved, polished and dielectric coated to form high reflectivity cavity mirrors. A diode pumped CW Nd:YAG laser operating at 1064 nm is locked to this resonator and pumps it with $\approx$ 100 mW of power. The doubler has a conversion efficency greater than 50%. The squeezed light, at 532 nm, is picked off with a dichroic mirror and is detected at a balanced pair of detectors (a self-homodyne detector). See Figure 1b.

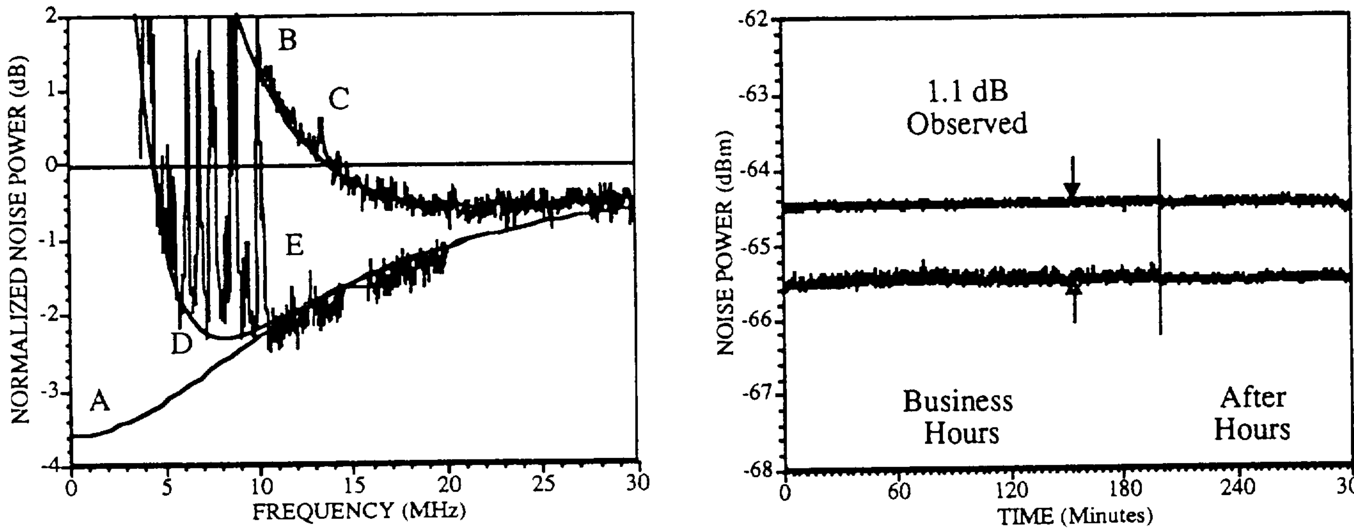


FIG. 2a. Theoretical and experimental noise spectra for doubler.
FIG. 2b. Reliability trace. Degree of squeezing is constant over a 5 hour period.

Figure 2a shows the results of a scan of the detection frequency. Trace A is the predicted squeezing for SHG when illuminated with a coherent state, an example of an unrealistic model based on a single system. It predicts best noise suppression at zero detection frequency. The width of the noise spectrum corresponds to the linewidth of the doubler. Trace B shows the experimental results, after allowing for the nonideal detection efficiency ($\approx$ 65 %). Whilst the agreement at large detection frequencies is reasonable, the prediction of good noise suppression at low frequencies is clearly wrong. The noise properties of the real laser dominate.

Trace C shows the results of a model which simultaneously describes the laser and the doubler: it is in excellent agreement with the measured results. The parameters for the laser model are derived from direct measurement of the laser output, no futher adjustment to the parameters are required when used in the coupled model. To access greater squeezing we placed a mode cleaner between the laser and the doubler and locked it to the laser. The prediction for the coupled system of three cavities (laser, mode cleaner and doubler) is shown in trace D, the corresponding experimental results are shown in trace E (again allowing for nonideal detection efficiency). Both the improvement in squeezing and the agreement between theory and experiment is excellent, apart from the frequency window from 5 to 10 MHz where we see a series of well defined technical noise spikes, most likely due to acoustic resonaces in the doubling crystal. Figure 2b shows the results of the reliability test. Observed squeezing of 1.1 dB (2.2 dB inferred) was measured at 11.16 MHz over a 5 hour period. In all, these results demonstrate the validity of our model for coupled systems and show that bright squeezing greater than 2 dB can be reliably obtained.

4 Evaluating quadrature squeezed states of light

In the previous section we obtained excellent quantitative agreement between theory and experiment. Curiously neither the theoretical model nor the experimental results we used truly quantified the state of the light - information was thrown away. In this section we examine this issue in some detail.

Broadly speaking there are two classes of models, *full* and *linearised*. The starting point for both is the same, the difference arises in the approximations made in the latter to evaluate the effect of the nonlinearity. Either model will give a two dimensional probability or quasiprobability distribution that describes the state of the light. (In the remainder of this discussion we will consider the Q representation and its corresponding Q function.) *Full* models use a full quantum mechanical description, covering both average mean values and fluctuations, to describe the complete state of light. Due to computing and mathematical limitations, these models are mainly used to describe states of low photon number (such as squeezed vacuum). The resulting Q functions may be aysmmetric and may show negative curvature. In *linearised* models, the mean values of the quadratures are evaluated by solving the semiclassical equations. The fluctuations are treated as perturbations, and only terms linear in fluctuations are considered. This allows consideration of high photon states, but limits the model to predicting only symmetrical Q functions. As we will see, the standard measurement taken with a homodyne detector is well matched to the simplified predictions of the linearised theory.

Now consider the experiment. In CW squeezing measurements the experimental signal is the phase dependent noise current from the homodyne detector. This is normally analysed with a spectrum analyser to give the phase and frequency dependent variance of the noise current, $V_{current}(\phi,\omega)$. For an arbitrary state of light no direct and unique mathematical conversion exists between the measured noise variances of the light and the predicted Q function. However, some important features of the Q function can be inferred.

For a coherent state ($\langle\Delta X_1^2\rangle = \langle\Delta X_2^2\rangle$, $\langle\Delta X_1^2\rangle\langle\Delta X_2^2\rangle = 1$) the probability distribution is a symmetric two dimensional Gaussian with a full width half maximum, $\delta X(\phi,\omega) = 1$, centred around the point given by the long term averages $\underline{X1}$, $\underline{X2}$ (where ϕ and ω are the detection angle and frequency respectively). By convention a contour is drawn at the full width half maximum of this probability distribution. For any projection angle the root mean square value of the distribution (i.e. the square root of the variance) is trivially equal to the separation of the contour from the centre of the distribution. The contour is a circle.

Squeezed states are those where the symmetry between X1 and X2 has been broken by some nonlinear process. In other words the fluctuations in X1 and X2 are no longer independent but are correlated. For a mininum uncertainty squeezed state ($\langle\Delta X_1^2\rangle \neq \langle\Delta X_2^2\rangle$, $\langle\Delta X_1^2\rangle\langle\Delta X_2^2\rangle = 1$) the distribution is still a two dimensional Gaussian, but the contour is now an ellipse. Note that such a two dimensional Gaussian function has Gaussian cross section for any angle ϕ. Once an elliptical contour is assumed, which is true for any mininum uncertainty state, measured variance $V_{current}(\phi,\omega)$ and contour $\delta X(\phi,\omega)$ can still be related point by point.

For a squeezed state with excess noise ($\langle\Delta X_1^2\rangle\langle\Delta X_2^2\rangle > 1$) the shape of the Q function can vary significantly from the previous cases. Without specifying the specific squeezing process, no simple assumption about the shape or the symmetry of the contour nor the shape of the various cross sections through the Q function, can be made. The connection between $V_{current}(\phi,\omega)$ and

$\delta X(\phi, \omega)$ is no longer local. The value of $V_{current}(\phi, \omega)$ depends on all parts of the probability distribution. The projection of the entire function, not just one specific cross section at ϕ, must be taken into account when determining the contour points (and thus the Q function shape) from the variance $V_{current}(\phi, \omega)$.

There are three courses in such a situation. To date the most common course has been to simply assume that the Q distribution is Gaussian / the contour is an ellipse. Whilst unsatisfactory, by definition this gives good agreement with the linearised models most often used to describe experiments as they only produce Gaussian distributions / elliptical contours. In fact one can only interpret the variance $V_{current}(\phi, \omega)$ as the limit to the extent of the distribution function in the direction ϕ. The second course then is to obtain a rough idea of the contour for the Q function by taking every value of $V_{current}(\phi, \omega)$ and converting and plotting it as to two tangents with the separation $[V_{current}(\phi, \omega)]^{1/2}$. The actual distribution will lie inside the perimeter bounded by these tangents. The shape and size of the contour can then be estimated from the plot. This can be done easily with typical variance data[9].

The third approach is to measure the Q function directly and was pioneered by the group of Raymer et. al. [10] with pulsed sources of squeezed light. At fixed ϕ, many pulses are recorded and a full histogram of the energy of the pulses is constructed. This gives not only the variance of the fluctuations but the full distribution function at that angle. Using data from various angles the Q function is tomographically reconstructed. Each pulse is a mode of the light and is constructed of a complex mixture of frequencies. In CW squeezing the measured squeezing, and therefore the measured Q function, is highly frequency dependent. (The intracavity squeezing value / probability distribution is for a mode of light - it can be related to the measured extracavity squeezing spectrum of the light field via the input/output formalism of Collett and Gardiner [11]). The analogous experiment is thus to look at only one frequency of the phase dependent noise current from the homodyne detector. This can then be sampled and digitised to build up a histogram of the photocurrent fluctuations. This is repeated for a number of angles and the histograms are then in the tomographical reconstruction. This technique was recently demonstrated successfully to analyse the squeezed vacuum / low photon squeezed light produced by a CW optical parametric amplifer / oscillator [12].

To conclude, mininum uncertainty states are well described by linearised theories, and well evaluated by current measurment techniques. States with excess noise, such as a Kerr squeezed state, cannot be accurately described by a linearised model - interesting (non Gaussian) features are lost. Furthermore, current measurement techniques will also miss these interesting features. New models and experimental techniques are required. Table I summarises the salient points.

TABLE I Summary of Section 4.

	Mininum Uncertainty State	State with excess noise
Theory	Linearised theory / quadratic Hamiltonian gives exact result	Linearised theory is not sufficient. Full quantum theory / higher order higher order Hamiltonian required
Distrib. charac.	$\langle\Delta X_1^2\rangle\langle\Delta X_2^2\rangle = 1$	$\langle\Delta X_1^2\rangle\langle\Delta X_2^2\rangle > 1$
	Q function is a 2 dimensinal Gaussian. Any cross section is gaussian	Q function has an arbitrary shape
	Contour line is an ellipse	Contour shape is arbitrary
	Contour/Q function defined by the two parameters r and ϕ_0	Definition of contour/Q function requires many parameters
	Distance from contour to centre is $\delta X(\phi) = 1/2V(\phi)$	Distance from contour to centre can be greater or smaller than $1/2V(\phi)$
	Conversion of $V(\phi)$ to contour is unique	No unique conversion of $V(\phi)$ to contour
Detect.	Homodyne detector and spectrum analyser gives V_{min} and V_{max} which define contour and Q function uniquely.	Requires tomography to describe Q function. Conditional distribution constructed from homodyne output for a given LO angle. Tomography requires a range of LO anles.
Nonlin. system	Any system with nonlinearity constant across distribution function	System where nonlineariy varies across distribution function (e.g. singularity)

5 Conclusion

Strong squeezing of bright, short wavelength, light has been demonstrated and found to be extremely reliable. We have developed models that describe the behaviour of, and account for the interaction between, the various elements in a realistic system and find excellent agreement with experiment. We conclude that current theory and measurement techniques will need to be extended to properly evaluate the next generation of nonclassical light experiments.

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