Motion of Cesium Atoms in the One-Dimensional Magneto-Optical Trap*

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Abstract

We calculated the force to which Cs atoms are subjected in the one-dimensional magneto-optical trap (1D-MOT) and properties of this force are also discussed. Several methods to increase the number of Cs atoms in the 1D-MOT are presented on the basis of the analysis of the capture and escape of Cs atoms in the 1D-MOT.

PACs: 3280P; 3270N; 4250

1. INTRODUCTION

Laser cooling and trapping of neutral atoms is one of the most active research fields in physics in this decade . Several kinds of neutral atom traps have been achieved experimentally [1] ,one of which is the magneto-optical trap(MOT) whose basic principles were discussed in detail in Refs.[2] and [3]. The MOT is formed in the intersection of three orthogonal pairs of counter propagating laser beams with opposite circular polarization. A pair of anti-Helmholtz coils is used to generate an inhomogeneous magnetic field. whose zero point coincides with the center of the intersection region where a stable potential well is formed. Atoms confined in this volume will experience a damping force and a restoring force. MOT has become almost a standard technique for obtaining large number of cold atoms due to its large depth, large trap size and long trap lifetime. A lot of theoretical and experimental work has been carried out since Raab et al.[2] first achieved MOT in 1987. In 1990, Wieman et al.[4] built a MOT into which for the first time Cs atoms were loaded directly from low pressure Cs vapor in a quartz cell and this greatly promoted the research on MOT. At present about 10^{10} atoms can be trapped in MOT with a density of 10^{11} atoms/cm^3 [5] and temperatures below the Doppler limit [6] . In recent years, the research of MOT has been concentrated on two aspects: (1)investigating the properties of MOT with the aim of increasing the number and density of trapped atoms and lowering the temperature so as to optimize the performance of MOT as a source of cold atoms; (2)using MOT to carry out some fundamental or applied research work such as atomic fountain[7] , cold atoms collision[8] , atomic interferometry[9] and Bose-Einstein condensation, etc.

A simple case of a fictitious atom with a $J_g = 0 \rightarrow J_e = 1$ transition in the 1D-MOT

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In this paper we use a simple method to deal with this problem and have got some valuable results.

In section 2 we calculate the force as well as the capture and escape of Cs atoms in 1D-MOT ; in section 3 we continue by analyzing the features of the force and we present several measures to increase the number of Cs atoms in MOT; finally in section 4 we give a conclusion and some comments on our future work.

2. CALCULATION OF THE FORCE AND THE MOTION OF Cs ATOMS

A. Calculation of the force

The force that an atom experiences in a laser field is[10]

$$F = -\langle \frac{dV}{dx} \rangle \tag{1}$$

where < > represents the mean value, $V = \vec{D} \cdot \vec{E}$ is given by the interaction Hamiltonian between the induced electric dipole $\vec{D}$ of the atom and the electric field vector $\vec{E}$ of the laser. The authors of Ref.[10] analyzed the force of a fictitious atom with $J_g = 1 \rightarrow J_e = 2$ transition under $\sigma^+ - \sigma^-$ configuration. The transition used for cooling and trapping of Cs atoms is $6S_{1/2}, F_g = 4 \rightarrow 6P_{3/2}, F_e = 5$ and the schematic diagram of the corresponding energy levels is shown in Fig. 1[11] .

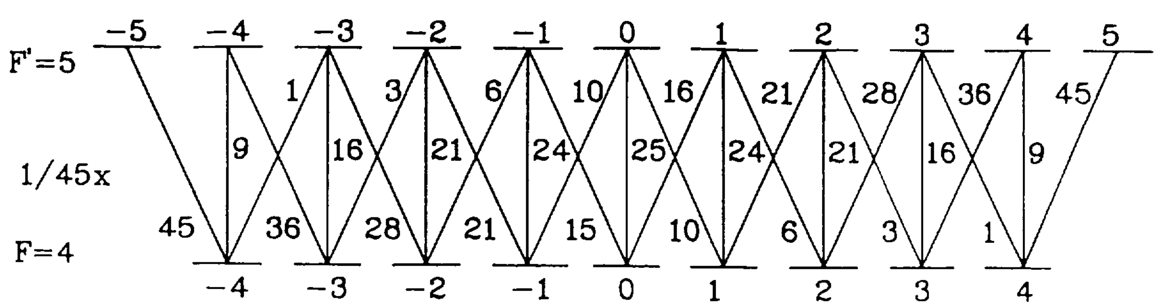


Fig. 1 Energy levels of Cs $6S_{1/2}, F_g = 4 \rightarrow 6P_{3/2}, F_e = 5$

There are several kinds of mechanisms which contribute to the total force exerted on Cs atoms in the case of complicated energy level structure . The most important two are , namely , the scattering force and the polarization gradient force[1] . By extending the result of Ref.[10] to the case $F_g \rightarrow F_e = F_g + 1$ (the contribution of the excited-state population is taken into account in the calculation) , we have

$$f = f_{scatter} + f_{grad} \tag{2}$$

$$f_{scatter} = \frac{\hbar k \Gamma}{2} \sum_{m=-F_g}^{F_g} [(\rho_m^{(g)} - \rho_{m+1}^{(e)}) S_{m,m+1} (C_m^{m+1})^2$$

$$-(\rho_m^{(g)} - \rho_{m-1}^{(e)}) S_{m,m-1} (C_m^{m-1})^2] \tag{3}$$

$$f_{grad} = \frac{\hbar k \Gamma}{2} \sum_{m=0}^{F_g} C_m^{m-1} C_{m-2}^{m-1} [(S_{m-2,m-1} - S_{m,m-1}) \mathrm{Re}(\tilde{\rho}_{m,m-2})$$

$$-\frac{2}{\Gamma}(\delta_{m-2,m-1} S_{m-2,m-1} + \delta_{m,m-1} S_{m,m-1}) \mathrm{Im}(\tilde{\rho}_{m,m-2}) \tag{4}$$

where $\rho_m^{(g)} = < F_g, m_g | \rho | F_g, m_g >, \rho_m^{(e)} = < F_e, m_e | \rho | F_e, m_e >$ are diagonal elements of the density matrix, $S_{m,m\pm1} = \dfrac{\Omega^2 / 2}{\delta_{m,m\pm1}^2 + \Gamma^2 / 4}$ is the saturation factor, $\Omega = 2dE\hbar^{-1}$ is the Rabbi frequency, $(C_{m_g}^{m_e})^2$ is corresponding normalized transition probabilities shown in Fig.2, $_{m,m\pm1} = \delta \mp kV + [mg_g - (m \pm 1)g_e]\mu_B X \dfrac{dB}{dx}$ is the relative frequency detaining, is the frequency detaining, k is the optical wave vector, v is the atomic velocity, μ_B is the Bohr magneton, g_g and g_e are the g factors of the ground state and the excited state, respectively. , and $\tilde{\rho}_{m,m-2} = \langle F_g, m | \rho | F_g, m-2 \rangle \exp(-2ikVt)$ represents the coherence between sublevels of the ground state.

In formula (3) , $\frac{\Gamma}{2} \sum_{m=-F_g}^{F_g} (\rho_m^{(g)} - \rho_{m+1}^{(e)}) S_{m,m+1} (C_m^{m+1})^2$ represents the scattering rate for σ^+ photons by spontaneous emission and $\frac{\Gamma}{2} \sum_{m=-F_g}^{F_g} (\rho_m^{(g)} - \rho_{m-1}^{(e)}) S_{m,m-1} (C_m^{m-1})^2$ is that for σ^- photons. The atom will obtain a momentum $\hbar k$ or $-\hbar k$ if it scatters a σ^+ photon or a σ^- photon , respectively . The net effect is that the atom will experience a scattering force $f_{scatter}$.

From formula (4) , we know that f_{grad} is due to the off-diagonal element $\rho_{m,m-2}$, that is , the coherence between sublevels of the ground state. Fig.2 shows the stimulated absorption-stimulated emission process which contributes to the coherence under σ^+ -σ^- configuration.

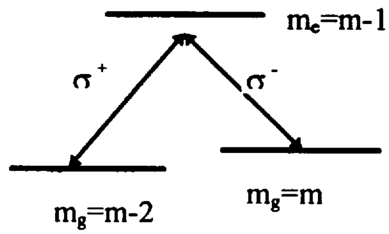


Fig.2 Stimulated absorption-stimulated emission process between sublevels of the ground state of Cs under σ^+-σ^- configuration.

The authors of Ref.[10] also pointed out that f_{grad} would contribute dominantly to the force experienced by an atom only when the condition $(\delta_{m_g,m_g\pm1} - \delta) < \Gamma'$ (Γ' refers to the optical pumping rate. The condition can also be written as $\Delta V=\Gamma'/k$, $\Delta X=\hbar\Gamma'(g_e\ \mu_B\ dB/dx)^{-1}$) is satisfied. We may neglect the effect of f_{grad} , if this condition is not fulfilled. Taking the Cs atom for example, with Γ'<0.5MHz under the condition δ= -10 MHz , I = 4mW/cm^2 , dB/dx = 1mT/cm , g_g ($6S_{1/2}$, F = 4) = 0.25 , g_e ($6P_{3/2}$, F' = 5) = 0.4 , the velocity range is ΔV<0.45m/s and the spatial range is ΔX<1mm. Such a small interaction range makes f_{grad} negligible and we can keep only the term of $f_{scatter}$ when discussing the process of capture and escape of Cs atoms.

It can be seen from formula (3) that $f_{scatter}$ only relates to the diagonal elements of density matrix so we can use rate equations to calculate it. Taking into account the effect of spontaneous emission, stimulated emission and stimulated absorption, we have the following rate equations[11]

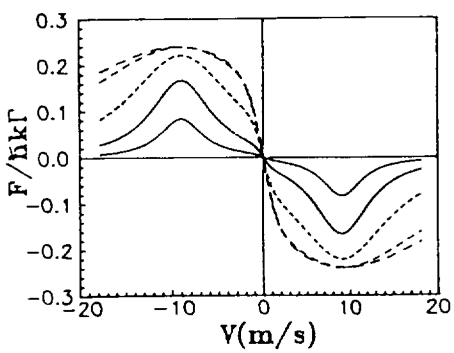
$$\dot{\rho}_m^{(g)} = -\frac{\Gamma}{2}[S_{m,m+1}(C_m^{m+1})^2(\rho_m^{(g)} - \rho_{m+1}^{(e)}) + S_{m,m-1}(C_m^{m1})^2(\rho_m^{(g)} - \rho_{m-1}^{(e)})] + \Gamma\sum_{q=-1}^{1}(C_m^{m+q})^2\rho_{m+q}^{(e)} \tag{5}$$

$$\dot{\rho}_m^{(e)} = \frac{\Gamma}{2}[S_{m-1,m}(C_{m-1}^{m})^2(\rho_{m-1}^{(g)} - \rho_m^{(e)}) + S_{m+1,m}(C_{m+1}^{m})^2(\rho_{m+1}^{(g)} - \rho_m^{(e)})] - \Gamma\rho_m^{(e)} \tag{6}$$

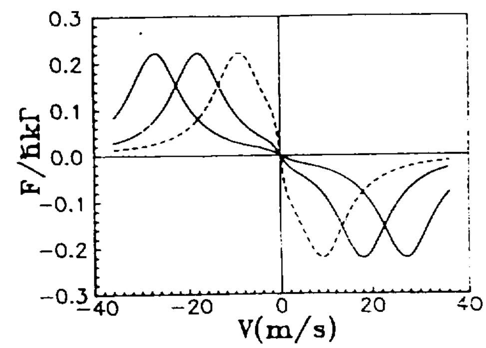
and the normalization condition

$$\sum_{m_e}\rho_{m_e}^{(e)} + \sum_{m_g}\rho_{m_g}^{(g)} = 1 \tag{7}$$

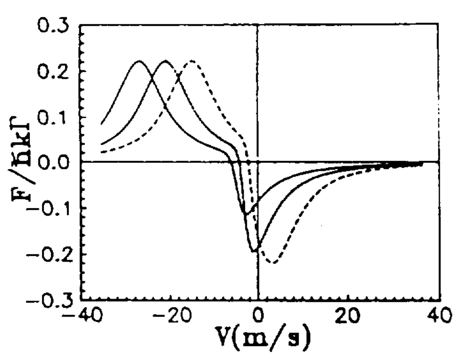
After obtaining the steady solutions $\rho_m^{(e)}$ and $\rho_m^{(g)}$ to the above equations we can calculate the scattering force exerted on a Cs atom under different physical conditions. Fig.(a) shows the influence of light intensity;Fig.3(b) is the variation of force with different laser detuning;Fig.3(c) shows the forces as a function of the magnetic field gradient at a fixed point in space. In these figures, the X-axis represents velocity in m/s and the Y-axis is the force in units of $\hbar k\Gamma$ (it equals to 4×10^{-21} N for the transition Cs $6S_{1/2} \rightarrow 6P_{3/2}$).



(a) For curves in the direction of the arrow, from lower to upper, I=0.27, 1.1 , 4.4 ,18 , 27 mW/cm ^{2}, at Δ = - 10.6 MHz, B=0T



(b) For curves in the direction of the arrow, from right to left, Δ = - 10.6, -21.2 , -42.4 MHz, at I=4.4mW/cm^2, B=0T



(c) For curves in the direction of the arrow, from right to left , dB/dx=0.5 , 1 , 1.5 mT/cm , at I = 4.4 mW/cm^2 , and Δ = -10.6 MHz. The distance to the center of the trap is 1cm.

Fig.3 The force experienced by a Cs atom under different conditions

Fig.4 shows the position dependence of the scattering force on a Cs atom with different velocities . The X-axis is the position and the Y-axis represents the force as in Fig.3 .

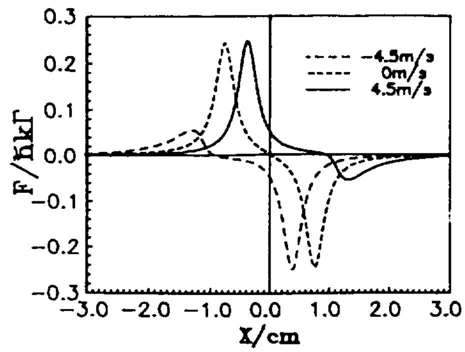


Fig.4 The scattering force of a Cs atom with different velocities. For curves in the direction of the arrow, from left to right $V = -4.5, 0, 4.5$ m/s, at $I = 1.1\mathrm{mW/cm^2}$, $\Delta = -10.6$MHz, $dB/dx = 1$mT/cm.

From Fig. 3(a) it can be seen if an atom is stopped at the center of the MOT and has a small velocity, we can obtain

$$f_{scatter} = -\alpha V \tag{8}$$

From Fig. 4, we derive that if an atom has zero velocity and is near the trap center, then

$$f_{scatter} = -kX \tag{9}$$

Therefore when both X and V are relatively small, we may rewrite $f_{scatter}$ approximately as

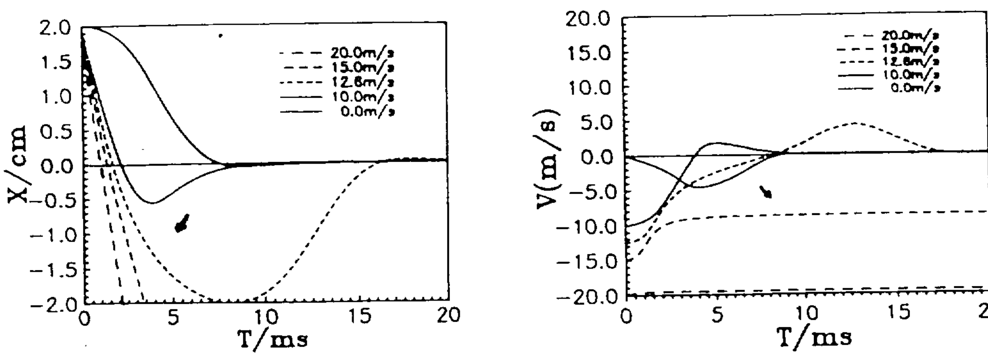
$$f_{scatter} = -kX - \alpha V \tag{10}$$

where K and α are the string constant and damping coefficient respectively. This is the typical form of force in a potential well with damping.

B. Calculation of the capture and escape of Cs atoms

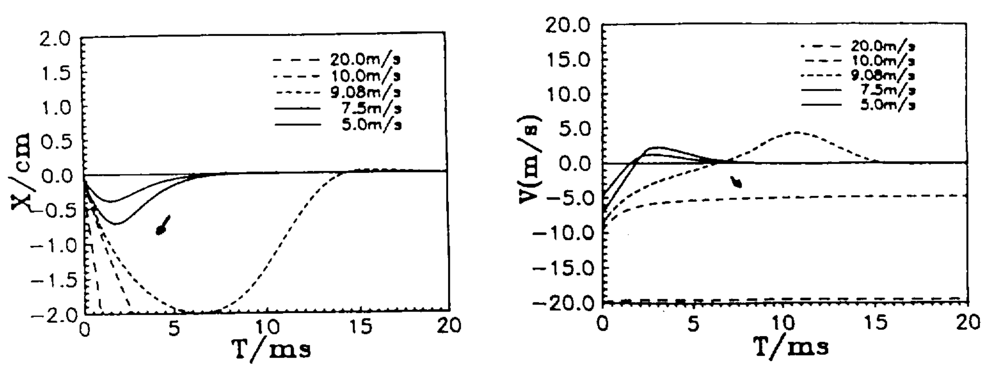
Before the calculation, we would like to explain several related concepts and introduce two important parameters. In practice the MOT is three dimensional and is formed at the intersection of six laser beams. The size of the MOT is determined by the radius of the laser beams. We will then also use the radius of laser beam as the spatial range of 1D-MOT. The atom is considered to be trapped if it finally stops at the trap center. The maximum velocity of atoms captured at the edge of the MOT is defined as the capture velocity V_c. Mainly due to collisions with fast background atoms, trapped atoms at the center of the MOT may obtain enough initial velocity to be knocked out of the trap. The minimum initial velocity is defined as the escape velocity V_e.

For the calculation of the parameters of motion the Cs atom is treated as classical particle . Its dynamical behavior obeys the Newton's second law . The force is obtained by the approach discussed above . After integrating the equations of motion by the Runge-Kutta algorithm . we obtain curves showing variation of the velocity and position of a Cs atom in the process of capture and escape . Fig . 5(a) shows the variation of the position in the capture process . The motion of Cs atoms with different initial velocities is given where the atoms' initial position is supposed to be at the edge of the trap . Fig .5(b) shows the variation of the velocity in the same process . Finally Fig . 6 presents the same curves in the escape process . Here the atoms' initial position is assumed to be at the trap's center



(a) Variation of position (b) Variation of velocity

Fig .5 The variation of position and velocity in the process of capture . For curves in the direction of the arrow , from upper to lower , the initial velocity is V_i = 0 , 10 , 12.58 , 15 , 20 m/s , the radius of the laser beam is 2cm , I = 4.4 mW/cm^2 , Δ = - 10.6 MHz , dB/dx = 1mT/cm . From these curves , we obtain V_c = 12.58 m/s .



(a) Variation of position (b) Variation of velocity

Fig .6 The variation of position and velocity in the process of escape . For curves in the direction of the arrow , from upper to lower , V_i = 5 ,7.5,9.08 ,10, 20 m/s . The parameters of laser are the same as those in Fig .5. From these curves we see that the escape velocity evaluates to be V_e = 9.08 m/s .

V_c and V_e are determined respectively in the following way : first , we set the velocity range to be [0m/s, 20m/s] , which , according to our calculation , cover the value of V_c

and V_e ; then , the velocity range is step by step decreased by the dichotomy ; finally , we get the approximate values of V_c and V_e for a velocity range smaller than 0.01m/s . The magnitude of V_c and V_e is related closely to the number of trapped atoms

$$N_{ss} = \frac{r^2}{\sigma} \cdot \left(\frac{V_c}{\mu} \right)^4 \quad [5] \qquad (11)$$

where r is the radius of the laser beam , μ is the average velocity and σ is the cross section for a trapped atom to be knocked out of the trap by a background atom.Obviously , σ decreases with V_e . From (11) , we know that for a large number of trapped atoms it is beneficial to increase V_c and V_e by changing the trap parameters .

3 . DISCUSSION

A . The properties of the force in MOT

A Cs atom experiences three kinds of forces in MOT . These are the gravitational force the magnetostatic force and the optical force . The magnetostatic force is caused by the inhomogeneous magnetic field . When dB/dx = 2 mT/cm , a cs atom in the $F_g = 4, m_g = 4$ sublevel obtains an acceleration of about g (gravitational acceleration) . The optical force is due to the interaction between the light and the atom . For $\Delta = -10.6$ MHz and I = $4 mW/cm^2$, the acceleration caused by this force is about 103 g , which is much greater than that by the gravitational force and the magnetostatic force . Therefore , we could neglect the gravitational and the magnetostatic force .

As discussed above,both $f_{scatter}$ and f_{grad} contribute to the optical force in the 1D-MOT. The capture and escape of atoms are mainly determined by the former due to its large interaction range of velocity and space . Even though f_{grad} has a smaller interaction range than that of $f_{scatter}$ but it acts in this very small range much stronger on the atom [10] . When atoms in MOT reach a steady state , they concentrate at the trap center with very small velocity amplitudes . In this situation , the temperature and density of atomic cloud are determined by f_{grad} .

B . Contribution of Zeeman effect and Doppler effect

In MOT the origin of $f_{scatter}$ is due to the different relative detuning for σ^+ and σ^- photons . This imbalance between the absorbed photons from opposite directions results in the net scattering force . But an atom in the MOT can not tell whether the relative detuning is caused by the Zeeman effect or by the Doppler effect , because there is certain equivalence between these two effects on their contribution to the net force .When Zeeman splitting between $\Delta m = 2$ sublevels of a Cs atom excited state in compensated by Doppler shift , i.e.

$$\hbar^{-1} g_e \mu_B \frac{dB}{dx} X = kV \qquad (12)$$

then the relative detuning for σ^+ light is the same as that for σ^- , Therefore the scattering rate of photons is equal in both directions , under which circumstance the atom experiences no force ,

$$f = 0 \tag{13}$$

By substituting (12), (13) into (10) , we obtain

$$K = g_e \mu_B (\hbar k)^{-1} \frac{dB}{dx} \cdot \alpha \tag{14}$$

which indicates the equivalence between the Zeeman effect and the Doppler effect , i.e. K is proportional to α . It also shows that they both are dependent on the parameters of the laser field (detuning and intensity) . However they are different in that K is proportional to dB/dx whereas α is independent of dB/dx , which indicates that the scattering force originates in the inhomogeneous magnetic field . Under the condition of small V and X the calculation is simplified and we have obtained the approximate formula for α and K (see appendix for detail)

$$\alpha \approx -0.9644 \frac{\hbar k^2 \delta S_0 \Gamma}{(1+S_0)(\delta^2 + \Gamma^2/4)} \tag{15}$$

$$K \approx -0.9644 \frac{g_e \mu_B k \delta S_0 \Gamma}{(1+S_0)(\delta^2 + \Gamma^2/4)} \cdot \frac{dB}{dx} \tag{16}$$

C . The relation of the force on the parameters of MOT

We will concentrate our discussion on the effect of the laser intensity I , the frequency detuning Δ , the radius r of the laser beam and the gradient of the magnetic field dB/dx on the scattering force in MOT .

From Fig.3(a) we know that for small intensities the force increases sharply with the laser intensity . When $I > I_0 \delta^2 \Gamma^{-2} = 4.4 mW/cm^2$ (i.e. $S_0 = 1$. The saturated intensity for the cycling transition $F_g = 4, m_g = 4 \rightarrow F_e = 5, m_e = 5$ is $I_0 = 1.1 mW/cm^2$) , the intensity has only a small influence on the force . $S_{m,m\pm1}$ increases with the laser intensity but the population difference $\rho_m^{(g)} - \rho_{m\pm1}^{(e)}$ will become small due to the saturation effect . Therefore the influence of laser intensity is diminished due to the compensation of these two effects with each other . Fig .3(b) shows that the capture range of velocity increases with laser detuning . Since the volume of the MOT is determined by the radius r of the laser beam , increasing r may increase the volume efficiently ; Fig .3(c) shows the effect of the magnetic field gradient . At a point far from the trap center , atoms will be cooled to non-zero velocity . The curve of the force is asymmetric about its zero point , which is caused by the difference of g-factor between the upper and the lower state . (Upper state $6P_{3/2}, F = 5, g_e = 0.4$; lower state $6S_{1/2}, F = 4, g_g = 0.25$) . From Fig.3(c) we know that increasing the gradient dB/dx tends to decrease the damping force, whereas the string constant K increases with dB/dx(See [16]). This situation sets a certain limit on the value of dB/dx.

D. Several measures to increase the number of trapped atoms

We have known that increasing light intensity will not increase the number of trapped atoms if $I > I_0 \delta^2 \Gamma^{-2}$. Under the condition of $I \approx I_0 \delta^2 \Gamma^{-2}$, enlarging r (the radius of the laser beam) may increase the volume of MOT and increasing δ (laser detuning) may increase the velocity capture range. After r and δ being determined, we use the condition $\delta_{m,m\pm1} \leq 0$ as the limit on the value of dB/dx. Among all the relative detunings , $\delta_{-4,-5}(V = 0 , X = r) = \delta + \hbar^{-1} r\mu_B$ dB/dx is the biggest one. If it is negative, i.e., (note that $\delta < 0$) , the condition will be satisfied.

All in all, we think that the following methods may increase the number of trapped atoms: (1) increasing the detuning; (2) enlarging the radius of the laser beam as possible as to keep $I \approx I_0 \delta^2 \Gamma^{-2}$; (3) $dB/dx = -\hbar \delta(\mu_B r)^{-1}$ when r and δ are determined.

4. CONCLUSION

The properties of MOT have been studied profoundly for years. On the basis of these studies, we used a simple method to calculate the force and motion of Cs atoms in 1D-MOT. The approximate formulas for α and K are given in the case of small velocity and volume and the characteristics of the force and the contribution of Zeeman effect and Doppler effect are also discussed. The condition to increase V_c and V_e upon which we may build an efficient MOT are given.

However in this paper we don't discuss the problems about the atomic density and equilibrium temperature in MOT, because the approximate method we used does not include f_{grad} and atomic momentum diffusion. Thus it cannot be applied to calculate the atomic density and temperature. Furthermore the theory of density and temperature in MOT is quite complicated because it requires the consideration of various effects, which are not easy to evaluate. Anyway the related work is in proceeding.

The authors are gratefully indebt to Prof. Wu Jike of Department of Mechanics, Peking University and Dr. Zhang Wenqing of Institute of Physics, Academia Science Sinica for their instruction and help with the computer program.

APPENDIX

At first, we give the cited definition of the saturation factor

$$S_0 = \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4} \tag{a.1}$$

and the relative detuning when X is small can be rewritten as follows

$$\delta_{m_g, m_g \pm 1} = \delta \mp kV \tag{a.2}$$

Finally the approximate population difference between the upper and the lower state is

$$\rho_{m_g}^{(g)} - \rho_{m_g \pm 1}^{(e)} \approx \frac{1}{1 + S_0} \rho_{m_g}^{(0)} \tag{a.3}$$

where $\rho_{m_g}^{(0)}$ is the population of sublevels of the ground state in the case of very weak light intensity under σ^+-σ^- configuration and the calculated results are presented below

$$\rho_4^{(0)} = \rho_{-4}^{(0)} = 0.3864 \quad , \qquad \rho_3^{(0)} = \rho_{-3}^{(0)} = 0.0498$$

$$\rho_2^{(0)} = \rho_{-2}^{(0)} = 0.0336 \quad , \qquad \rho_1^{(0)} = \rho_{-1}^{(0)} = 0.0188$$

$$\rho_0^{(0)} = 0.0172$$

After combining (3), (a.2) and (a.3), we have

$$f_{scatter} = \frac{\hbar k \Gamma}{2(1 + S_0)} \cdot \left[\frac{\Omega^2/2}{(\delta - kV)^2 + \Gamma^2/4} \sum_{m_g} \rho_{m_g}^{(0)} (C_{m_g}^{m_g+1})^2 - \right.$$

$$\left. \frac{\Omega^2/2}{(\delta + kV)^2 + \Gamma^2/4} \sum_{m_g} \rho_{m_g}^{(0)} (C_{m_g}^{m_g-1})^2 \right] \tag{a.4}$$

By substituting the values of $\rho_{m_g}^{(0)}$ and corresponding C-G coefficients

$$\sum_{m_g} \rho_{m_g}^{(0)} (C_{m_g}^{m_g-1})^2 = \sum_{m_g} \rho_{m_g}^{(0)} (C_{m_g}^{m_g-1})^2 = 0.4822 \tag{a.5}$$

and using the approximate condition $\delta - kV \approx \delta + kV \approx \delta$ for very small V as well as formula (a.1) , finally we obtain

$$f_{scatter} \approx \frac{0.9644 \hbar k^2 \delta S_0 \Gamma}{(1 + S_0)(\delta^2 + \Gamma^2/4)} \cdot V \tag{a.6}$$

$$\alpha \approx -0.9644 \cdot \frac{\hbar k^2 \delta S_0 \Gamma}{(1+S_0)(\delta^2+\Gamma^2/4)} \tag{a.7}$$

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