FOCK STATE GENERATION
FROM THE NONLINEAR KERR MEDIUM

W. Leoński and R. Tanaś

Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University,
Grunwaldzka 6, 60-780 Poznań, Poland.

Abstract

We discuss a system comprising a nonlinear Kerr medium in a cavity driven by an external coherent field directly or through the parametric process. We assume that the system is initially in the vacuum state, and we show that under appropriate conditions, i.e., properly chosen detuning and intensity of the driving field, the one or two-photon Fock states of the electromagnetic field can be achieved.

1 One-photon state generation

The model discussed here contains a nonlinear Kerr medium, described as an anharmonic oscillator, placed in a lossless cavity driven by an external coherent field. The coupling of the cavity field with the external field is governed by the following Hamiltonian in the interaction picture (we use units of $\hbar = 1$):

$$\hat{H}_{ext.} = \epsilon \left(\hat{a} + \hat{a}^\dagger \right) \ , \tag{1}$$

where ϵ denotes the strength of the coupling, whereas $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the cavity field, respectively. The Hamiltonian corresponding to the dynamics of the nonlinear Kerr medium in the cavity can be written as follows :

$$\hat{H}_{Kerr} = \frac{\lambda}{2}\hat{n}\left(\hat{n} - 1 \right) \ , \tag{2}$$

where λ is proportional to the third-order nonlinear susceptibility of the medium and $\hat{n}$ is the photon number operator. Our aim here is to determine the time evolution of the system. We assume that the system is initially in the vacuum state $|0\rangle$. Moreover, we assume that the external field driving the cavity according to (1) is weak, i.e. $\epsilon \ll \lambda$. In consequence, we can treat the problem perturbatively with respect to the small parameter ϵ.

Let us express the state of the system in a Fock basis:

$$|\Psi(t)\rangle = \sum_{j=0}^{\infty} a_j(t) |j\rangle \ . \tag{3}$$

This state vector obeys the Schrödinger equation with the Hamiltonians expressed by eqs. (1,2):

$$i\frac{d}{dt} |\Psi(t)\rangle = (H_{Kerr} + H_{ext.}) |\Psi(t)\rangle \ . \tag{4}$$

Applying the standard procedure to the state vector (3) and the Hamiltonians (1,2), we obtain a set of equations for the probability amplitudes a_j. Although this set of equations is infinite, it can be shown [1] that due to the degeneracy of the Hamiltonian (2) and the weakness of the driving field the system dynamics is restricted to the subspace of the degenerate states. In consequence, the evolution of the systems starts from the vacuum $|0\rangle$ and the only state that can be essentially populated with the driving field, according to (1), is the one photon state $|1\rangle$. The crucial point of our considerations is the fact that the unperturbed Hamiltonian for the Kerr process (2) produces degenerate states $|0\rangle$ and $|1\rangle$. In practice, we deal here with a situation analogous to that discussed in the paper [2] and we can write the following equations of motion for the probability amplitudes:

$$i\frac{d}{dt}a_0(t) = \epsilon a_1 ,$$

$$i\frac{d}{dt}a_1(t) = \epsilon a_0 . \tag{5}$$

Assuming $a_0(t = 0) = 1$ and $a_1(t = 0) = 0$ we get the following solution for the probability amplitudes

$$a_0 = i\cos(\epsilon t) ,$$

$$a_1 = \sin(\epsilon t) . \tag{6}$$

We treat eq.(6) as the zero-order solution. For this order the amplitude $a_2 = 0$. To obtain the formula for a_2 we need higher order solutions. We write the first-order formula for a_2:

$$a_2 = -\frac{\epsilon\sqrt{2}}{\lambda}\sin(\epsilon t) + \mathcal{O}(\epsilon^2) , \tag{7}$$

where we have removed all terms proportional to ϵ^2. Obviously, we are in a position to perform this perturbative procedure due to the fact that the coupling (1) is weak, i.e. ($\epsilon \ll \lambda$). Moreover, since we are interested in finding the time evolution of the probabilities rather than the amplitudes a_j, we neglect the influence of the dynamics of the state $|2\rangle$ on the system as being proportional to ϵ^2.

To verify these results we shall now perform a numerical experiment and compare its results with those based on formulas (6). This will be done similarly as in the paper [3].

The history of our system is governed by the unitary evolution operator $\hat{U}(t)$ defined as follows:

$$\hat{U}(t) = \exp(-i\hat{H}t) . \tag{8}$$

Hence, the state vector $|\Psi(t)\rangle$ for arbitrary time t can be expressed as:

$$|\Psi(t)\rangle = \hat{U}(t)|0\rangle . \tag{9}$$

For numerical calculations we use the number state basis, which is truncated as to obtain sufficient numerical accuracy.

Fig.1 shows the probabilities of finding the system in the vacuum $|0\rangle$ and one-photon states $|1\rangle$. We assume that for the time $t = 0$ the field was in the vacuum state

$(a_0(t = 0) = 1)$, and that the coupling (1) is weak, i.e., $\epsilon = \pi/50 \ll \lambda$ (in units of $\lambda = 1$).
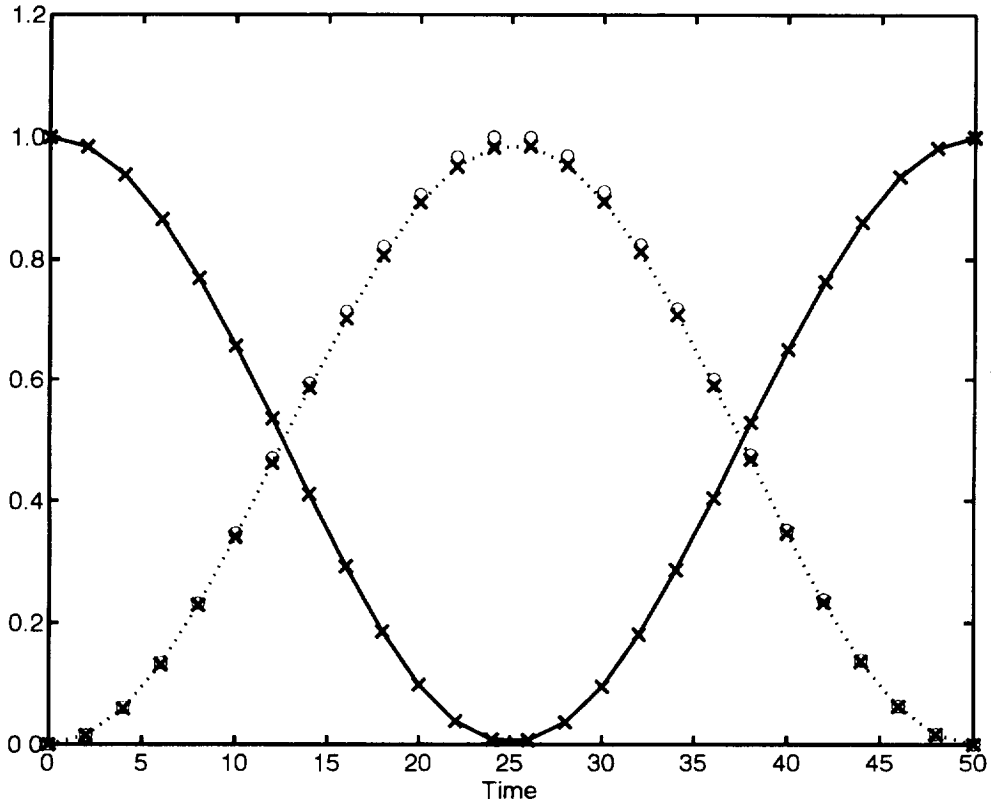

FIG.1 Analytical solutions for the probabilities of the vacuum (solid line) and one-photon (dotted line) states, and the mean number of photons (circle marks) obtained from the numerical experiment. The parameter $\epsilon = \pi/50$ (all parameters are measured in units of $\lambda = 1$). X-marks correspond to the probabilities found in the numerical experiment.

We see that our analytical results (solid and dashed lines) agree perfectly with those generated in the numerical experiment (star marks). The system starts to evolve from the vacuum and after the time $t = 25$ the probability $|a_1|^2 = 1$. This means that at this moment of time the field is in the pure one-photon state. For longer times the system returns to its initial state and starts to evolve in the same way as from $t = 0$. Moreover, we have plotted in Fig.1 the time dependence for the mean number of photons $n(t)$ (dotted line)

$$n(t) = \langle \Psi(t = 0)| \, \hat{U}^\dagger \hat{a}^\dagger \hat{a} \hat{U} \, |\Psi(t = 0)\rangle \tag{10}$$

found in our numerical experiment. It is seen that the behavior of $n(t)$ reflects the evolution of the probabilities and oscillates between 0 ad 1. One should keep in mind, however, that if we increase the strength of the external coupling the picture changes drastically. For this situation the perturbation procedure breaks down. In consequence, as it is visible from the numerical

experiment, higher n-photon Fock states start to play a significant role. Fig.2 shows the probability amplitudes for $\epsilon = \pi/15$. We see that the influence of the amplitude corresponding to the two-photon state becomes visible and perturbs the dynamics of the vacuum and one-photon states significantly. Of course, results of the numerical experiment become different from those obtained analytically under assumption of week coupling.
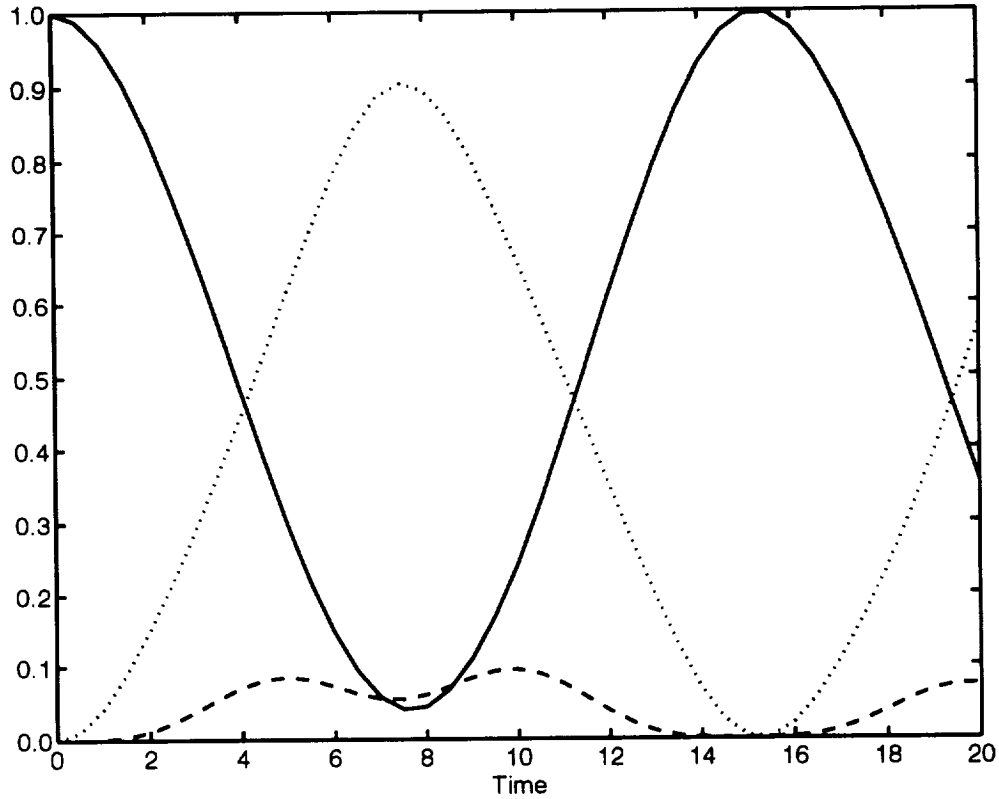

FIG. 2. The probability amplitudes corresponding to the vacuum (solid line), one-photon (dotted line) and two-photon (dashed line) states. The strength $\epsilon = \pi/15$ and the remaining parameters are the same as in Fig.1.

2   Two-photon state generation

Now, we consider a system containing the nonlinear Kerr medium which is parametrically excited by the electromagnetic field. The parametric excitation seems to be more suitable for the experimental realization of the model than the previous one. In this case the system is governed by the following Hamiltonian:

$$H = \frac{\chi}{2} n (n - 2) + \epsilon \left(\left(a^\dagger \right)^2 + (a)^2 \right) , \tag{11}$$

where the $\hat{n}(\hat{n} - 1)$ is replaced by $\hat{n}(\hat{n} - 2)$. This replacement can be justified by the appropriate choice of the detuning. With such a choice of the detuning the states $|0\rangle$ and $|2\rangle$ are degenerate,

and the parametric process, second term in (11), couples resonantly the two states. This suggests that the dynamics of the system will be restricted to the two states if the coupling is sufficiently weak. Except for a special choice of the detuning, the system discussed here resembles that discussed by Milburn [4], and Milburn and Holmes [5]. However, their model involved series of ultra-short excitations, whereas in this paper we assume continuous excitation.

Applying the same procedure as that for the one-photon state generation case we get the following equations for the probability amplitudes:

$$i\frac{d}{dt}a_0(t) = \epsilon\sqrt{2}a_2 ,$$

$$i\frac{d}{dt}a_2(t) = \epsilon\sqrt{2}a_0 . \tag{12}$$

We again assume that $a_0(t = 0) = 1$. In consequence the solutions for the amplitudes a_0 and a_2, to which the dynamics is restricted, are of the following form:

$$a_0 = i\cos\left(\epsilon\sqrt{2}t\right) ,$$

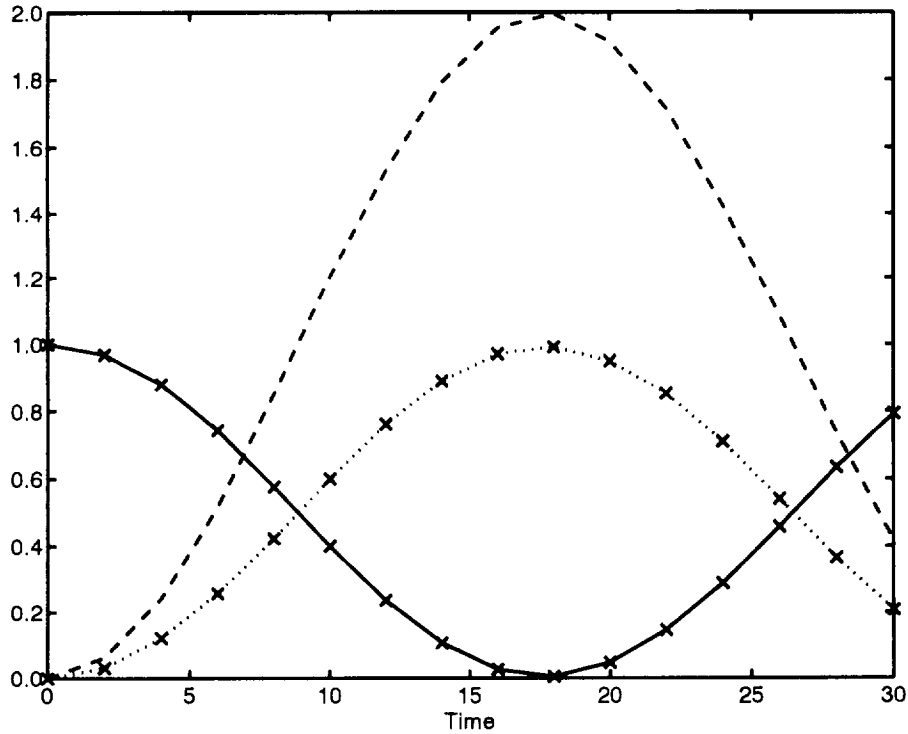$$a_2 = \sin\left(\epsilon\sqrt{2}t\right) . \tag{13}$$

FIG. 3. Analytical solutions for the probabilities of the vacuum (solid line) and two-photon (dotted line) states, and the numerically found mean number of photons (dashed line). The parameters $\epsilon = \pi/50$, $\lambda = 1$. Marks correspond to the numerical experiment results.

Obviously, formulas (13) are the zero-order solutions analogously as for the one-photon state (eq.(6)). Moreover, we shall perform numerical experiment and compare its results with those of eq.(13) again. For this case the unitary evolution operator $\hat{U}$ is constructed on the basis of the Hamiltonian defined in (11). Fig.3 depicts the probability amplitudes for the vacuum $|0\rangle$ and two-photon states $|2\rangle$ obtained from the eq.(13) and from the numerical experiment. We see very good agreement between the perturbative analytical results and those obtained from the experiment again. The system starts its evolution from the vacuum state and after the time $t = \pi / \left(2\sqrt{2}\epsilon\right) \simeq 17.7$ the two-photon Fock state is reached. Moreover, the numerical results show that the probability for the four-photon $|4\rangle$ state is proportional to $\epsilon^2 \simeq 3 \cdot 10^{-3}$ and can be neglected for the case discussed here.

3 Conclusions

We have shown here that it is possible to generate the one-photon and two photon Fock states by the use of nonlinear Kerr media placed in a lossless cavity driven by a week external field. This generation is associated with resonant transitions between two Fock states and can be described analytically using standard perturbative procedure. Moreover, we have performed numerical experiments that show very good agreement with the analytical solutions. Of course, our considerations are based on a very simple model, and one should realize that many difficulties, for instance damping processes, can obscure the model and make it difficult to realize in practical experiments. Although it was not the aim of this paper to investigate the influence of such obstacles, one should keep in mind the fact of their existence. A short discussion of these problems was given in [3].

Acknowledgments

References

[1] W. Leoński, S. Dyrting, and R. Tanaś, Coherence and Quantum Optics VII, Rochester, 1995, and in preparation

[2] W. Leoński, Acta.Phys.Slov. **45**, 383 (1995).

[3] W. Leoński and R. Tanaś, Phys.Rev. A **49**, R20 (1994).

[4] G. J. Milburn, Phys.Rev. A **41**, 6567 (1990).

[5] G. J. Milburn and C. A.R. Holmes, Phys.Rev. A **44**, 4704 (1991).