OPTIMAL SIGNAL FILTRATION IN OPTICAL SENSORS WITH NATURAL SQUEEZING OF VACUUM NOISES

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Abstract

The structure of optimal receiver is discussed for optical sensor measuring a small displacement of probe mass. Due to nonlinear interaction of the field and the mirror a reflected wave is in squeezed state (natural squeezing) two quadratures of which are correlated and therefore one can increase signal-to-noise ratio and overcome the SQL. A measurement procedure realizing such correlation processing of two quadratures is clarified. The required combination of quadratures can be produced via mixing of pump field reflected from the mirror with local oscillator phase modulated field in dual-detector homodyne scheme. Such measurement procedure could be useful not only for resonant bar gravitational detector but for laser longbase interferometric detectors as well.

Measurement of small gravitational force acting on high quality mechanical oscillator is of great importance in modern physics. However in usual measurement scheme there is a Standard Quantum Limit (SQL) on force resolution [1] :

$$F_0 \geq F_Q = (2/\hat{\tau})(M\hbar\omega_\mu)^{1/2}, \tag{1}$$

where F_0 is force amplitude and we suppose that gravitational signal has the following form

$$F_s(t) = F_0 \sin\omega_0 t, 0 \leq t \leq \hat{\tau}, \tag{2}$$

ω_μ and M are resonant frequency and mass of probe oscillator $(\omega_\mu \approx \omega_0)$.

There are measurement procedures which allow to achieve the sensitivity larger than SQL [1, 2, 3] . For example in scheme with external squeezing of vacuum noises [2, 3] one can achieve the sensitivity

$$F_0 \geq g^{-1} F_Q,$$

where g is squeezing coefficient. In this contribution a scheme with internal (natural) squeezing is proposed for optical sensor.

Optical sensor in the most simple modification is a mirror attached to mechanical resonator and illuminated with coherent pump field. Usual schemes measuring the phase of reflected pump field (i.e. one quadrature component) have the limit of resolution of force acting on mechanical system according to (1). On the other hand reflected wave is in squeezed state (natural squeezing) due to nonlinear interaction of the field and the mirror and a coefficient of squeezing could be large

[4] . For such system two quadratures of reflected wave are correlated. One quadrature consists of the signal (gravitational displacement of mechanical oscillator) and the noise and the other only of the noise. Using correlation of noises in two quadratures one can increase signal-to-noise ratio and overcome the SQL. A photodetector in this scheme must measure linear combination of quadratures of reflected wave with weight functions rather than the phase of reflected wave. Such combination of quadratures can be produced via mixing of reflected wave with phase modulated local oscillator field.

The similar scheme was analized earlier in [5, 6] . However the modulation of local oscillator does not depend there on gravitational signal form therefore the optimal filtration procedure cannot be realized.

Let the reflection coefficient of the mirror be $r \approx 1$. The incident and reflected fields one can take in the following form:

$$\begin{aligned} E_1(x,t) &= Re(A(x,t)\exp\{j(\omega_0 t - kx)\}) \\ E_2(x,t) &= Re(B(x,t)\exp\{j(\omega_0 t + kx)\}) \end{aligned} \tag{3}$$

where $A(x,t)$ and $B(x,t)$ are complex amplitude operators of the fields ($k = (\omega_0/c)$. Let for simplicity

$$A(0,t) = A_0 + a(t) - jb(t) \tag{4}$$

where $A_0 = (2P_0 z_0/S)^{1/2}$ - amplitude of the pump with power $P_0, z_0 = 120\pi$ [ohm] - resistance of free space, S - cross section of pump beam, $a(t)$ and $b(t)$ - operators of field quadrature components with correlation matrix (we assume this field in the vacuum state)

$$\begin{aligned} < a(t)a(t+\tau) > &= < b(t)b(t+\tau) >= N_0\delta(\tau), \\ < a(t)b(t+\tau) > &= 0 \\ N_0 = \hbar\omega_0 z_0/(2S) & \end{aligned} \tag{5}$$

In linear approximation ($| kx | \ll 1, | a(t), b(t) | \ll A_0$) one can obtain (constant term is omitted)

$$\tilde{B}(0,t) = \tilde{B}_1(t) + j\tilde{B}_2(t) \tag{6}$$

$$\tilde{B}_1(t) = -a(t)$$

$$\tilde{B}_2(t) = b(t) + 2A_0 k G(p)(F + \lambda a) \tag{7}$$

where $G(p) = [\, M(p^2 + 2\alpha p + \omega_\mu^2)\,]^{-1}$ - mechanical oscillator transfer function, $p = d/(dt), 2\alpha = H/M, \omega_\mu^2 = K/M; \lambda = SA_0/(z_0 c), M, K$ and H are dynamical parameters of mechanical oscillator. Force $F(t)$ acting on mechanical oscillator have the following form

$$F(t) = F_s(t) + F_\mu(t), \tag{8}$$

where $F_s(t)$ is signal force and $F_\mu(t)$ is zero mean white Gaussian process with covariance function

$$< F_\mu(t)F_\mu(t+\tau) >= N_\mu\delta(\tau), \quad (9)$$

The process $F_\mu(t)$ corresponds to zero temperature thermal noise of mechanical oscillator or any other white noise.

Using eqs. (6,7) one can obtain spectral density matrix for quadrature components $\tilde{B}_1$ and $\tilde{B}_2$:

$$\begin{aligned} W_{11}(\omega) &= N_0, \qquad W_{22}(\omega) = N_0 + (2A_0k)^2(\mid G(j\omega)\mid)^2(N_\mu + \lambda^2 N_0) \\ W_{12}(j\omega) &= -N_0(2A_0k)\lambda G(j\omega), \qquad W_{12}(j\omega) = W_{21}^*(j\omega) \end{aligned} \quad (10)$$

From eq. (10) one can see that the noises in two quadrature components $\tilde{B}_1$ и $\tilde{B}_2$ have nonzero correlation $W_{12}(j\omega)$.

Optimal receiver on the base of the vector output signal $[\tilde{B}_1(t), \tilde{B}_2(t)]^T$ must construct a following functional [7] (time of observation $0 < t < T$)

$$Z = C_0\int_0^T [\,\tilde{B}_1(t)\rho_1(t) + \tilde{B}_2(t)\rho_2(t)\,]\,dt \quad (11)$$

where C_0 is arbitrary scale coefficient, $\rho_1(t)$ and $\rho_2(t)$ are the reference signals defined by the following system of integral equations

$$\begin{aligned} \int_0^T [\,K_{11}(t-\tau)\rho_1(\tau) + K_{12}(t-\tau)\rho_2(\tau)\,]\,d\tau &= 0 \\ \int_0^T [\,K_{21}(t-\tau)\rho_1(\tau) + K_{22}(t-\tau)\rho_2(\tau)\,]\,d\tau &= < \tilde{B}_2(t) >, \end{aligned} \quad (12)$$

where $K_{mn}(t-\tau) = < \tilde{B}_m(t)\tilde{B}_n(\tau) >|_{F_s=0}$ - are correlation matrix elements of time stationary noises in quadrature components $\tilde{B}_1$ and $\tilde{B}_2, m, n = 1, 2$.

Using a transformation of variables in eq. (11)

$$\rho_1(t) = Q(t)\cos\varphi(t), \qquad \rho_2(t) = Q(t)\sin\varphi(t) \quad (13)$$

one can obtain

$$Z = C_0\int_0^T Q(t)\Theta_0(t)dt = C_0\int_0^T \Theta(t)dt \quad (14)$$

where

$$\Theta_0(t) = [\,\tilde{B}_1(t)\cos\varphi(t) + \tilde{B}_2(t)\sin\varphi(t)\,], \qquad \Theta(t) = Q(t)\Theta_0(t) \quad (15)$$

Let a field of local oscillator be $E(t) = E_0(t)\cos(\omega_0 t + \varphi_0(t))$. Then for a photocurrent $I_{ph}(t)$ in dual detector scheme one can obtain for $\varphi_0(t) = \varphi(t)$ [8]

$$I_{ph}(t) \propto Re\,[\,\tilde{B}(0,t)E^*(t)\,] = E_0(t)\Theta_0(t) \quad (16)$$

This photocurrent acts on an input of the optimal receiver.

From eqs. (14)-(16) one can consider that for local oscillator field without amplitude modulation $E_0(t) = const$ the optimal receiver can be realized either in the form of correlation receiver with reference signal $\propto Q(t)$ or in the form of concordant filter with transfer function [7]

$$G_{opt}(t) = K_0 Q(t - t_0),$$

where K_0 is arbitrary scale coefficient, t_0 is time of observation.

For local oscillator field with amplitude modulation $E_0(t) \propto Q(t)$ the optimal receiver can be realized in the form of ideal integrator (cf. (14)-(16)).

The calculation of reference signals $\rho_1(t)$ and $\rho_2(t)$ considerably simplifies in the assumption of unlimited time of observation. For $T \to \infty$ one can obtain from eq. (12) the reference signals $\rho_{i\omega}(j\omega), i = 1, 2$:

$$\begin{aligned} \rho_{1\omega}(j\omega) &= -[W_{12}(j\omega)/\Delta(\omega)]Y_\omega(j\omega) \\ \rho_{2\omega}(j\omega) &= [W_{11}(\omega)/\Delta(\omega)]Y_\omega(j\omega), \end{aligned} \tag{17}$$

where

$$\begin{aligned} \Delta(\omega) &= W_{11}(\omega)W_{22}(\omega) - \mid W_{12}(j\omega) \mid^2 \\ Y_\omega(j\omega) &= 2A_0 k G(j\omega) F_{s\omega}(j\omega) \end{aligned} \tag{18}$$

and $W_{mn}(j\omega)$ is the spectral density matrix corresponding to $K_{mn}(\tau)$ (cf. (10), (12)),$F_{s\omega}(j\omega)$ is the spectrum of the signal $F_s(t)$.

Taking into account eqs. (13), (17) one can obtain optimal modulation functions for the scheme

$$\begin{aligned} Q(t) &= [\rho_1^2(t) + \rho_2^2(t)]^{1/2} \qquad \varphi(t) = arctg\,[\rho_2(t)/\rho_1(t)] \\ \rho_1(t) &= -(2\pi)^{-1}\int_{-\infty}^{\infty} W_{12}(j\omega)Y_\omega(j\omega)\exp\{j\omega t\}d\omega/\Delta(\omega) \\ \rho_2(t) &= (2\pi)^{-1}\int_{-\infty}^{\infty} W_{11}(\omega)Y_\omega(j\omega)\exp\{j\omega t\}d\omega/\Delta(\omega) \end{aligned} \tag{19}$$

and $\rho_1(t), \rho_2(t)$ depend on the signal form according to eq. (19).

The signal-to-noise ratio [7] for the measurement of Z (cf. eq. (14)) in the assumption of unlimited time of observation one can obtain from eqs. (17), (18)

$$\begin{aligned} S/N &= (2\pi)^{-1}\int_{-\infty}^{\infty} \mid Y_\omega(j\omega) \mid^2 W_1(\omega) d\omega/\Delta(\omega) \\ &= (2\pi)^{-1}\int_{-\infty}^{\infty} \mid F_{s\omega}(j\omega) \mid^2 \left[N_\mu + \kappa|G(j\omega) \mid^{-2}\right]^{-1} d\omega \end{aligned} \tag{20}$$

where $\kappa = N_0(2A_0k)^{-2} \propto 1/P_0$. When the pump power increases the influence of the noises due to vacuum fluctuations decreases and the sensitivity (20) approachs the limit which depends only on the dissipation in mechanical system.

It is worth mentioning that this scheme is useful not only for resonant mechanical oscillator but for longbase laser interferometric detector as well. In this case the transfer function $G(p)$ in eq. (6) must be specified for free masses gravitational antenna and the noise $F_\mu(t)$ in eqs. (7), (8) will be a thermal noise of free masses suspension.

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