The quantum phase-dynamical properties of the squeezed vacuum state intensity-couple interacting with the atom

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Abstract
The Phase-dynamical properties of the squeezed vacuum state intensity-couple interacting with the two-level atom in an ideal cavity are studied using the Hermitian phase operator formalism. Exact general expressions for the phase distribution and the associated expectation value and variance of the phase operator have been derived. we have also obtained the analytic results of the phase variance for two special cases–weakly and strongly squeezed vacuum. The results calculated numerically show that squeezing has a significant effect on the phase properties of squeezed vacuum.

1 Introduction

The squeezed state exhibits phase sensitive noise properties. Therefore, it is important to examine the phase properties of squeezed state of light. Recently, Sanders et al[1], and Yao[2], and Fan et al[3] have studied phase properties of the ideal squeezed state using Susskind and Glogower phase-operator formalism[4]. Vaccaro et al[5] have re-examined the phase properties of the squeezed vacuum state, particularly, the weakly squeezed vacuum state using the phase-operator formalism of Pegg and Barnett[6-8]. However, they have not considered the field-atom interaction. Dung et al[9] and Fan et al[10-11] have studied the phase properties of a coherent light interacting with a two-level atom. Bužek[12] has studied the time evolution of the squeezing and the atomic population inversion in the Jaynes- Cummings model(JCM) with intensity-dependent coupling with the squeezed vacuum state.

In the present paper, we will study phase properties in the JCM with intensity-dependent coupling with a light field initially prepared in the squeezed vacuum state using Pegg-Barnett phase-operator formalism. The results calculated numerically show that in such a model how the squeezing have an effect on phase properties.

2 The model

The model Hamiltonian for the JCM with the intensity-dependent coupling in the rotating-wave approximation is [13]

$$H = \hbar\omega_0 S^Z + \hbar\omega N + \hbar g(S^+R + S^-R^+), \tag{2.1}$$

where $N = a^+a, R = a\sqrt{N}, R^+ = \sqrt{N}a^+$; a^+ and a is the creation and annihilation operators of the field mode of frequency ω, S^Z and $S^\pm$ are the pseudospin operators for the two-level atom of frequency ω_0, g is the atom-field coupling constant. The commutation relations for N, R and R^+ are

$$[R, R^+] = 2N + 1, [R, N] = R, [R^+, N] = -R^+. \tag{2.2}$$

We assume the initial state of the light field to be the squeezed vacuum state $|0,\xi >= S(\xi)|0>$, where $S(\xi)$ is the squeeze operator and ξ is complex squeeze parameter:

$$S(\xi) = \exp[\frac{1}{2}(\xi^* a^2 - \xi a^{+2})]. \tag{2.3}$$

$$\begin{aligned} &\xi = |\xi|e^{i\beta}, r = |\xi|, \beta = 2\eta, \\ &0 \leq r < \infty, 0 \leq \beta \leq 2\pi. \end{aligned} \tag{2.4}$$

We can find the $|n>$ state representation of the state $|0,\xi>$ [14] :

$$|0,\xi >= \sum_{n=0}^{\infty} Q_n|2n>, \tag{2.5}$$

$$Q_n = (\mathrm{sech} r)^{1/2}\frac{[(2n)!]^{1/2}}{n!}(-\frac{1}{2}e^{2i\eta}\tanh r)^n. \tag{2.6}$$

If the atom is supposed to be in the excited state $|e>$ at the initial time, then the initial state $|\Psi(0)>$ of the system is

$$|\Psi(0) >= |0,\xi > \otimes|e >= \sum_{n=0}^{\infty} Q_n|e, 2n>, \tag{2.7}$$

In the resonant case, the exact solution for an initial state $|\Psi(0)>$ given by Eq.(2.7), is

$$|\Psi(t) >= \sum_{n=0}^{\infty} \exp[-i(E_e + 2n\hbar\omega)t]\cdot Q_n[\cos[(2n+1)gt]|e, 2n > -i\sin[(2n+1)gt]|g, 2n+1>], \tag{2.8}$$

where E_e is the energy of excited state of the atom.

3 The phase properties

Accooding to Pegg and Barnett[6-8], the Hermitian phase operator operates on a $(s+1)$ - dimensional subspace spanned by $(s+1)$ number states. The value of s can be made arbitrary large. A complete set of $(s+1)$ orthonormal phase states is defined by

$$|\theta_m >= (s+1)^{1/2}\sum_{n=0}^{S} \exp(in\theta_m)|n>, \tag{3.1}$$

where phase $\theta_m = \theta_0 + 2\pi m/(s+1), m = 0, 1, 2, \cdots, s$. The Hermitian phase operator is

$$\hat{\Phi}_\theta = \sum_{m=0}^{S} \theta_m |\theta_m >< \theta_m|, no(3.2)$$

The phase states $|\theta_m>$ are eigenstates of $\hat{\Phi}_\theta$ with the eigenvalues θ_m. We see that the eigenvalues θ_m are restricted to lie within a phase window θ_0 and $\theta_0 + 2\pi$, where θ_0 is an arbitrary real number. It has to be noted that after all expectation values of phase variables of light field have been calculated in the finit $(s+1)$ dimensional space, s is allowed to tend to infinity. The phase distribution of the state given by Eq.(2.8), is

$$P(\theta_m, t) = | < \theta_m|\Psi(t) > |^2, \tag{3.3}$$

with the expectation value and the variance

$$< \hat{\Phi}_\theta >= \sum_m \theta_m P(\theta_m, t), \tag{3.4}$$

$$< \Delta\Phi_\theta^2 >= \sum_m (\theta_m - < \hat{\Phi}_\theta >)^2 P(\theta_m, t). \tag{3.5}$$

Now we choose the reference phase $\theta_0 = \eta - \pi s/(s+1)$, and introduced a new phase label $\mu = m - s/2$, which ranges in integer steps from $-s/2$ and $s/2$. When s tends to infinity we replace $\mu 2\pi/(s+1)$ by $d\theta$ and $2\pi/(s+1)$ by θ. Then we find a continuous phase distribution

$$P(\theta, \tau) = \frac{1}{2\pi}\{1 + 2\text{sech}r \sum_{n,n',n>n'} \frac{[(2n)!(2n')!]^{1/2}}{n!n'!}(-\frac{1}{2}\tanh r)^{n+n'} \cos[2(n-n')\theta] \cos[2(n-n')\tau]\}, \tag{3.6}$$

where $\tau = gt, P(\theta_m, \tau)$ is normalized according to

$$\int_{-\pi}^{\pi} P(\theta, \tau) d\theta = 1, \tag{3.7}$$

The numerical calculation results of formula (3.6) are shown in Fig.1. In Fig.1, the phase distributions are plotted against θ in the polar coordinate system. It is seen from Fig.1, that as $r = 0$, the phase distribution has slways circle shape for any values of τ. As r is not equal to zero, the bifurcation of the phase distribution appears. At $\tau = 0$, as r is increased, the circle shape splits into two separate leaves (Fig.1(a)). As the interaction is turned on, this circle shape splits into four separate leaves which rotate and change its shape (Fig.1(b) and (c)). The larger the squeezing parameter is, the more obvious the phase distribution splits. At $\tau = \pi/2$, the four satellite distributions become again two satellite distributions and rotate by $\pi/2$ from the state of $\tau = 0$(Fig.1(d)). At $\tau = \pi$, the shape of $P(\theta, \tau)$ is the same as that at $\tau = 0$ (Fig.1(a)), and so on.

Using Eq.(3.6), and replacing the summation in Eqs.(3.4) and (3.5) by an approrite integral for the variable θ, over the range $-\pi$ to π, and taking into accout $\theta_m = \theta + \eta$, in the limit as s tends to infinity, we obtain

$$< \hat{\Phi}_\theta >= \eta = \beta/2, \tag{3.8}$$

$$< \Delta\Phi_\theta^2 >= \frac{\pi^2}{3} + \text{sech}r \sum_{n>n'} \frac{[(2n)!(2n')!]^{1/2}}{n!n'!} \cdot (-\frac{1}{2}\tanh r)^{n+n'} \frac{1}{(n-n')^2} \cos[(2(n-n')\tau], \tag{3.9}$$

The numerical calculation results of the variance of phase given by formula (3.9) are illustrated in Fig.2(a). As $r=0$, the phase variance of vacuum state is equal to $\pi^2/3$. which reflects random phase character. As $r \neq 0$, the phase variance shows periodic oscilation around $\pi^2/3$. The larger the squeezing parameter is, the larger the oscillational amplitude of phase variance is. The phase variance is calculated numerically as a function of r and plotted in Fig.3 for different values of τ. We see that the variance is departure from $\pi^2/3$ as r is increased.

In the limits of small r (weakly squeezed vacuum) and larger r (strongly squeezed vacuum), from Eq.(3.9), we can find respectively

$$<\Delta\Phi_\theta^2>=(1/3)\pi^2 - 2^{-1/2} r\cos(2\tau) + 1/4(3/8)^{1/2} r^2 \cos(4\tau) + O(r^3), \tag{3.10}$$

$$<\Delta\Phi_\theta^2>=\frac{1}{3}\pi^2 - A_1\cos(2\tau) + \frac{1}{4}A_2\cos(4\tau) - \frac{1}{9}A_3\cos(6\tau) + \frac{1}{16}A_4\cos(8\tau) + \cdots, \tag{3.11}$$

where

$$A_1 = \tanh r(1 - \frac{1}{2}\frac{\cosh r}{\sinh^2 r} + \frac{1}{2}\frac{1}{\sinh^2 r}), \tag{3.12}$$

$$A_2 = \tanh^2 r(1 - \frac{5}{12}\frac{\cosh^3 r}{\sinh^4 r} - \frac{3}{8}\frac{\cosh r}{\sinh^2 r} + \frac{5}{12}\frac{\cosh^2 r}{\sinh^4 r} + \frac{7}{12}\frac{1}{\sinh^2 r}), \tag{3.13}$$

$$A_3 = \tanh^3 r(1 - \frac{3}{8}\frac{\cosh^5 r}{\sinh^6 r} - \frac{5}{16}\frac{\cosh^3 r}{\sinh^4 r} - \frac{21}{64}\frac{\cosh r}{\sinh^2 r} + \frac{3}{8}\frac{\cosh^4 r}{\sinh^6 r} + \frac{1}{2}\frac{\cosh^2 r}{\sinh^4 r} + \frac{5}{8}\frac{1}{\sinh^2 r}, \tag{3.14}$$

$$A_4 = \tanh^4 r(1 - \frac{39}{112}\frac{\cosh^7 r}{\sinh^8 r} - \frac{63}{224}\frac{\cosh^5 r}{\sinh^6 r} - \frac{245}{896}\frac{\cosh^3 r}{\sinh^4 r} - \frac{539}{1792}\frac{\cosh r}{\sinh^2 r}$$

$$+\frac{39}{112}\frac{\cosh^6 r}{\sinh^8 r} + \frac{51}{112}\frac{\cosh^4 r}{\sinh^6 r} + \frac{61}{112}\frac{\cosh^2 r}{\sinh^4 r} + \frac{73}{112}\frac{1}{\sinh^2 r}). \tag{3.15}$$

The numerical calculation results of the variance of phase given by formulas (3.10) and (3.11) are illustrated in Fig.2(b). We see that Fig.2(b) coincides well with Fig.2(a) plotted by exact formula (3.9).

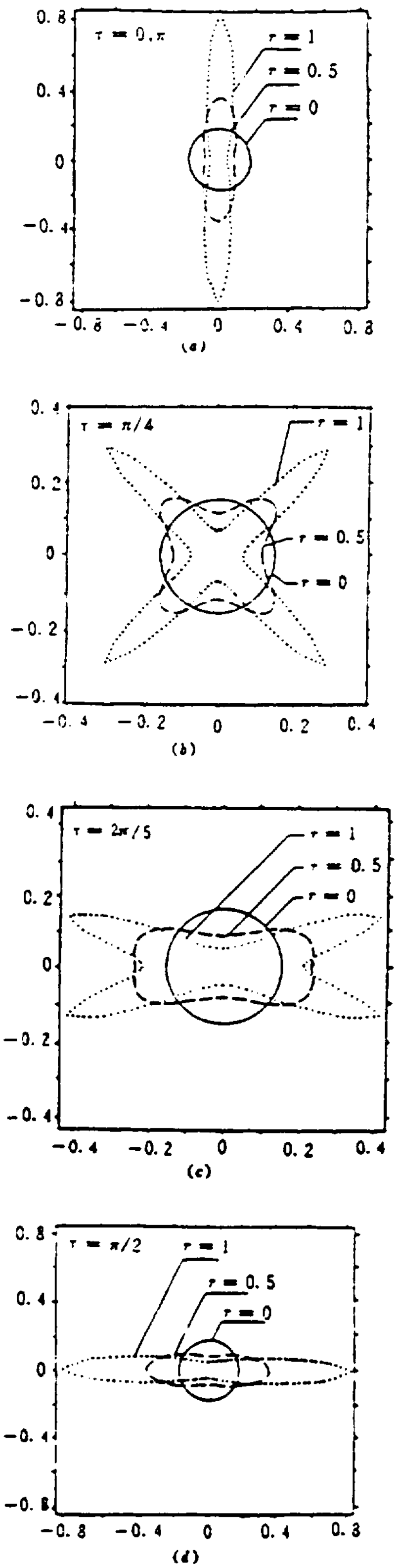

Fig.1. The phase distribution $P(\theta,\tau)$ plotted against θ in the polar coordinate system for various values of τ and different values of r.

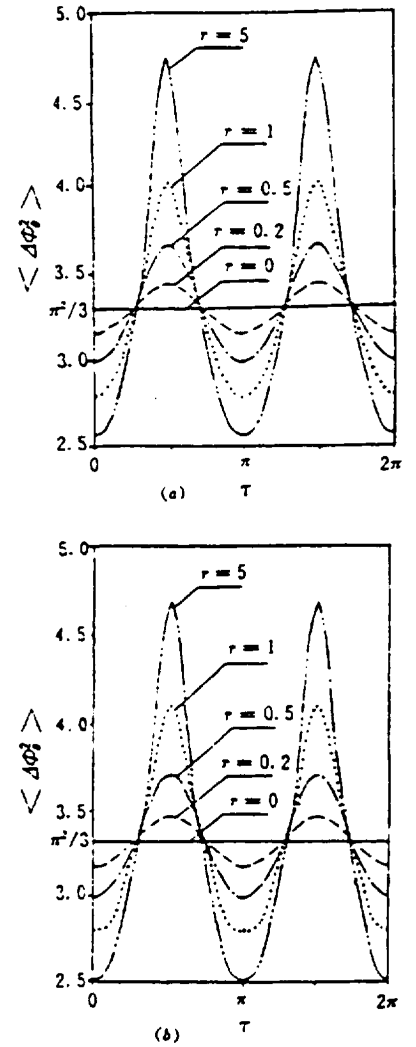

Fig.2. Plot of the phase variance $< \Delta\Phi_\theta^2 >$ as a function of τ for different values of r. (a) according to exact formula (3.9), (b) according to approximate formula (3.10) for $r = 0.2, 0.5$ and (3.11) for $r = 1, 5$.

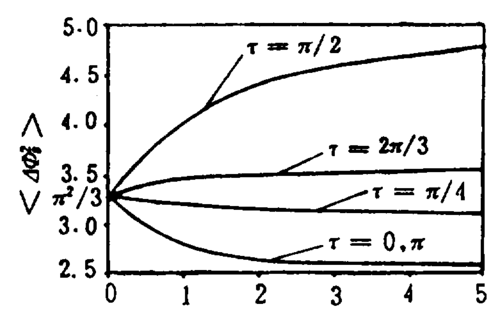

Fig.3. Plot of the phase variance $< \Delta\Phi_\theta^2 >$ as a function of r for different values of τ.

4 Conclusion

Using the Hermitian phase-operator formalism, the phase propeties of the squeezed vacuum intensity-couple interacting with an atom have been obtained. We have found that the bifurcation of the phase distribution appears as squeezing parameter is not equal to zero. At the initial time the phase distribution splits into two symmetric distributions, that is, two satellite leaves in a polar representation. As the interaction is turned on, the phase distribution splits into four symmetric distribution, that is, four satellite leaves which rotate and change its shape in a polar representation. At the scaled time $\tau = gt = \pi/2$, the four satellite leaves become again two satellite leaves and rotate by $\pi/2$ from the state of $\tau = 0$. We have also found that the phase variance shows periodic oscilation around $\pi^2/3$ as squeezing parameter is not equal to zero. This reflects the fact that the squeezed vacuum has the non-random phase character because of the squeezing.

Acknowledgments

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References

[1] B.C.Sanders, S.M.Barnett and P.L.Knight, Optics Comm.58,290 (1986)

[2] D.M.Yao, Phys.Lett.A122,77 (1987)

[3] H.Y.Fan and H.R.Zaidi, Optics Comm.68,143 (1988)

[4] L.Susskind and J.Glogower, Physics 1,49 (1964)

[5] J.A.Vaccaro and D.T.Pegg, Optics Comm.70,529 (1989)

[6] D.T.Pegg and S.M.Barnett, Europhysics Lett.6,483 (1988)

[7] S.M.Barnett and D.T.Pegg, J.Mod.Opt.36,7 (1989)

[8] D.T.Pegg and Barnett, Phys.Rev.A39,1965 (1989)

[9] H.T.Dung, R.Tanaś and A.S.Shumovsky, Optics Comm.79,462 (1990); J.Mod.Opt.38,2069 (1991)

[10] A.F.Fan, Optics Comm.98,340 (1993)

[11] A.F.Fan and Z.W.Wang, Phys.Rev.A 49,1509 (1994)

[12] V.Bužek, Phys.Lett.A 139,231 (1989)

[13] B.Buck and C.V.Sukumar, Phys.Lett. A 81,132 (1981)

[14] J.N.Hollenhorst, Phys.Rev. D 19,1669 (1979)