STEADY-STATE SQEEZING IN THE MICROMASER CAVITY FIELD

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The micromaser has been extensively studied as a source of radiation fields having various nonclassical properties. One such propety is the sub-Poissonian nature of the radiation field which has been experimentally demonstrated in this dynamics [1].This device involves a microwave cavity of high quality factor Q and cooled down to sub-Kelvin temperature. In the conventional micromaser, two-level Rydberg atoms in their upper states are pumped into the cavity at such a rate that at most one atom at a time is allowed to interact with the cavity field. The very first atom encounters the field at thermal equilibrium. After the interaction, the field evolves under the influence of its own reservoir until the next atom enters the cavity. The process repeats itself with a repeatation time $t_c = \tau + t_{cav}$ where τ is the atom-field interaction time and t_{cav} is the duration between successive atoms. τ is fixed whereas t_{cav} is random following a Poissonian pump with an average $\bar{t}_c = 1/R$ where R is the number of atoms entering the cavity per second. We find that the cavity field is coupled to the atom during τ only whereas it is coupled to its reservoir during the entire duration t_c. Hence, the nonclassical nature of the field would be very sensitive to the cavity field damping.

The theory of micromaser proposed in Ref.[2] is capable of

including the effect of reservoir-induced interactions even during the short duration τ on the evolution of the cavity field. Its influence on the normalized variance $v = [(\langle n^2 \rangle - \langle n \rangle^2)/\langle n \rangle]^{1/2}$ ($v < 1$ for sub-Poissonian fields) is clearly evident in the Fig.(1). The spontaneous emission constant between the two masing levels $\gamma = 4400$ Hz (dotted), 4.4 Hz (dashed), and 0.0 (full). The dash-dot-dot curve represents the results for the ideal situation during τ i.e no damping whatsever.

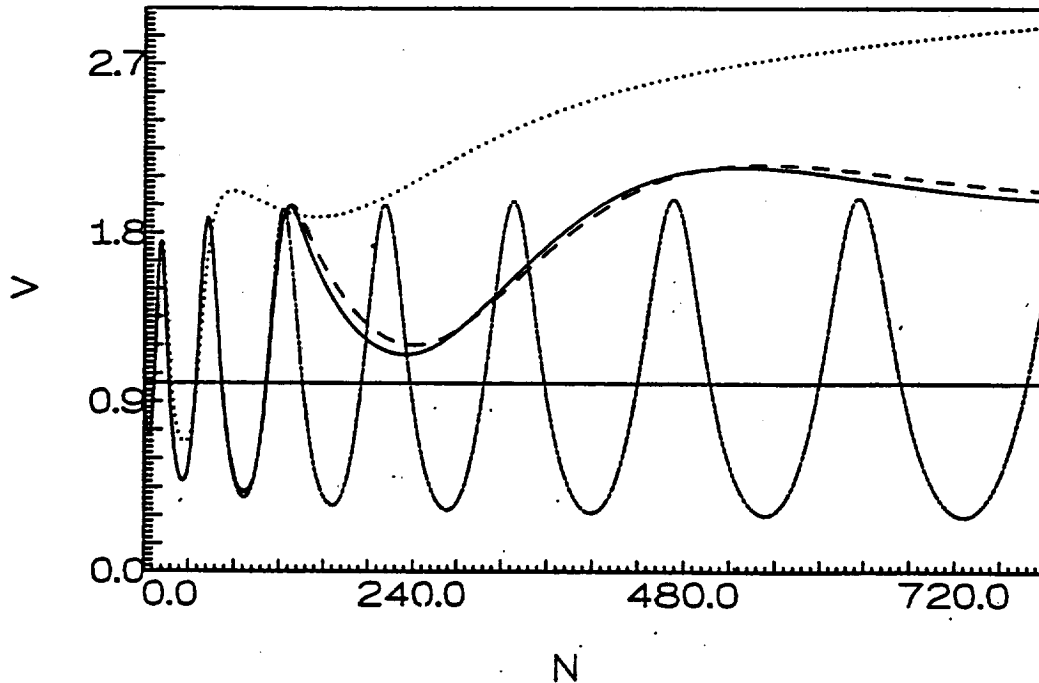Fig.1. Variation of v with N, the number of atoms

streamed through the cavity during photon lifetime

for the experimental setup in Ref.[1]. $\tau = 35$ μsec.,

and cavity temperature T=0.5 K.

The above description is for the atoms being in their upper states at the time of entering the cavity. In stead, if the atoms are in a coherent superposition of two masing levels at the beginning of interaction, it can induce a phase information to

the cavity field. This may result in the squeezing of a quadrature of the radiation field.

The two-level Rydberg atoms, at the time of entering the cavity, are in a coherent superposition

$$|\psi> = \alpha|a> + \beta|b> \qquad (1)$$

of their upper ($|a>$) and lower ($|b>$) states. We assume through out this paper that α and β are real. The transition frequency between the levels is in resonance with the single eigenmode of the cavity at frequency ω. We represent the atom by the Pauli pseudo-spin operators obeying the commutation relation $[s^+, s^-] = 2s^z$. The cavity field is represnted by the annihilation (creation) operator a ($a^\dagger$) which satisfy the commutation ralation $[a, a^\dagger] = 1$. The atom-field iteraction is then given by the well-known Jaynes-Cummings Hamiltonian [3]

$$H = g(s^+a + s^-a^\dagger) \qquad (2)$$

where g is is the coupling constant.

In a frame rotating at ω the equation of motion for the composite atom-field system can be taken to be

$$\dot{\rho} = -i[H,\rho] - \varkappa\{(1+\bar{n}_{th})(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a)$$
$$+ \bar{n}_{th}(aa^\dagger\rho - 2a^\dagger\rho a + \rho aa^\dagger)\}. \qquad (3)$$

The effect of the heat-bath at the temperature $T > 0$ is introduced through the terms in the Planck function $\bar{n}_{th}$. $\varkappa = \omega/2Q$ is half the bandwidth of the eigenmode. The effect of atomic reservoir is not included here as it has been seen in Ref.(2b) that the significant influence comes from the cavity damping.

We follow the procedure in Ref.(2) to get an expression for time derivative of the density operator for the field only in photon number representation i.e. $\rho_{n,m} = <n|\rho|m>$. The resulting expression gives, in the steady-state, the continued fractions

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for $W_{n,m} = \rho_{n,m} / \rho_{n-1,m-1}$ for all n and m which is

$$W_{n,m} = Z^-_{n,m} / [Z_{n,m} + Z^+_{n,m} W_{n+1,m+1}] \qquad (4)$$

where z, $z^{\pm}$ are all functions of system parameters. We need $\rho_{n,n}$, $\rho_{n,n+1}$ and $\rho_{n,n+2}$ to obtain variances X and Y in the two quadratures defined by $a_x = (a+a^{\dagger})/2$ and $a_y = (a-a^{\dagger})/2i$ respectively. X or Y < 0 indicates squeezing in that quadrature.

Our numerical study of X and Y as a function of $g\tau$, pump rate N and other parameters reveals interesting results for $g\tau = \pi$ which we display in Fig. (2) for a cavity with $\varkappa/g = 0.81 \times 10^{-5}$. The experiment in Ref. [1] had the same value of $\varkappa/g$ but the cavity
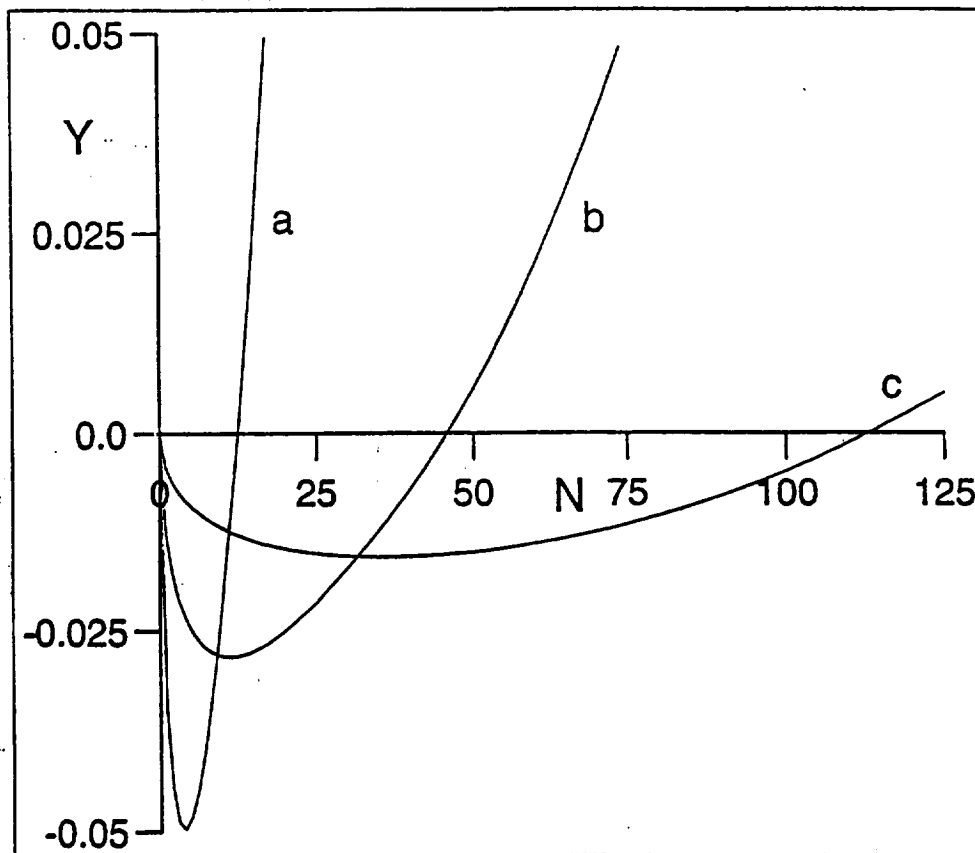

Fig.2. Squeezing Y as a function of dimensionless pump rate N=R/2$\varkappa$. The curves are for the atom-field inter-action set by $g\tau = \pi$. $|\beta|^2 = 0.1$.

temperature was at $T=0.5$ K. Figure 2 indicates that T needs to be further reduced to observe squeezing in the cavity field.The curves a, b and c in the Fig. (2) are for $T=0.224$ K, 0.15 K and 0.11 K respectively. The corresponding thermal photons at the cavity mode frequency [1] are $\bar{n}_{th}=0.01$, 0.001 and 0.0001. Fig. (2) shows that the squeezing in a_y quadrature persists for higher pump rate N for lower temperature.

The photon distribution of the cavity field is, in general, peaked at various n at temperatures such as in Fig. (2). These states $|n\rangle$, are known as trapped states [4].These can be easily analysed for an ideal cavity ($Q=\infty$) which reveal that,in the case of polarized atoms, a trapped vacuum state $|0\rangle$ of the cavity field results for $g\tau=\pi$ and for $T \simeq 0.1$ K. With cavity dissipation included, it is difficult to notice directly the trapped states of the cavity field in Eq. (4). However, from our numerical study, we notice in the case of Fig.(2c), for lower pump rate N, the cavity field is *almost* in the trapped vacuum state and the uncertainty product of its two quadratures a_x and a_y is close to that for a minimum uncertainty state.

In conclusion, we have shown that the radiation field in the micromaser cavity presently in operation [1] may be squeezed if pumped with polarized atoms. With both α and β considered real, we show squeezing in the a_y quadrature. In Refs. [5], the a_x quadrature has been shown squeezed as α and β are out of phase by $\pi/2$.However, it is difficult to make a quantitative comparison with the results in Ref. [5] as the squeezing there has been studied in various types of fields which are assumed to be present in an ideal cavity, that is $Q=\infty$ during the entire repeatation time t_c. In the present paper, the squeezing is in the steady-state field evolved from the action, same as in

conventional micromaser, with the effect of cavity dissipation included during the entire $t_c = \tau + t_{cav}$. The atoms at the time of entering the cavity is prepared, instead of being in the upper state, in a polarized state such that $\beta \ll \alpha$.

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