Amplitude and transverse quadrature component squeezing of coherent light in high Q cavity by injection of atoms of two-photon transition

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Abstract

The amplitude and transverse quadrature component squeezing of coherent light in high Q cavity by injection of atoms of two-photon transition are studied.

The Golubev-Sokolov master equation and generating function approach are utilized to derive the exact variances of photon number and of transverse quadrature component as function of t. The correlation functions and power spectrums of photon number noise and of output photon current noise are also investigated.

1 Introduction

In this work, the amplitude squeezing as well as the transverse quadrature component squeezing of coherent light in high Q cavity by injection of atoms are investigated. The interaction is assumed to be two-photon transition type and the initial mean photon number N is assumed large.

The interaction interval τ for individual atom is taken the faverable value π/g, where g denotes the effective coupling constant between the atom and the single mode light. This value makes each incoming atom to emit two photons during passing the cavity.

Our approach is based on Golubev-Sokolov master equation[1]. Since this equation was doubted by Benkert and Rzazewski[2] for it may give negative probabilities. We will do some discussion on it first. To our view, even if Golubev-Sokolov equation does not have the meaning as a common differential equation, it is able to give correct mean values, variances and correlation functions of appropriate quantities, when it is utilized along with generating function method. In this work, this approach is not only used to derive photon number variance, the power spectrums of steady photon number noise and of output photon current noise, but also is generalized to study the squeezing of transverse quadrature component.

In the investigation of photon number variance, we find that, in the case that the steady mean value of photon $\overline{n}_s$ is much larger than the initial mean value of photon N, the ratio $\langle\Delta n(t)^2\rangle/\langle n(t)\rangle$ will first drop to a value which is much smaller than its steady state value 1/2, and then turns up to approach 1/2.

In the investigation of squeezing of transverse quadrature component, we get that its variance square is expressed by $\langle n(t)\rangle/4N$, hence the correspondent steady value $\overline{n}_s/4N$ maybe either smaller on larger thatn the standard value 1/4. It is interesting to note, this steady value is related to the initial parameter N. Furthermore it does not depend on whether the injection is regular or poissonian.

2 Model, Golubev-Sokolov master equation and generating function approach

We assume that the initial state of light in the high-Q cavity is single mode coherent light with mean photon number $N \gg 1$. The injected two level atoms are in upper level, they interact with the cavity field by resonant two-photon transition: $\omega_0 = 2\omega$.

The change of density matrix of photon field due to its interaction with a single atom initially in upper level is described by

$$(\hat{u}\rho)_{mn} = \rho_{mn}\cos(g\tau\sqrt{(m+1)(m+2)})\cos(g\tau\sqrt{(n+1)(n+2)}) \\ +\rho_{m-2,n-2}\sin(g\tau\sqrt{m(m-1)})\sin(g\tau\sqrt{n(n-1)}) - \rho_{mn}. \tag{1}$$

For lage N, m and n for important ρ_{mn} are also large, so that $g\tau\sqrt{(m+1)(m+2)}$ may be approximated[3] by $g\tau(m+\frac{1}{2})$, etc. If we take the value of τ as $\frac{\pi}{g}$ then eq.(1) turns out to be

$$(\hat{u}\rho)_{mn} = (-1)^{m-n}\rho_{m-2,n-2} - \rho_{mn} \tag{2}$$

which means each atom emits two photons during passing the cavity, namely the quantum efficiency of photon production equals one.

We assume that the atoms enter the cavity one by one and at most one atom in the cavity every moment. Therefore after injection of k atoms, ρ will change to $(1+\hat{u})^k\rho$.

If the injection is of poissonian statistics with r as mean injection rate, the average number of injected atom during the interval $t \to t+dt$ will equal $K = r\Delta t$. Thus[1]

$$\rho(t+\Delta t) = \sum e^{-K}\frac{K^k}{k!}(1+\hat{u})^k\rho = e^{r\hat{u}\Delta t}\rho(t), \tag{3}$$

leading to

$$\frac{d\rho(t)}{dt}_{pump} = r\hat{u}\rho. \tag{4}$$

This is just the pumping term in the well known Scully-Lamb master quation.

For regular pumping, k itself is a definite number, $k = r\Delta t$ with r denoting the injection rate.Therefore

$$\rho(t+\Delta t) = (1+\hat{u})^{r\Delta t}\rho(t) = e^{r\Delta t ln(1+\hat{u})}\rho(t), \tag{5}$$

which leads to[1]

$$\frac{d\rho(t)}{dt}|_{pump} = r[ln(1+\hat{u})]\rho(t) \tag{6}$$

for regular injection. By adding the cavity damping term, Golubev and Sokolov got the equation

$$\frac{d\rho_{mn}(t)}{dt} = r[ln(1+\hat{u})\rho(t)]_{mn} + \Gamma[-\frac{1}{2}(m+n)\rho_{mn}(t) + \sqrt{(m+1)(n+1)}\rho_{m+1,n+1}(t)], \quad (7)$$

in which Γ denote the cavity damping and the thermal photon is assumed negligable.

Benkert and Rzazewski found[2] that this equation gives negative ρ_{nn} when it is solved by letting $\frac{d}{dt}\rho_{nn} = 0$ to derive the steady values of ρ_{nn}. Let us see where this problem might come from. For regular injection $r\Delta t$ equals k therefore must be larger than 1. Thus Δt cannot be taken as arbitrarily small. This in turn means eq.(7) may not be a differential equation of common sense, one ought to avoid by setting $\frac{d\rho}{dt}$ be zero to get the steady value of ρ. Because of stepwise increase of rt, the strickly steady value of ρ may not exist.

In practice, one usually only needs to calculate the expectation values, varicances or correlation functions of some relavant quantities. In this case it is better to evaluate these values directly rather than through evaluating ρ_{mn} first. Generating function aproach is especially good for this purpose. In this work this approach will be used not noly to study the amplitude squeezing (photon number squeezing) but also generalized to study the squeezing of transverse quadrature component.

3 Photon number squeezing[4]

Golubev and Sokolov, as well as some other authors, expanded the loguritthm $ln(1+\hat{u})$ and trancated at the second order of $\hat{u}$:

$$log(1+\hat{u}) \cong \hat{u} - \frac{1}{2}\hat{u}^2. \quad (8)$$

W.-h.Tan[5] and the present author[6] has shown independently that for evaluating the variance square $\langle \Delta n^2(t) \rangle$, this treatment is correct. The result so obtained is identical to the exact solution, but it is not so for evaluating $\langle \Delta n(t)^3 \rangle$. In general, for calculating of $\langle \Delta n(t)^l \rangle$, one needs to expand $ln(1+\hat{u})$ to l terms to get the correct value[6].

As did in Ref[1], we introduce the generating function for $\langle \Delta n^2(t) \rangle$ as

$$G(z,t) = \sum_{n=0}^{\infty} \rho_{nn}(t) z^n, \qquad z \leq 1. \quad (9)$$

By utilization of eqs.(7) and (8), we get the equation for $G(z,t)$ as

$$\frac{\partial G(z,t)}{\partial t} = -\frac{r}{2}(3-z^2)(1-z^2)G(z,t) + \Gamma(1-z)\frac{\partial G(z,t)}{\partial z}. \quad (10)$$

This is a partial difference equation of first order, its general solution is expressed by one of its special solution mutiplied by the general solution of equation $\frac{\partial G(z,t)}{\partial t} = \Gamma(1-z)\frac{\partial G(z,t)}{\partial z}$. The latter will be determined by the requirement of initial condition. The desired solution so obtained expressed by

$$G(z,t) = G(y,0)e^{f(z,t)}, \quad (11.1)$$

where

$$y = 1 + (z-1)e^{-\Gamma t}, \tag{11.2}$$

$$f(z,t) = \frac{r}{2\Gamma}[3(z-y) + \frac{3}{2}(z^2-y^2) - \frac{1}{3}(z^3-y^3) - \frac{1}{4}(z^4-y^4)]. \tag{11.3}$$

The values of $\langle n(t)\rangle$ and $\langle \Delta n^2(t)\rangle$ are easily obtained from $G(z,t)$:

$$\langle n(t)\rangle = \frac{\partial G(z,t)}{\partial z}|_{z=1} = \frac{2r}{\Gamma} + \left(N - \frac{2r}{\Gamma}\right)e^{-\Gamma t}, \tag{12.1}$$

$$\begin{aligned}\langle \Delta n^2(t)\rangle &= \frac{\partial^2 G(z,t)}{\partial z^2}|_{z=1} + \frac{\partial G(z,t)}{\partial z}|_{z=1} - \left[\frac{\partial G(z,t)}{\partial z}\right]^2_{z=1} \\ &= \frac{r}{\Gamma} + \left(N - \frac{2r}{\Gamma}\right)e^{-\Gamma t} + \frac{r}{\Gamma}e^{-2\Gamma t}.\end{aligned} \tag{12.2}$$

The steady values of $\langle \Delta n(t)\rangle$ and $\langle \Delta n^2(t)\rangle$ exist.By letting $t = \infty$, one gets

$$\langle n\rangle_s = \frac{2r}{\Gamma}, \quad \langle \Delta n^2(t)\rangle_s = \frac{r}{\Gamma}. \tag{13}$$

If we define $\eta(t)$ as $\langle \Delta n^2(t)\rangle / \langle n(t)\rangle$, then its steady value η_s will be $\frac{1}{2}$, the same as one photon-transition subpoissonian lasers.

Eqs.(12) can be checked in the special case of ideal cavity $(\Gamma = 0)$[3].

The $\eta(t)$ defined above has different behavior for $x(\equiv \overline{n_s}/N) > 1$ or < 1. In the latter case $\eta(t)$ drops from its initial value and monotonically tends to the steady value 1/2. In the former case $\eta(t)$ first drops down to a minimum value η_{min} less than 1/2 and then turns up to approach 1/2. For $x \gg 1$, $\eta_{min} \simeq \sqrt{\frac{2}{x}} \ll 1$, therefore the correspondent state may be closed to the photon number eigen state.

The steady state correlation function $g(t)$ defined as following

$$g(t) = tr\left[\rho_s \hat{a}_H^\dagger(0)\hat{a}_H^\dagger(t)\hat{a}_H(t)\hat{a}_H(0)\right], \quad t > 0 \tag{14}$$

can also be evaluated by a generating function $F(z,t)$. $F(z,t)$ satisfies the same differential equation as eq.(10), but has different initial conditions:

$$F(z,0) = \sum_n (n+1)\rho^{(s)}_{n+1,n+1} z^n. \tag{15}$$

The $g(t)$ so attained is

$$g(t) = \frac{\partial F(z,t)}{\partial z}|_{z=1} = \langle n\rangle_s^2 - \frac{1}{2}\langle n\rangle_s e^{-\Gamma t}, \qquad t > 0. \tag{16}$$

The power spectrum of the steady state output photon current noise is related to $g(t)$, in the case that the damping of the cavity field is mainly due to output, its expression will be

$$P_I(\omega) = \Gamma\langle n\rangle_s \frac{\omega^2}{\omega^2 + \Gamma^2}. \tag{17}$$

The correlation function $\langle \Delta n(t_1)\Delta n(t_2)\rangle$ for arbitrary t_1 and t_2 can also be calculated by similar approach[4]. ¿From it we obtain the power spectrum of steady state photonnumber noice as

$$P_n(\omega) = \langle n\rangle_s \frac{\Gamma}{\omega^2 + \Gamma^2}, \tag{18}$$

which mainly lies in low frequency region, in contrast to $P_I(\omega)$ given above.

4 The squeezing of transverse quadrature component.

We are now generalizing the generating function approach to investigate the squeezing of quadrature components of $\hat{a}$.

Let α, the eigen value of $\hat{a}$ for the initial photon state, be real number, then $\hat{a}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$, $\hat{a}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$ will be the longitudinal and tranverse quadrature components respectively.

The mean value of longitudinal quadrature component is given by

$$\langle a_1(t)\rangle = \sum_n \sqrt{n+1}\rho_{n,n+1}(t).$$

In our model $(\hat{u}\rho)_{n,n+1} = -\rho_{n-2,n-1} - \rho_{n,n-1}$, which absolute value is not small as compared with $|\rho_{n,n+1}|$. Actually it is almost twice as large as $|\rho_{n,n+1}|$. And the sign of $[(1+\hat{u})^k\rho]_{n,n+1}$ varies alternately between positive and negative as k varies. Because of these features, the evolution of $\langle \hat{a}_1(t)\rangle$ could not be described by differential equations. The situation of transverse quadrature component is different. In our case $\langle a_2(t)\rangle$ remains to be zero. We may generalize the generating function method to investigate its variance square, which is expressed by

$$\langle \Delta a_2(t)^2\rangle = \frac{1}{4} + \frac{1}{2}\sum_n \rho_{nn}(t) - \frac{1}{2}\sum_n \sqrt{(n+1)(n+2)}\rho_{n,n+2}(t). \tag{19}$$

As before, $\sum_n \sqrt{(n+1)(n+2)}\rho_{n,n+2}(t)$ may be approximated by $\sum_n (n+\frac{3}{2})\rho_{n,n+2}(t)$. Define

$$G_2(z,t) = \sum_n \rho_{n,n+2}(t)z^n, \quad z \le 1, \tag{20}$$

then

$$\sum_n (n+\frac{3}{2})\rho_{n,n+2}(t) = \frac{\partial G_2(z,t)}{\partial z}|_{z=1} + \frac{3}{2}G_2(z,t)|_{z=1}. \tag{21}$$

We see that only first order derivative appears in eq.(21), therefore it is enough[6],[4] to take just one term in the expansion of $ln(1+\hat{u})$. The equation of $G_2(z,t)$ can be derived accordingly, solving it as before, we get $\sum_n (n+\frac{3}{2})\rho_{n,n+2}(t)$, which in turn yields

$$\langle \Delta a_2(t)^2\rangle = \frac{\langle n\rangle_s}{4N} + \frac{1}{4}(1 - \frac{\langle n_s\rangle}{N})e^{-\Gamma t} = \frac{\langle n(t)\rangle}{4N}. \tag{22}$$

This result may also be checked in the special case of ideal cavity. Setting $e^{-\Gamma t} \cong 1 - \Gamma t$ in eq.(22), we get

$$\langle \Delta a_2(t)^2\rangle = \frac{1}{4} + \frac{rt}{2N},$$

which is the same as that given in Ref[3] by a completely different approach.

One may show from eq.(22) that $\langle \Delta a_2(t)^2\rangle - \frac{1}{4}$ may be positive or negative, depending on whether $\langle n_s\rangle/N$ is larger or smaller than one. The steady value of $\langle \Delta a_2(t)^2\rangle$ is given by

$$\langle \Delta a_2(t)^2\rangle_s = \frac{\langle n\rangle_s}{4N}, \tag{23}$$

which may be much larger or much smaller than 1/4. The latter means, in certain sense, deep suppression of phase noise.

It is interesting to note that the stationary value $\langle \Delta a_2(t)^2 \rangle_s$ is still related to the initial parameter N.

It is also interesting to note that $\langle \Delta a_2(t)^2 \rangle$, unlike $\langle \Delta n(t)^2 \rangle$, has no concern with whether the injection is regular or poissonian, since in the above derivation, $ln(1+\hat{u})$ is allowed to be replaced by $\hat{u}$.

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