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Reflection spectrum of two level atoms by an evanescent laser wave *

Tan Weihan and Li Qingning
Department of Physics, Shanghai University,
Shanghai Institute of Optics and Fine Mechanics, Academia Scinica
(P.O.Box.800-211,Shanghai 201800,China)

Abstract

An exact solution and numerical caculation of the reflection of two level atoms by atomic mirror are presented. The curve of reflection coefficient against Rabi frequency caculated shows some new features, and the physical machanism underlying is analysed.

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1 INTRODUCTION

One of the fundamental problems in atomic optics is the reflection and diffraction of two level atoms by an evanescent laser wave–atomic mirror[1~5]. Via an adiabatic dressed-state approximation the problem was studied by Deutschmann , Ertmer and Wallis[6] . In this paper an exact solution is presented by using the method given in one of the authors previous paper [7]. The curve of reflection coefficient against the Rabi frequency shows some new features, the physical machanism involved is analysed.

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2 The Schrödinger equation, Wave Function, Normalization , and Solution

A schematic diagram for an atomic mirror is shown in Fig.1. An atomic beam incident upon the surface of a dielectric interacting with the evanescent wave in the x–y plane. The total Hamitonian H reads

$$H = H_a + \frac{1}{2m}(p_x^2 + p_y^2) - \vec{\mu}\cdot\vec{\varepsilon} \tag{1}$$

Where H_a , depending on the coordinate $\vec{q}$, is the internal energy, $\frac{1}{2m}(p_x^2 + p_y^2)$ represents the translation energy of atom as a whole, and $-\vec{\mu}\cdot\vec{\varepsilon}$ denotes the atom–laser coupling energy. The Schrödinger equation of the atom reads

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi + H_a(\vec{q})\psi - \vec{\mu}\cdot\vec{\varepsilon}\psi \tag{2}$$

The solution of Eq.(2) has the form

$$\begin{aligned} \psi &= u_e(x,y)\phi_e(\vec{q})\exp\left(-i\frac{E_e+E_g}{2\hbar}t - i\frac{\omega t}{2} - \frac{iEt}{\hbar}\right) \\ &+ u_g(x,y)\phi_g(\vec{q})\exp\left(-i\frac{E_e+E_g}{2\hbar}t + i\frac{\omega t}{2} - \frac{iEt}{\hbar}\right) \end{aligned} \tag{3}$$

Substituting Eq.(3) into Eq.(1), we obtain

$$\begin{aligned} Eu_e &= \left(-\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + \frac{p_e^2}{2m} - \frac{\hbar\Delta}{2}\right)u_e - \mu\varepsilon e^{-\eta y}u_g, \qquad \Delta = \omega - \frac{E_e - E_g}{\hbar} \\ Eu_g &= \left(-\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + \frac{p_g^2}{2m} + \frac{\hbar\Delta}{2}\right)u_g - \mu\varepsilon e^{-\eta y}u_e, \qquad \eta = k_0\sqrt{n^2\sin^2\theta - 1} \end{aligned} \tag{4}$$

Now we introduce the Rabi frequency $\Omega = \frac{2\mu\varepsilon}{\hbar}$, the normalization frequency $\Omega_0 = \hbar\eta^2/m$, and adopt the normalization

$$\begin{aligned} \frac{T_{ey}}{\hbar\Omega_0/2} &= \frac{E + \hbar\Delta/2 - p_e^2/2m}{\hbar\Omega_0/2} = \gamma_1 \\ \frac{T_{gy}}{\hbar\Omega_0/2} &= \frac{E - \hbar\Delta/2 - p_g^2/2m}{\hbar\Omega_0/2} = \gamma_2 \\ \frac{\hbar^2/2m}{\hbar\Omega_0/2}\frac{d^2}{dy^2} &= \frac{1}{\eta^2}\frac{d^2}{dy^2} \Rightarrow \frac{d^2}{dy^2}, \qquad \frac{\Omega}{\Omega_0} \Rightarrow \Omega \end{aligned} \tag{5}$$

$$\gamma_1 - \gamma_2 = \frac{\hbar\Delta - (\hbar\xi)^2/2m - p_g\hbar\xi/m}{\hbar\Omega_0/2}, \qquad \xi = k_0 n \sin\theta$$

After nomalization, Eq.(4) assumes the forms

$$\frac{d^2}{dy^2}u = -\bar{\gamma}u + Me^{-y}u \tag{6}$$

where

$$u = \begin{pmatrix} u_e \\ u_g \end{pmatrix}, \qquad \bar{\gamma} = \begin{pmatrix} \gamma_1 & \\ & \gamma_2 \end{pmatrix}, \qquad M = \begin{pmatrix} & -\Omega \\ -\Omega & \end{pmatrix}$$

Now we rewrite Eq.(6) in the form of first order differential Eqs.

$$\frac{du}{dy} = v, \qquad \frac{dv}{dy} = -\bar{\gamma}u + Me^{-y}u \tag{7}$$

or briefly

$$\frac{dw}{dy} = -\Gamma w + Ne^{-y}w \tag{8}$$

where

$$w = \begin{pmatrix} u \\ v \end{pmatrix}, \quad \Gamma = \begin{pmatrix} & -1 \\ \bar{\gamma} & \end{pmatrix}, \quad N = \begin{pmatrix} & \\ M & \end{pmatrix} \tag{9}$$

The Laplace transformation of $w(y)$ can be written as

$$\tilde{w}(s) = \int_0^\infty e^{-sy} w(y) dy \tag{10}$$

which leads to

$$\begin{aligned} \tilde{w}(s) = \frac{w(0)}{s+\Gamma} + \frac{1}{s+\Gamma}N\tilde{w}(s+1) = \Big(\frac{1}{s+\Gamma} \\ + \frac{1}{s+\Gamma}N\frac{1}{s+1+\Gamma} + \frac{1}{s+\Gamma}N\frac{1}{s+\Gamma+1}N\frac{1}{s+\Gamma+2} + \cdots\Big)w(0) \end{aligned} \tag{11}$$

When the inverse transformation of Eq.(11) is evaluated, the solutions $u(y)$, $v(y)$ can be derived immediately

$$\begin{pmatrix} u_e \\ u_g \\ v_e \\ v_g \end{pmatrix} = \begin{pmatrix} I_1 & I_2 & I_3 & I_4 \\ \tilde{I}_2 & \tilde{I}_1 & \tilde{I}_4 & \tilde{I}_3 \\ \frac{dI_1}{dy} & \frac{dI_2}{dy} & \frac{dI_3}{dy} & \frac{I_4}{dy} \\ \frac{d\tilde{I}_2}{dy} & \frac{d\tilde{I}_1}{dy} & \frac{d\tilde{I}_4}{dy} & \frac{d\tilde{I}_3}{dy} \end{pmatrix} \begin{pmatrix} u_e(0) \\ u_g(0) \\ v_e(0) \\ v_g(0) \end{pmatrix} \tag{12}$$

where $u_e(0)$, $u_g(0)$, $v_e(0)$, $v_g(0)$ is the boundary values of $u_e(y)$, $u_g(y)$, $v_e(y)$, $v_g(y)$ at the target surface $y = 0$.

3 The boundary conditions and the reflection coefficient for atomic wave

3.1 Spontaneous emission

The spontaneous transition of excited atoms to the ground state for large y_m yeilds the condition for excited state wave function

$$\begin{gathered} u_e(y_m) \simeq 0, \quad v_e(y_m) \simeq 0 \\ y_m \gg 1, \qquad \frac{p_{ey}}{m} T_1 \times k_0 \sqrt{n^2 \sin^2\theta - 1} \end{gathered} \tag{13}$$

where 1 is the thickness of evanescent laser wave, T_1 is the life time of atom and p_e/m the velocity departure from the target. The typical datas are, $\lambda \simeq 0.5\mu$, $T_1 = 10^{-8} sec, p_{ey}/m \simeq 0.5 m/sec$, $p_{ey}/m T_1 \times k_0\sqrt{n^2 \sin^2\theta - 1} \simeq 1.73$, setting $y_m \simeq 7$, the inequality Eq.(13) is satisfied well. Using Eq.(13) to eliminate u_{e0}, v_{e0} in Eq.(12), we obtain

$$u_g = u_{g1} u_{g0} + u_{g2} v_{g0} \tag{14}$$

3.2 Perfect adsorption of the atoms transmitted the target surface, non recoil

This implies that, near the target surface, the ground state atoms have the travelling wave structure for small y

$$u_g(y) = u_{g0} e^{i\sqrt{\gamma_2} y} = (\cos(\sqrt{\gamma_2} y) + i \sin(\sqrt{\gamma_2} y)) u_{g0} \tag{15}$$

Comparison with the analytical solution u_g for small y

$$u_g(y) = \cos(\sqrt{\gamma_2} y) u_{g0} + \frac{\sin(\sqrt{\gamma_2} y)}{\sqrt{\gamma_2}} v_{g0} \tag{16}$$

gives

$$v_{g0} = i\sqrt{\gamma_2} u_{g0} \tag{17}$$

Substituting this relation (17) into Eq.(12), we have

$$\begin{gathered} u_g(y) = (u_{g1}(y) + i\sqrt{\gamma_2} u_{g2}) u_{g0} = u_{g0} \rho_g e^{i\theta_g} \\ \rho_g = \sqrt{u_{g1}^2 + \gamma_2 u_{g2}^2}, \quad \theta_g = tan^{-1} \frac{\sqrt{\gamma_2} u_{g2}}{u_{g1}} \end{gathered} \tag{18}$$

3.3 In the region of $y_m \gg 1$

The wave structure of $u_s(y)$ may be also considered as the supperposition of incoming wave $|A|e^{i(\sqrt{\gamma_2}y+\varphi)}$ and the reflected wave $|B|e^{-i(\sqrt{\gamma_2}y+\varphi)}$, i.e.

$$u_s(y) = |A|e^{i(\sqrt{\gamma_2}y+\varphi)} + |B|e^{-i(\sqrt{\gamma_2}y+\varphi)} = \rho_{AB}e^{i\varphi_{AB}} = u_{s0}\rho_s e^{i\theta_s}$$
$$\rho_{AB} = \sqrt{|A|^2 + |B|^2 + 2|AB|\cos 2(\sqrt{\gamma_2}y + \varphi)} = |u_{s0}|\rho_s \tag{19}$$

which gives $\rho_{ABmax} = |A| + |B| = |u_{s0}|\rho_{max}$ at $\sqrt{\gamma_2}y + \varphi = n\pi$, and $\rho_{ABmin} = |A| - |B| = |u_{s0}|\rho_{min}$ at $\sqrt{\gamma_2}y + \varphi = (n + 1/2)\pi$. Thus, the reflection coefficent R can be written as

$$R = \frac{|B|}{|A|} = \frac{\rho_{ABmax} - \rho_{ABmin}}{\rho_{ABmax} + \rho_{ABmin}} = \frac{\rho_{max} - \rho_{min}}{\rho_{max} + \rho_{min}} \tag{20}$$

4 Numerical calculation and discussion

4.1 Parameters

Refering to Eq. (5), the normalized parameters used in the calculation are

$$\gamma_1, \gamma_2 = \begin{cases} 1.96,\ 12.6 & \textit{negative detuning} \\ 12.6,\ 1.96 & \textit{positive detuning} \end{cases}$$
$$y_m = 7.0, \qquad \Omega = 25.0 \tag{21}$$

4.2 Reflection coefficient caculated from Fig.2(a), (b)

$R = \dfrac{253.89 - 1.09}{253.89 + 1.09} = 0.991$ for positive detuning

$R = \dfrac{5.156 - 0.928}{5.156 + 0.928} = 0.695$ for negative detuning.

4.3 Reflection coefficient R agaist Rabi frequency Ω Fig. 3

1. The Rabi frequency Ω very small, the reflection cofficients R approaches to zero in the cases of either positive or nagative detuning

2. The reflection coefficient R for positive detuning is much higher than that for negative detuning.

3. The R curve for negative detuning displays some oscillating features with it's maxima at $\Omega \simeq 12.5, 25, 37.5, 50 \cdots$, and the interval between successive maxima is $\Delta\Omega \simeq 12.5$.

4.4 The physical machenism

We introduce a relative phase shift δ_l between the real and imaginary part of wave function u_g in Eq.(15), during the atoms are departing from the target surface

$$u_g(y) = u_{g0}[\cos(\sqrt{\gamma_2}y - \delta_l) + i\sin(\sqrt{\gamma_2}y + \delta_l)]$$

$$\begin{aligned} \rho_g &\propto \sqrt{\cos^2(\sqrt{\gamma_2}y - \delta_l) + \sin^2(\sqrt{\gamma_2}y + \delta_l)} \\ &= \sqrt{1 + \sin(2\sqrt{\gamma_2}y)\sin(2\delta_l)} \end{aligned} \tag{22}$$

$$R = \frac{\sqrt{1 + |\sin(2\delta_l)|} - \sqrt{1 - |\sin(2\delta_l)|}}{\sqrt{1 + |\sin(2\delta_l)|} + \sqrt{1 - |\sin(2\delta_l)|}}$$

The maxima of R occur at $\delta_l \simeq (n + 1/2)\pi/2, \quad n = 0, 1, \cdots$. The interval between the successive maxima δ_l is $\Delta\delta_l \simeq \pi/2$. The comprison of $\Delta\delta_l$ with the observed interval $\Delta\Omega \simeq 12.5$ reminds us that the phase shifts δ_l induced are proportional to the Rabi frequency Ω, after $\delta_l = \pi/4$. In the inital stage, $\Delta\Omega = 0 \sim 12.5$, the phase shifts induced, $\Delta\delta_l = 0 \sim \pi/4$, is relatively small in comparision with $\Delta\delta_l = \pi/2$ after $\delta_l = \pi/4$.

In conclusion, the reflection coefficient R of two level atoms by evanescent laser wave is studied through analytical solution and numerical caculation. The curve R versus Ω shows that $R < 0.1$ when $\Omega < 2.5$ and $R > 0.7$ when $\Omega > 37.0$. Especially, in the case of negative detuning, an oscillatory feature with a period $\Delta\Omega = 12.5$ appears.

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Figure Captions

Fig.1. Schematic diagram for an atomic mirror.

Fig.2. The variation of ρ_g versus y

(a) for positive detuning, $\gamma_1 = 12.6, \gamma_2 = 1.96, \eta = 1, \Omega = 25.0$

(b) for negative detuning, $\gamma_1 = 1.96, \gamma_2 = 12.6, \eta = 1, \Omega = 25.0$

Fig.3. The variation of reflection coefficients R versus Rabi frequency Ω

(a) for positive detuning, $\gamma_1 = 12.6, \gamma_2 = 1.96, \eta = 1$

(b) for negative detuning, $\gamma_1 = 1.96, \gamma_2 = 12.6, \eta = 1$

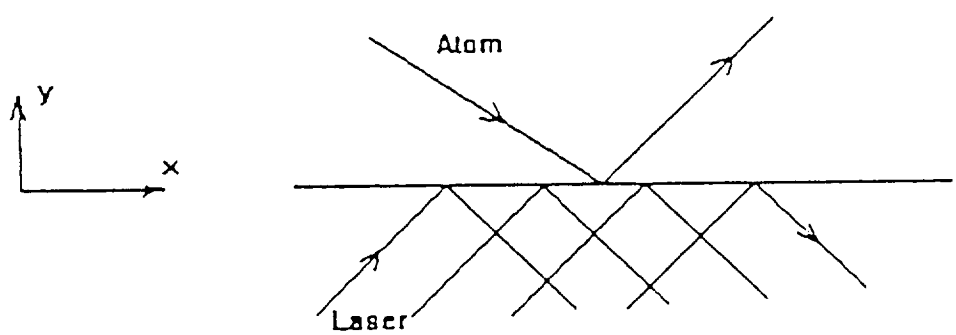
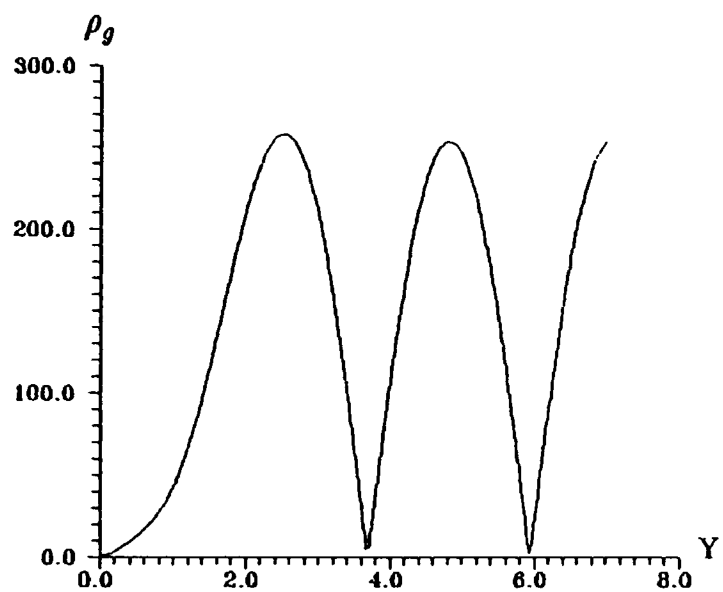
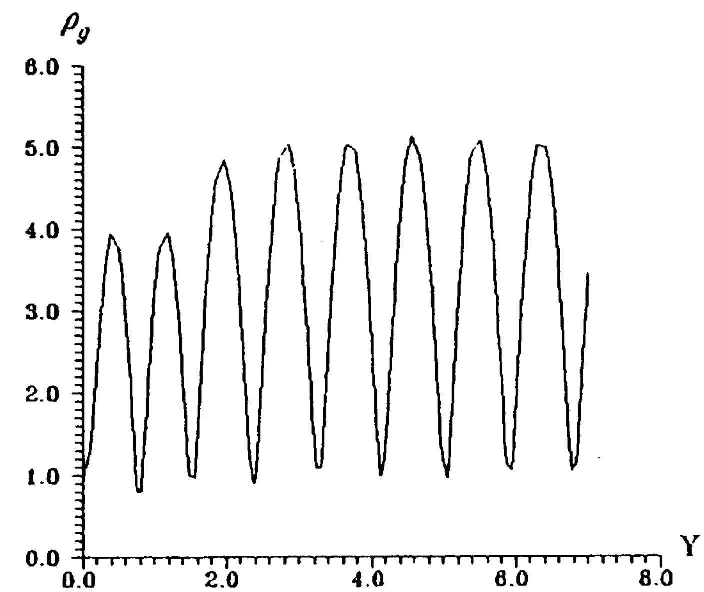


Fig. 1



ρ_g
300.0
200.0
100.0
0.0
0.0
2.0
4.0
6.0
8.0
Y

Fig.2(a)



ρ_g
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.0
2.0
4.0
6.0
8.0
Y

Fig.2(b)

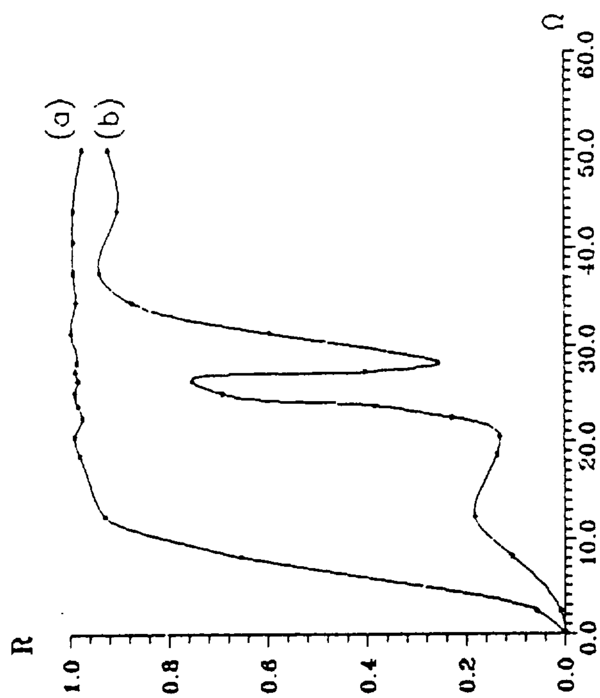


Fig.3