Application of Twin Beams in Mach-Zehnder Interferometer

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Abstract

Using the twin beams generated from parametric amplifer to drive the two port of a Mach-Zehnder interferometer, it is shown that the minimum detectable optical phase shift can be largly reduced to the Heisenberg limit$(1/n)$ which is far below the Shot Noise Limit$(1/\sqrt{n})$ in the large gain limit. The dependence of the minimum detectable phase shift on parametric gain and the inefficient photodetectors has been discussed.
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1 Introduction

As well known, the output signal of the Mach-Zehnder interferometer is sensitive to the relative phase shift between two fields travelling down separated paths. The interferometers can be used in the precision measurements such as optical gravitational wave and gyroscopy detector[1 − 2]. The sensitivity of interferometers relies on the ability to resolve extremely small relative shifts in the two path lengths. The smallest detectable phase shift in principle is determined by the quantum properties of the illuminating field.

Usually the coherent state light is injected into one port of the standard intereferometer and the other one left unused. In this case the vacuum noise must enter the interferometer and the effect of zero-point fluctuations in the vacuum is amplified by the mean intensity of the laser[3 − 4], so that the minimum detectable phase shift is limited by the shot noise limit, i.e. $(\theta_{SNL} = 1/\sqrt{n})$ rad, here n is photon numbers of the input coherent state during the measurment interval. Therefore increasing the strength of the input laser light can increase the resolution of the interferometer, which requires the huge and expensive laser sources and sometimes it is not available. One possible device for enhancing the sensitivity is to replace the vacumm state with squeezed light in interferometer. In the experiment with squeezed vacua performed by Min Xiao et. al[5], an improvement in the signal-to-noise ratio of 3.0dB ralative to the SNL has been achieved. M.J.Holland and K.Burnett show that the Heisenberg limit of sensitivity can be realized by driving the interferometer with two Fock state lights[6].

A research group of Kastler-Brossel lab in France has successfully generated the twin beams with optical parametric osclillator above thresheld and a perfect quantum noise suppression on the difference between the intensities of the two generated beams has been demonstrated[7]. We suggest a device in which twin beams with same polarization orientation are respectively injecetd into the two ports of Mach-Zhender interferometer instead of the usual coherent and vacuum states, therefore the sensitivity of the interferometer is improved to approach the Heisenberg limit of $1/n$.

2 Interferometer with twin-beams

The arrangment of the system is illustrated in Fig.1, the signal and idler mode of twin beams are injected into on the first beam splitter (B_1) of M-Z interferometer through the two ports. M is a phase shift medium set in one of the paths. A half-wave plate($\lambda/2$) is used to align the polarization. The intensities of output fields are detected by D_1 and D_2, at last the fluctuation spectrum of different photocurrent is analysed by spectrum analyzer.

the ralationship between the input and output field operators of the interferometer is:

$$c = e^{(\frac{i\theta}{2})}[\cos(\frac{\theta}{2})a_1^{out} + i\sin(\frac{\theta}{2})a_2^{out}] \tag{1}$$

$$d = e^{(\frac{i\theta}{2})}[\cos(\frac{\theta}{2})a_2^{out} + i\sin(\frac{\theta}{2})a_1^{out}] \tag{2}$$

Where θ is the measured phase shift, a_1^{out} and a_2^{out} are the mode operators of twin beams generated by the optical parametric amplifer with same polarization orientation . The output operators of amplifer is ralated to the input operators by the following formula:

$$a_1^{out} = \sqrt{G}a_1^{in} + \sqrt{G-1}a_2^{+in} \tag{3}$$

$$a_2^{out} = \sqrt{G}a_2^{in} + \sqrt{G-1}a_1^{+in} \tag{4}$$

G is the power gain of amplifer.

The intensity difference measured is proportional to:

$$\begin{aligned} I_- &= c^+c - d^+d \\ &= \cos\theta(a_1^{+out}a_1^{out} - a_2^{+out}a_2^{out}) - i\sin\theta(a_2^{+out}a_1^{out} - a_1^{+out}a_2^{out}) \end{aligned} \tag{5}$$

Taking $\alpha_1^{in} = \alpha_2^{in} = \alpha$ we obtain:

$$< I_- >= 2|\alpha|^2\sin\theta[\sqrt{G(G-1)}\sin 2\phi + (2G-1)\sin\phi] \tag{6}$$

$$\begin{aligned} V < I_- > &=< (I_-)^2 > - < (I_-) >^2 \\ &= 2(\cos\theta)^2|\alpha|^2 + \sin\theta\cos\theta|\alpha|^2 B + (\sin\theta)^2[|\alpha|^4 C - |\alpha|^4 A + |\alpha|^2 E + D] \end{aligned} \tag{7}$$

Where

$$D = 4G(G-1)$$

$$B = 4\sqrt{G(G-1)}\sin 2\phi$$

$$A = 4[\sqrt{G(G-1)}\sin 2\phi + (2G-1)\sin\phi]^2$$

$$C = 2[G^2 + (G-1)^2 + 3G(G-1)] - 2G(G-1)\cos 4\phi$$
$$+4[G\sqrt{G(G-1)} + (G-1)\sqrt{G(G-1)}]\cos\phi - 2[G^2 + (G-1)^2]\cos 2\phi$$
$$-2[2G\sqrt{G(G-1)} + 2(G-1)\sqrt{G(G-1)}]\cos 3\phi$$
$$E = 8[G\sqrt{G(G-1)} + (G-1)\sqrt{G(G-1)}\cos\phi + 2[G^2 + (G-1)^2 + 6G(G-1)]$$

The Signal-to-Noise Ratio(SNR) is defined by

$$SNR = \frac{< I_- >}{\sqrt{V(I_-)}} \geq 1 \tag{8}$$

Therefore

$$|\alpha|^4 A(\sin\theta)^2 \geq 2|\alpha|^2(\cos\theta)^2 + |\alpha|^2 B \sin\theta\cos\theta$$
$$+ [|\alpha|^4 C - |\alpha|^4 A + |\alpha|^2 E + D](\sin\theta)^2 \tag{9}$$

We get

$$|\alpha|^4 A \geq 2|\alpha|^2(\frac{\cos\theta}{\sin\theta})^2 + |\alpha|^2 B\frac{\cos\theta}{\sin\theta} + [|\alpha|^4 C - |\alpha|^4 A + |\alpha|^2 E + D] \tag{10}$$

We set $\sin\theta \sim \theta, \cos\theta \sim 1$ for the small θ, $|\alpha|^2$ is the average photon numbers (n) of the incident fields for parametric amplifier.

Then we have

$$[2n^2 A - n^2 C - nE - D]\theta^2 - nB\theta - 2n \geq 0 \tag{11}$$

Because of $2n^2A - n^2C - nE - D > 0$, the solution of equation is

$$\theta \geq \frac{B + \sqrt{B^2 + 8[(2A-C)n - E - \frac{D}{n}}}{2(2A-C)n - E - \frac{D}{n}} \tag{12}$$

According to the equation (12) the minimum detectable phase shift (θ_{min}) as a function of n is illustrated in Fig.2. The solid line is the Heisenberg limit, the dashed line and dot-dashed line illustrate the minimum detectable phase shift calculated with $G = 2.3 \times 10^7$ and $G = 2.0 \times 10^7$. We can see that for $G = 2.3 \times 10^7$ and small photon numbers n, the minimum detectable phase shift is gratually approach to the Heisenberg limit. The larger the G is, the smaller the minimum detectable phase shift is. Bright twin beams of wavelength $1.06\mu m$ with power of 3mw has been experimently obtained. With the twin beams of 3mw the minimum detectable phase shift of $10^{-16} rad$ can be easily realized in the interferometer suggested by us, but if using the coherent state the incident power of 1000kw must be demanded.

3 Inefficient photodetection

A detector with quantum efficiency η is equivalent to a beamsplitter which mixes the input mode(a) with a vacuum mode(v), then the output mode from beamsplitter is detected by a perfectly efficient detector[8].

For brevity, setting that the quantum efficiencies of the photodetectors D_1 and D_2 are equal, i.e $\eta_1 = \eta_2 = \eta$, then the anihilation operators for detected modes by D_1 and D_2 are given by:

$$j = \eta^{1/2} c + (1-\eta)^{1/2} v \tag{13}$$

$$k = \eta^{1/2} d + (1-\eta)^{1/2} v \tag{14}$$

j and k are the anihilation operators for the inefficient detector with η.

The analyzed photocurrent and its variance are:

$$< j^+ j - k^+ k >= \eta < c^+ c - d^+ d > \tag{15}$$

$$\begin{aligned}[\Delta(j^+ j - k^+ k)]^2 &= \eta^2 < (c^+ c - d^+ d)^2 > -\eta^2 < c^+ c - d^+ d >^2 \\ &+ \eta(1-\eta)[< c^+ c > + < d^+ d >]\end{aligned} \tag{16}$$

The SNR is

$$SNR = \frac{< j^+ j - k^+ k >}{\sqrt{[\Delta(j^+ j - k^+ k)]^2}} \geq 1 \tag{17}$$

$$\begin{aligned}[\Delta(j^+ j - k^+ k)]^2 &= \eta^2 n^2 (C - A)(\sin\theta)^2 + \eta^2 n B \sin\theta \cos\theta \\ &+ 2\eta^2 n (\cos\theta)^2 + \eta^2 n E (\sin\theta)^2 + \eta^2 D (\sin\theta)^2 \\ &+ \eta(1-\eta) n F \sin\theta + \eta(1-\eta) n M + \eta(1-\eta) H\end{aligned} \tag{18}$$

Where

$$F = 2[(2G-1)\sin\phi + \sqrt{G(G-1)}\sin 2\phi]$$

$$M = 4[(2G-1) + 2\sqrt{G(G-1)}\cos\phi]$$

$$H = 4(G-1)$$

From eq. (17) (18) we get the inequality:

$$A\prime\theta^2 - B\prime\theta - C\prime \geq 0 \tag{19}$$

where $A\prime = (2A - C)n - E - D/n$
$B\prime = B + \frac{\eta(1-\eta)}{\eta^2} F$
$C\prime = \frac{\eta(1-\eta)}{\eta^2} M + 2 + \frac{\eta(1-\eta)}{\eta^2} \frac{H}{n}$
The solution of eq.(19) is:

$$\theta \geq \frac{B\prime^2 + \sqrt{B\prime^2 + 4A\prime C\prime}}{2A\prime} \tag{20}$$

Fig.3 shows the dependence of the minimum detectable phase shift θ_{min} on the power gain respectively for detector with $\eta = 1$ and detector with $\eta = 0.99$. When the gain(G) increase, the θ_{min} decrease, i.e. the sensitivity of interferometer is raised. The effect of inefficiency is very severe. The physical origin of above results is that the quantum correlation degree between the twin beams a_1^{out} and a_2^{out} depends positively on G and η. Therefore the quantum correlation between twin beams is the key to realize high sensitivity detection.

4 Conclusion

We have shown that the Heisenberg limit can be met in the detection of phase shift by using twin beams as the input fields of the interferometer. The dependence of the minimum detectable phase shift on the gain of parametric amplifier which produces the twin beams and the quantum efficiency of detectors has been presented. The sensitivity of the suggested device is always higher than SNL and can tend to the Heisenberg limit for appropriate parameters.

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Figure captions

Fig.1 The diagram of interferometer with twin beams

Fig.2 The minimum detectable phase shift vs the numbers of incident photons

Solid line corresponds to Heisenberg limit

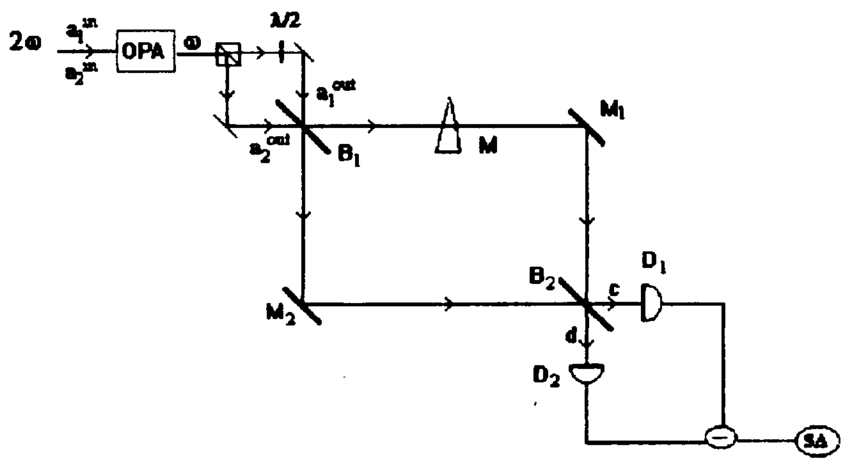
Dashed line for $G = 2.3 \times 10^7$

Dot-dashed for $G = 2.0 \times 10^7$

Fig.3 The minimum detectable phase shift vs the power gain of amplifer with $n = 2 \times 10^{18}$

Solid line for $\eta = 1$

Dotted line for $\eta = 0.99$



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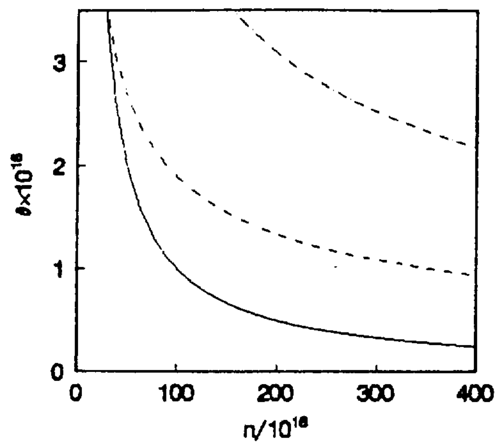

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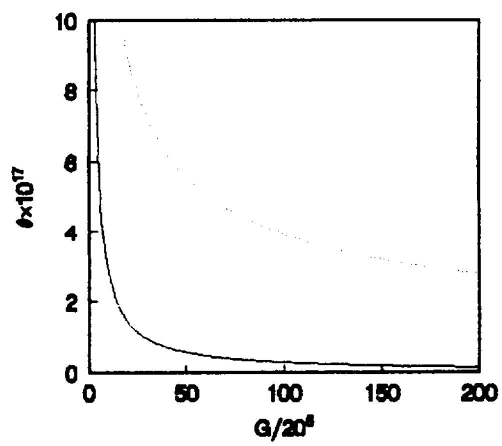

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