Cooperative effects in a one photon Micromaser .

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Abstract

For small photon numbers, trapping states are difficult to detect due to the influence of the collective effects. We find that if the atoms are injected with atomic polarization, the micromaser becomes more insensitive to these effects. In particular, the squeezing properties of the cotangent states are basically unchanged.

Recent work studied the effects of having two simultaneous and fully inverted atoms in the one photon micromaser cavity, and found that the trapping states were strongly disrupted by these effects for a low photon number[2, 3].

Here we will describe the cooperative effects of polarized atoms on the trapping states and squeezing properties of the cotangent states[4][1].

The Hamiltonian of the two atom field system, in the Dipole and Rotating wave approximations is:

$$V_I = \hbar g \left\{ \left(a\sigma_1^\dagger + a^\dagger \sigma_1 \right) \otimes 1_2 + \left(a\sigma_2^\dagger + a^\dagger \sigma_2 \right) \otimes 1_1 \right\}, \qquad (1)$$

where g is the one atom- field coupling constant and σ_1 and σ_2 the atomic Pauli spin operators for atom one and two respectively. Here we assume

exact resonance between the field and the atoms.

Let us denote by $|e\rangle_i$ and $|g\rangle_i$, $i = 1, 2$ the excited and ground state of the i-th atom.

The system can be described by the basis:

$$\{|e\rangle_1|e\rangle_2, |e\rangle_1|g\rangle_2, |g\rangle_1|e\rangle_2, |g\rangle_1|g\rangle_2\}$$

The time evolution operator of the system can be calculated in a simple way. It is:

$$U_2(\Delta\tau) = \begin{pmatrix} A & -iaS & -iaS & B \\ -iSa^\dagger & D & E & -iSa \\ -iSa^\dagger & E & D & -iSa \\ B^\dagger & -ia^\dagger S & -ia^\dagger S & \tilde{A} \end{pmatrix} \tag{2}$$

where

$$A = 1 + \frac{a(C-1)a^\dagger}{\Lambda} \qquad B = \frac{a(C-1)a}{\Lambda} \qquad D = \frac{1}{2}(1+C) \tag{3}$$

$$E = -\frac{1}{2}(1-C) \qquad C = \cos(\sqrt{2}g\Delta\tau\sqrt{2n+1}) \qquad \Lambda = 2n+1 \tag{4}$$

$$S = \sin(\sqrt{2}g\Delta\tau\sqrt{2n+1})/\sqrt{4n+2}, \qquad \tilde{A} = 1 + a^\dagger(C-1)a/\Lambda, \tag{5}$$

where $\Delta\tau$ is the interaction time during which the 2 atoms are present in the cavity.

We assume that at $t = 0$ a first atom enters the cavity and a second one Δt seconds later.

The state of the system at the instant just before the first atom leaves the cavity is given by:

$$\rho(\Delta t + \Delta\tau) = U_2(\Delta\tau)\rho_{af}(\Delta t)U_2^{\dagger}(\Delta t)$$

$$\rho_{af}(\Delta t) = U_1(\Delta t)\rho_f(0) \otimes \rho_a(0)U_1^{\dagger}(\Delta t) \tag{6}$$

where $U_1(\Delta t)$ is the time evolution operator of the Jaynes-Cummings Model and ρ_a is given by:

$$\rho_a = \begin{pmatrix} |\alpha|^2 & \alpha^*\beta \\ \alpha\beta^* & |\beta|^2 \end{pmatrix}, \tag{7}$$

that describes the initial state of the atom. Next, we trace over the first atom and get the atom 2 -field density matrix. The evolution of the present system is governed by the Jaynes-Cummings model. Two possibilities arise. One is that the atom 2 leaves before a new atom enters the cavity or a new atom enters before atom 2 leaves.

We may define various sequences as described by Figure 1. Sequence (a) (010) corresponds to no atom-one atom-no atom sequence. Similarly, we may have a sequence (01210) (Figure 1-b) or (0121210) corresponding to Figure 1-c. We discard events containing three or more simultaneous atoms.

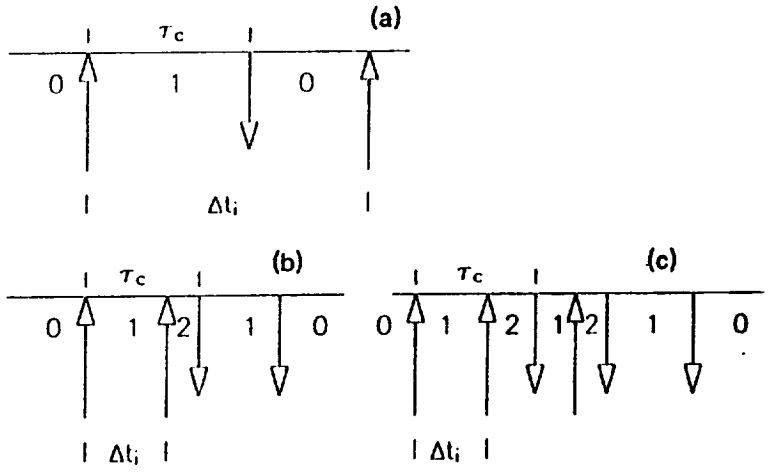

Figure1.Poissonian injection of atoms. The arrow pointing upwards indicates an atom entering the cavity and a downwards arrow means that it leaves. (a)$\Delta t_i \succ \tau_c$and we have either zero or one atom inside the cavity. (b)$\Delta t_i \prec \tau_c$ and $\Delta t_{i+1} \succ \tau_c$.(c)$\Delta t_i \prec \tau_c, \Delta t_{i+1} \prec \tau_c and \Delta t_{i+2} \succ \tau_c$.

For each of these sequences, it is possible, through a tedious but straightforward procedure, to write the field density matrix elements in terms of the relevant parameters of this system.

In order to numerically simulate the process described above in a realistic fashion,we consider that the atomic arrival obeys a Poisson distribution. We characterize the atomic flux by a parameter $p = \langle \Delta\tau \rangle / \tau_c$, where $\langle \Delta\tau \rangle$ is the average time between consecutive atoms and τ_c the atomic flight time through the cavity. We also define the usual one photon trapping condition by $\sqrt{N_u + 1} g\tau_c = q\pi$, q being an integer number.

Next, we describe some numerical results. The parameters used are $p = 15.6, N_u = 10, |\alpha|^2 = 0.9$.

In Figure 2-a we show the field density matrix elements after 1000 atoms crossed the cavity, and one can already see a small hill between n= 12 and 18, clearly indicating that a the trap had already a small leak. This effect is of course more dramatic, as one increases the atomic numbers to 2500 (Figure 2-b) and $N_{atom} = 5000$ (Figure 2-c). This set of three Figures clearly

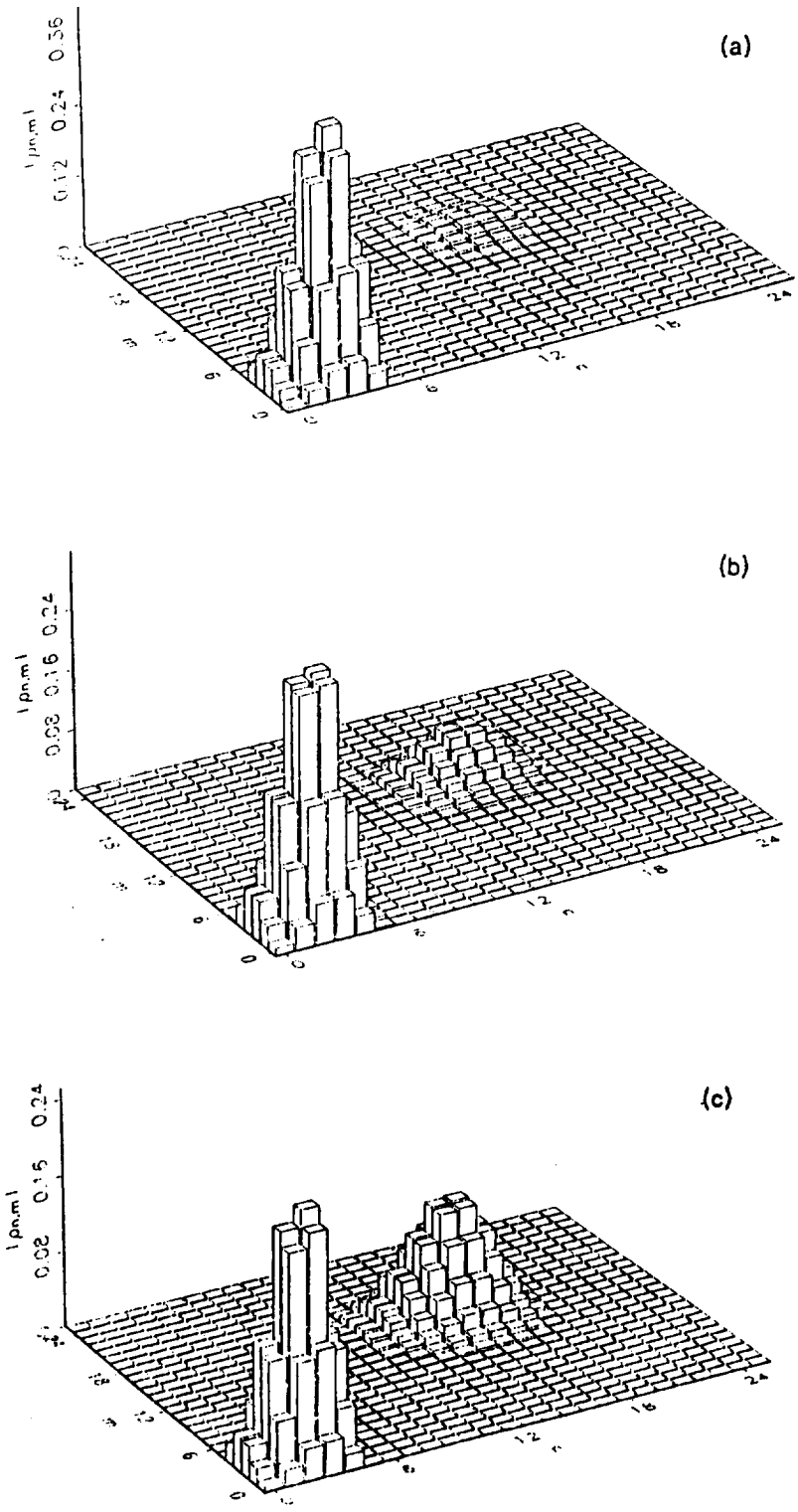

Fig. 2. Reduced field density matrix for $p = 15.6$, $N_u = 4$ and $|\alpha|^2 = .9$. (a) $N_{\text{atom}} = 1000$. (b) $N_{\text{atom}} = 2500$. (c) $N_{\text{atom}} = 5000$.

display the probability diffusion in phase space.

The most important result in this work is shown in Figure3, where the Y-quadrature variance is shown versus atomic Number. The dotted line corresponds to the squeezing of the cotangent states[4], and the full line to the present case. We observe that even for a relatively large atomic flux ($p = 15.6$), *the squeezing property of the cotangent states are extremely robust to the cooperative effects, that otherwise seem to be very destructive.*

In a future work, we would like to explore how the atomic measurement at the outside of the cavity affects all the properties of the field discussed here.

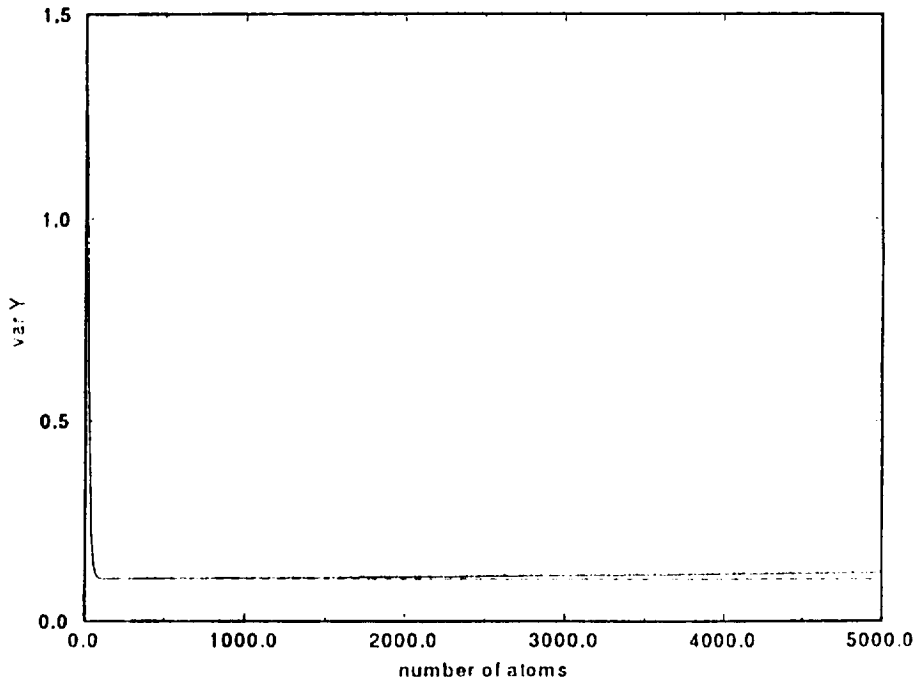

Figure3.Variance of the field quadrature Y versus the number of atoms of the cotangent state(dotted line) and the present case(full line). The parameters are the same as in Figure 2, except N_u=10.

References

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