The Amplitude Nth-Power Squeezing of Radiation Fields in the Degenerate Raman Process

Zhi-Ming Zhang
Department of Physics, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, PRC
Jin-Fang Pan
Department of Physics, Shanxi Normal University, Linfen, Shanxi 041004, PRC
Lei Xu
Institute of Nuclear Research, Academia Sinica, Shanghai 201800, PRC

Abstract

In this paper we study the amplitude Nth-power squeezing of radiation fields in the degenerate Raman process by using the modified effective Hamiltonian approach recently suggested by us. We found that if the field is initially in a coherent state it will not get squeezing for any Nth-power; if the field is initially in a squeezed vacuum, it may get Nth-power squeezing. The time evolution of the field fluctuation was discussed. Its dependences on power-order N, mean photon number $\bar{n}$, and squeezing angle ξ are analyzed.

1 Introduction

Squeezed states of radiation fields have been studied considerably in recent years. Besides the normal squeezing[1] it is also possible to define higher-order squeezing. Hong and Mandel[2] defined the 2Nth-order squeezing, and Hillery[3] introduced the amplitude squared squeezing. More recently, Zhang et al[4] suggested the amplitude Nth-power squeezing(ANPS), which includes the normal squeezing and the amplituide-squared squeezing as special cases. All these higher-order squeezing have been shown to be independent nonclassical features of radiation fields[5]. ANPS of radiation fields has been studied in many quantum optics systems[4–12].

On the other hand, the degenerate Raman process(DRP) is one of the most interesting two-photon interactions between atoms and radiation fields, and has been studied intensively[13–16]. Usually, this process was studied by the full microscopic Hamiltonian approach(FMHA)[13–14], and the effective Hamiltonian approach(EHA)[15]. Generally speaking, FMHA gives exact solution, but it may be too complicated to be used in some situations. Although EHA is simpler than FMHA, it loses a phase factor, it can not be used to deal with the quantities involving the off-diagonal elements of the density matrix. To overcome these shortages we have suggested a modified effective Hamiltonian approach(MEHA)[16].

In this paper we use MEHA to study ANPS of radiation fields in DRP.

2 The Degenerate Raman Process(DRP)

The DRP refers to the interaction between a Λ-type three level atoms and a single mode of a radiation field(Fig.1).

The modified effective Hamiltonian for DRP is [17]

$$H_{MEH} = H_{EH} + H_S \tag{1}$$

$$H_{EH} = \lambda a^{+} a(|e><g| + |g><e|) \tag{2}$$

is the effective Hamiltonian(when the detuning is very large, one can eliminate the upper level adiabatically and obtain it) and

$$H_S = -a^{+} a(\beta_1 |g><g| + \beta_2 |e><e|) \tag{3}$$

is the part representing the ac Stark shift of atomic levels. β_1 and β_2 are the Stark parameters for levels $|g>$ and $|e>$, respectively.

If the initial state for the atom-field system is

$$|\Psi(0)\rangle = \sum_{n=0}^{\infty} q_n[C_g(0)|g,n\rangle + C_e(0)|e,n\rangle] \tag{4}$$

we can express the state for a later time as

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} q_n[C_g^n(t)|g,n\rangle + C_e^n(t)|e,n\rangle] \tag{5}$$

¿From the time-dependent Schrödinger equation we can obtain $C_g^n(t)$ and $C_e^n(t)$.

The reduced density matrix for the field can be expressed as

$$\rho(t) = \sum_{n,n'=0}^{\infty} \rho_{nn'}(t)|n\rangle\langle n'| \tag{6}$$

$$\rho_{nn'}(t) = q_n q_{n'}^*[C_g^n(t)C_g^{n'*}(t) + C_e^n(t)C_e^{n'*}(t)] \tag{7}$$

Supposing initially the atom is in the state $|g\rangle$, i.e. $C_g(0) = 1$, and $C_e(0) = 0$, and let $g_1 = g_2 = g$ for simplisity, we get

$$\rho_{nn'}(T) = q_n q_{n'}^* exp[-i(n-n')T]cos(n-n')T \tag{8}$$

in which $T = \lambda t$. We see that the diagonal elements ρ_{nn} are independent of time and just the photon distribution function of initial field.

3 The Amplitude Nth-Power Squeezing(ANPS)

The amplitude Nth-power squeezing of a radiation field is defined in terms of the following quantities[4]

$$Z_1(N) = \frac{1}{2}(a^N + a^{+N}), \qquad Z_2(N) = \frac{1}{2i}(a^N - a^{+N}) \tag{9}$$

$Z_1(N)$ and $Z_2(N)$ satisfy the commutation relation and the uncertainty relation

$$[Z_1(N), Z_2(N)] = \frac{i}{2}[a^N, a^{+N}] \tag{10}$$

$$\langle(\Delta Z_1(N))^2\rangle\langle(\Delta Z_2(N))^2\rangle \geq \frac{1}{16}|\langle[a^N, a^{+N}]\rangle|^2 \tag{11}$$

The field is said to be Nth-power squeezed if

$$\langle(\Delta Z_i(N))^2\rangle < \frac{1}{4}\langle[a^N, a^{+N}]\rangle \qquad (i = 1,2) \tag{12}$$

Here we introduce a parameter named squeezed degree $S_i(N)$

$$S_i(N) = \frac{D_i(N)}{C(N)} \qquad (i = 1,2) \tag{13}$$

where $C(N)$ and $D_i(N)$ are defined as

$$C(N) = \langle[a^N, a^{+N}]\rangle, \qquad D_i(N) = 4\langle(\Delta Z_i(N))^2\rangle - \langle[a^N, a^{+N}]\rangle \tag{14}$$

Then the field is Nth-power squeezed if $D_i(N) < 0$, ($S_i(N) < 0$). $S_i(N) = -1$ corresponds to 100% squeezing. In the following section we will study ANPS in DRP. We will consider several kinds of initial field states.

4 ANPS in DRP

4.1. For an Initial Coherent State

$$|\alpha\rangle = \sum_{n=0}^{\infty} q_n^c |n\rangle, \qquad \alpha = \bar{n}^{\frac{1}{2}} e^{i\xi_c}$$

$$q_n^c = Q_n^c e^{in\xi_c}, \qquad Q_n^c = (e^{-\bar{n}}\frac{\bar{n}^n}{n!})^{\frac{1}{2}} \tag{15}$$

then we have

$$\rho_{nn'}^c(T) = Q_n^c Q_{n'}^c exp[-i(n-n')(T-\xi_c)]cos(n-n')T \tag{16}$$

We can find

$$D_1(N) = 4\bar{n}^N sin^2(NT) sin^2[N(T-\xi_c)]$$
$$D_2(N) = 4\bar{n}^N sin^2(NT) cos^2[N(T-\xi_c)] \tag{17}$$

We see that in a degenerate Raman process the field will not get Nth-power squeezing if it is initially in a coherent state.

4.2. For an Initial Squeezed Vacuum

$$|0_{sq}\rangle = \sum_{n=0}^{\infty} q_{2n}|2n\rangle, \qquad q_{2n} = Q_{2n}e^{in\xi}$$

$$Q_{2n} = (\frac{1}{\bar{n}+1})^{\frac{1}{4}}[-\frac{1}{2}(\frac{\bar{n}}{\bar{n}+1})^{\frac{1}{2}})]^n \frac{[(2n)!]^{\frac{1}{2}}}{n!} \tag{18}$$

where $\bar{n}$ is the mean photon number and ξ is the squeezing angle of the initial field. Then we have

$$\rho_{2n,2n'}(T) = Q_{2n}Q_{2n'}exp[-i(n-n')(2T-\xi)]cos(n-n')2T \tag{19}$$

We see that only even-photon-number states can be found in a squeezed vacuum. The photon-number distribution function is

$$P_{2n} = \rho_{2n,2n} = Q_{2n}Q_{2n} \tag{20}$$

For $N = odd = 2M-1 (M = 1,2,3,...)$ we can find

$$C(1) = 1$$

$$D_1(1) = 2\{\bar{n} - [\bar{n}(\bar{n}+1)]^{\frac{1}{2}} cos(2T-\xi)cos(2T)\}$$
$$D_2(1) = 2\{\bar{n} + [\bar{n}(\bar{n}+1)]^{\frac{1}{2}} cos(2T-\xi)cos(2T)\} \tag{21}$$
$$C(3) = 3(9\bar{n}^2 + 9\bar{n} + 2)$$

$$D_1(3) = 6\{\bar{n}^2(5\bar{n}+3) - 5[\bar{n}(\bar{n}+1)]^{\frac{3}{2}} cos(6T-3\xi)cos(6T)\}$$
$$D_2(3) = 6\{\bar{n}^2(5\bar{n}+3) + 5[\bar{n}(\bar{n}+1)]^{\frac{3}{2}} cos(6T-3\xi)cos(6T)\} \tag{22}$$

We can show that $[D_2(2M-1)]_{\xi=\pi} = [D_1(2M-1)]_{\xi=0}$ can be smaller than zero, but $[D_1(2M-1)]_{\xi=\pi} = [D_2(2M-1)]_{\xi=0}$ can not be smaller than zero. This shows that we can have squeezing in $Z_1(2M-1)$ components for $\xi = 0$ and in $Z_2(2M-1)$ components for $\xi = \pi$, but we have not squeezing in $Z_1(2M-1)$ components for $\xi = \pi$ and in $Z_2(2M-1)$ components for $\xi = 0$.

For $N = even = 2M (M = 1,2,3,...)$ we have

$$C(2) = 2(2\bar{n}+1)$$

$$D_1(2) = 2\bar{n}\{(3\bar{n}+1) + (\bar{n}+1)[3cos(4T-2\xi)cos(4T) - 2cos^2(2T-\xi)cos^2(2T)]\}$$
$$D_2(2) = 2\bar{n}\{(3\bar{n}+1) - (\bar{n}+1)[3cos(4T-2\xi)cos(4T) + 2sin^2(2T-\xi)cos^2(2T)]\} \quad (23)$$
$$C(4) = 24(10\bar{n}^3 + 15\bar{n}^2 + 7\bar{n} + 1)$$
$$D_1(4) = 6\bar{n}^2\{(35\bar{n}^2 + 30\bar{n} + 3) + (\bar{n}+1)^2[35cos(8T-4\xi) - 6cos^2(4T-2\xi)cos^2(4T)]\}$$
$$D_2(4) = 6\bar{n}^2\{(35\bar{n}^2 + 30\bar{n} + 3) - (\bar{n}+1)^2[35cos(8T-4\xi) + 6sin^2(4T-2\xi)cos^2(4T)]\} \quad (24)$$

We can show that $[D_2(2M)]_{\xi=\pi} = [D_2(2M)]_{\xi=0}$ can be smaller than zero, but $[D_1(2M)]_{\xi=\pi} = [D_1(2M)]_{\xi=0}$ can not be smaller than zero. This shows that we can have squeezing in $Z_2(2M)$ components for both $\xi = 0$ and $\xi = \pi$, but we can not get squeezing in $Z_1(2M)$ components for $\xi = 0$ and $\xi = \pi$.

We are also interested in the optimal squeezing.

$$[S(1)]_{min} = 2\{\bar{n} - [\bar{n}(\bar{n}+1)]^{\frac{1}{2}}\}$$
$$[S(2)]_{min} = -\frac{2\bar{n}}{2\bar{n}+1}$$
$$[S(3)]_{min} = \frac{2\{\bar{n}^2(5\bar{n}+3) - 5[\bar{n}(\bar{n}+1)]^{\frac{3}{2}}\}}{9\bar{n}(\bar{n}+1)+2}$$
$$[S(4)]_{min} = -\frac{2\bar{n}^2(5\bar{n}+4)}{10\bar{n}^3 + 15\bar{n}^2 + 7\bar{n} + 1} \quad (25)$$

We see that $[S(N)]_{min} \to 0$ when $\bar{n} \ll 1$, and $[S(N)]_{min} \to -1$ (100% squeezing) when $\bar{n} \gg 1$.

To see the features of the field fluctuation more clearly, we have done numerical calculation and drawn some figures(Fig.2-10). From these figures we see the follows:

1. Generally, the field fluctuation oscillates periodically, and the oscillation frequency is proportional to N(Fig.2-9).
2. For a given $\bar{n}$, the oscillation amplitude decreases as N increase (Fig.2-5).
3. For a given N, the oscillation amplitude increases as $\bar{n}$ increases, but $[S(N)]_{min}$ changes smaller as $\bar{n}$ increases (Fig.6-9). $S_{min} \to -1$ when $\bar{n} \gg 1$(Fig.10).

5 Conclusion

In this paper we have studied ANPS of radiation fields in DRP by using MEHA. We found that if the field is initially in a coherent state it will not get squeezing in any Nth-power; if the field is initially in a squeezed vacuum, it may get Nth-power squeezing. The relations between the time evolution of the field fluctuation with N, $\bar{n}$, and ξ are discussed.

References

1. R.Loudon and P.L.Knight,J.Mod.Opt.,34,709(1987)
2. C.K.Hong and L.Mandel,Phys.Rev.Lett.,54,323(1985); Phys.Rev.,32,974(1985)
3. M.Hillery, Opt.Commun.,62,135(1987); Phy.Rev.A,36,3796(1987)
4. Z.M.Zhang et al.,Phy.Lett.A,150,27(1990)
5. S.D.Du and C.D.Gong,Phys.Lett.A,168,296(1992); Phys.Rev.A,48,2198(1993)
6. Y.Zhan,Phys.Rev.A,46,686(1992)
7. J.Sun et al.,Phys.Rev.A,46,1700(1992)
8. A.Q.Ma et al.,Chin.J.Lasers B,2,257(1993)
9. H.H.Jiang and L.S.He,Acta Physica Sinica,42,1223(1993)
10. X.L.Wu,Acta Physica Sinica.43,1433(1994)
11. J.Wang et al.,Acta Optica Sinica,14,819(1994)

12.T.Song et al.,Chin.J.Quant.Electr.,11,46(1994)
13.J.C.Retamal et al.,Phys.Rev.A,45,1876(1992)
14.S.Y.Zhu et al.,Z.Phys.D,22,483(1992)
15.P.L.Knight,Phys.Scr.T,12,51(1986)
16.S.D.J.Phoenix, and P.L.Knight,J.Opt.Soc.Am.B,7,116(1990)
17.L.Xu and Z.M.Zhang,Z.Phys.B,95,507(1994)

Figiure Captions

Fig.1 Schematic diagram of the degenerate Λ-type three-level atom interaction with a single-mode field. ω: frequency of field; δ: atom field detuning.
Fig.2 S_1 vs T. $\bar{n}$=0.1 a: N=1; b: N=3
Fig.3 S_1 vs T. $\bar{n}$=1.0 a: N=1; b: N=3
Fig.4 S_2 vs T. $\bar{n}$=0.1 a: N=2; b: N=4
Fig.5 S_2 vs T. $\bar{n}$=1.0 a: N=2; b: N=4
Fig.6 $S_1(1)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n} = 1.0$; c: $\bar{n} = 5.0$
Fig.7 $S_2(2)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n} = 1.0$; c: $\bar{n} = 5.0$
Fig.8 $S_1(3)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n} = 1.0$; c: $\bar{n} = 5.0$
Fig.9 $S_2(4)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n} = 1.0$; c: $\bar{n} = 5.0$
Fig.10 $[S(N)]_{min}$ vs $\bar{n}$. a,b,c,d corresponde to N=1,2,3,4 espectively.

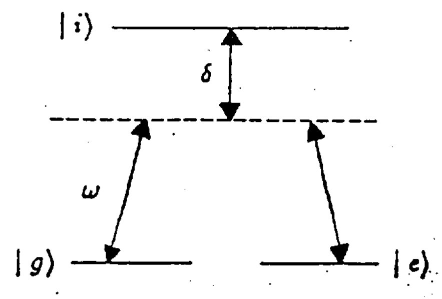

Fig.1

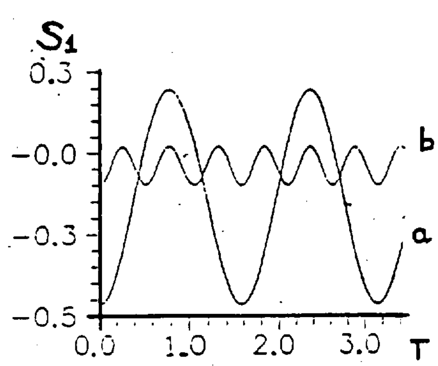

Fig.2

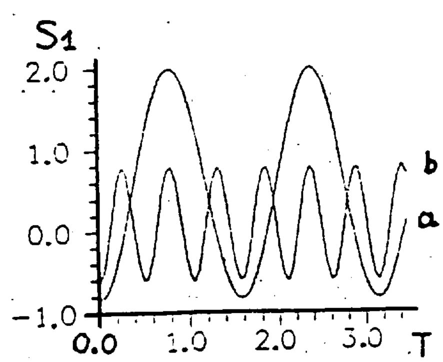

Fig.3

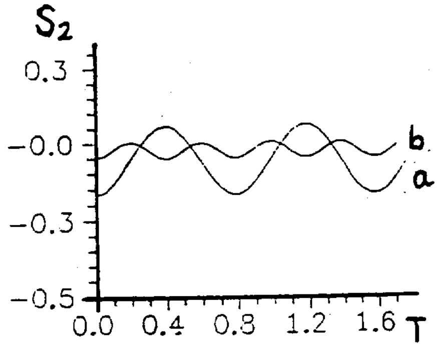

Fig.4

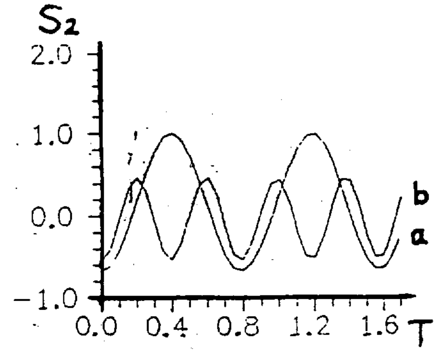

Fig.5

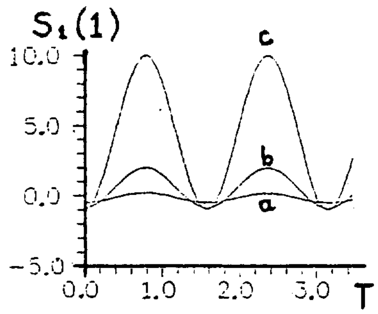

Fig.6

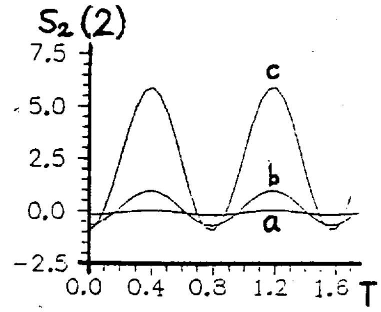

Fig.7

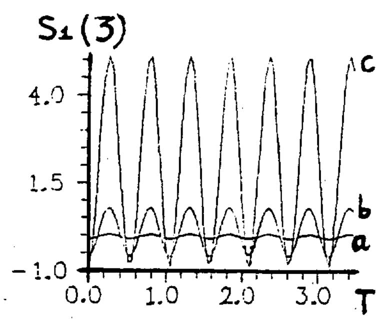

Fig.8

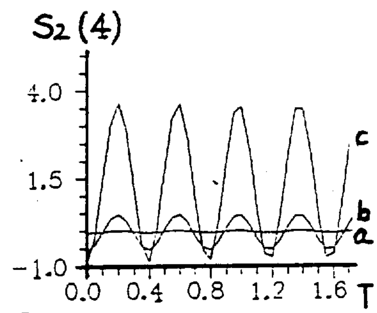

Fig.9

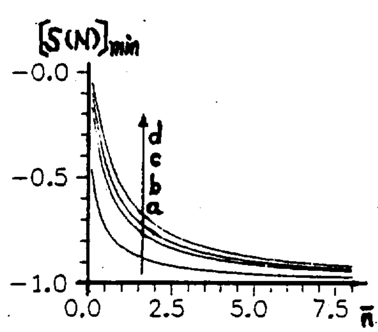

Fig.10