ARE THERE OPTICAL SOLITARY WAVE SOLUTIONS IN LINEAR MEDIA WITH GROUP VELOCITY DISPERSION ?

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Abstract

A generalized exact optical bright solitary wave solution in a three dimensional dispersive linear medium is presented. The most interesting property of the solution is that it can exist in the normal group-velocity- dispersion (GVD) region. In addition, another peculiar feature is that it may achieve a condition of "zero-dispersion" to the media so that a solitary wave of arbitrarily small amplitude may be propagated with no dependence on its pulse width.

1 Introduction

It is well known that there exist undistorted travelling wave solutions with arbitrarily shape in bulk linear media in the absence of dispersion effects. We-can call such a travelling wave solitary wave or soliton on the analogy of its definition in nonlinear science. In the presence of GVD, it has been proved that transmission of solitary wave or soliton can be achieved in cubic nonlinear media [1,2]. This research for optical solitons has attracted considerable attention because of not only the properties of preserving their shape and energy during propagation through a medium but their potential applications in ultra-high bit-rate optical communication and ultrafast signal-routing systems [3]. Mathematically, these optical solitons are a particular solution of the (1+1)-dimensional nonlinear Schrödinger equation (NLSE) or the equations, which can be transformed into (1+1)- dimensional NLSE. As is well-known, there exist two kinds of solitons in the (1+1)-dimensional NLSE: bright and dark solitons [1]. In physics, optical solitons can be classified as temporal and spatial solitons. In the case of temporal solitons, the GVD is balanced by self- phase modulation. In the spatial domain, a spatial soliton is better known as a "self-trapped beam", in which the self-focusing effect counteracts the diffraction [4]. In fact, the space-time analogy between dispersion pulse compression in time and optical-beam focusing in space has been pointed out early in 1969 [5-6]. When only diffraction or dispersion effects are considered, their governing equations are of the same structure under appropriate conditions. Now the four kinds of solitons (i.e. temporal bright, spatial bright, temporal dark,

spatial dark solitons) have been observed experimentally in optical fibers or in waveguides [2,7-13]. Besides the (1+1)-dimensional NLSE, it is necessary to deal with the higher-dimensional wave equation when a pulse propagate in optical media under the combined effect of diffraction and dispersion. In this case, one would expect that there exist the so-called light-bullets (i.e. stable, nondiffracting and nondispersing optical pulses) under certain conditions [14]. However, in contrast to (1+1)-dimensional NLSE, such a spatio-temporal solitonic solution has not yet been found even in theory due to the mathematical complexity of the higher- dimensional wave equations. On the other hand, the attempts of searching for multidimensional solitonic solutions in other kinds of optical media, such as exponential and quadratic media, have also been made [15-17]. Recently, we have proved, for the first time to our knowledge, that an envelope solitary wave solution mar exist in a two dimensional dispersive linear medium under certain appropriate conditions by taking into account the transverse effect and dispersion effect simultaneously [18]. In this paper, we will generalize the results in a three dimensional dispersive linear medium. It is proved that undistorted transmission of optical pulses in the above mentioned media may be realized even in the presence of GVD under appropriate conditions. Unlike the conventional bright solitary wave in cubic nonlinear media, the present bright solitary wave solution can be obtained in the normal (positive) GVD region. In addition, a peculiar feature of the solution is that it may achieve a condition of "zero-dispersion" to the media so that a solitary wave of arbitrarily small amplitude may be propagated with no dependence on its pulse width.

2 Governing Wave Equation

In the development that follows, we consider the propagation of pulses which are narrowly centered about a given frequency ω_0, and assume that the refractive index $n(\omega)$ is a slowly varying function of ω in the vicinity of ω_0 (which is generally true in situations of practical interest). It is convenient to represent the electric field intensity $\vec{E}(\vec{r}, t)$ by a product of an envelope and a rapidly oscillating terms:

$$\vec{E}(\vec{r}, t) = \hat{e}A(\vec{r}, t)e^{i(qz - \omega_0 t)} \tag{1}$$

where $\hat{e}$ is the polarization unit vector assumed to remain unchanged during pulse propagation, q the reference constant of propagation along z direction and ω_0 the carrier center frequency. Here we have restricted the development to be a scalar complex envelope function $A(\vec{r}, t)$.

Now let us consider the propagation of an optical pulse described by Eq.(1) in bulk dispersive homogeneous linear media. After removing the terms describing inhomogeneity and nonlinearity of media in Ref. [15], we can obtain the governing equation for the complex envelope function $A(\vec{r}, t)$. This three spatial and one temporal dimensions (3+1) linear wave equation with the GVD term included can be written in the form

$$\left[\nabla^2 - (k_0^{'2} + k_0 k_0^{''})\frac{\partial^2}{\partial t^2} + 2i(q\frac{\partial}{\partial z} + k_0 k_0^{'}\frac{\partial}{\partial t}) + k_0^2 - q^2\right] A(\vec{r}, t) = 0, \tag{2}$$

where $k \equiv \omega n(\omega)/c$ is the wave number, the primes indicate the derivatives with respect to ω, and the subscript 0 indicates evaluation at the carrier center frequency ω_0. Here, as is well

known (see, e.g.,[1]), k^2 is expanded around ω_0 in Taylor series and only terms up to second order are kept under the weak dispersion approximation (i.e., the refractive index is a slowly varying function of ω_0).

It is well-known that in the absence of GVD ($k_0'' = 0$), there are "complete" solitary wave solutions in Eq.(2). If the GVD does exist ($k_0'' \neq 0$), there will be no "complete" solitary wave solutions in Eq.(2). It is generally believed that the pulse shape will be distorted during its propagation. However, one will see in the following analysis that there may exist steady-state envelope solitary wave solutions in Eq.(2) under the combined action of transverse and dispersion effects.

3   A solitary wave solution and its property

In order to obtain a optical envelope solitary wave solution, let's introduce an ansatz with a hyperbolic secant function profile

$$A(\vec{r},t) = A_0 Sech(\frac{t - \vec{\alpha} \cdot \vec{r}}{\tau})e^{i(\vec{\beta} \cdot \vec{r} + \Delta \omega t)}, \tag{3}$$

where A_0 is the maximum amplitude of the optical envelope solitary wave solution. The parameter $\vec{\alpha}$ is the inverse of the group velocity, $\vec{\beta}$ describes the change of the wave vector, and $\Delta \omega$ is the frequency shift.

After substituting the ansatz (3) into Eq.(2), we can obtain three equations for the parameters $\vec{\alpha}$, $\vec{\beta}'$, and $\Delta \omega$:

$$\vec{\alpha} \cdot \vec{\alpha} = k_0'^2 + k_0 k_0'', \tag{4}$$

$$\vec{\alpha} \cdot \vec{\beta}' = (k_0'^2 + k_0 k_0'')\Delta \omega - k_0 k_0', \tag{5}$$

$$\vec{\beta}' \cdot \vec{\beta}' = (k_0'^2 + k_0 k_0'')\Delta \omega^2 - 2k_0 k_0' \Delta \omega + k_0^2 \tag{6}$$

where the parameter $\vec{\beta} = \{\beta_1, \beta_2, \beta_3\}$ has been replaced by $\vec{\beta}' = \{\beta_1, \beta_2, \beta_3 + q\}$.

If all of the parameters are reasonably chosen, we can expect to obtain the optical solitary wave solutions described by Eq.(3). Fortunately, one can prove that all of the parameters may physically choose reasonable value. Therefore, an optical envelope solitary wave solution can exist in Eq.(2).

According to the vector relation $|\vec{\alpha}||\vec{\beta}'| \geq \vec{\alpha} \cdot \vec{\beta}'$, substituting Eq.(4)-(6) into the relation, after tedious algebra calculation, we can obtain the condition:

$$k_0'' \geq 0. \tag{7}$$

This means that it is only in the normal (positive) dispersion region that there may exist optical envelope solitary wave solutions in a dispersive linear medium. This property is contrary to that of (1+1)-dimensional NLSE, in which the sech-like solitary wave solutions exist only in the anomalous (negative) GVD region. The existence of present solitary wave solution indicates that the physical effects of transverse confinement seems to counteract the effect of normal GVD. From Eq.(4)–(6) one can see that the parameters A_0 and τ are not included in them. This

implies that the maximum amplitude A_0 is independent of pulse half-width τ. Therefore, the optical envelope solitary wave solution (3) may propagate through a medium with an arbitrarily small amplitude. This means, such a solitary wave has no limitation of threshold.

Additionaly, in normal case, there is always dispersion in a practical medium, that means $k_0'' \neq 0$, this will lead to that the relation $|\vec{\alpha}||\vec{\beta'}| \neq \vec{\alpha} \cdot \vec{\beta'}$ is always satisfied. This implies that the propagating direction of envelope amplitude does not coincide with that of wavefront. Therefore, the optical envelope solitary wave solution (3) represents an inhomogeneous wave. The angle θ of the two directions between envelope amplitude and phase can be written by:

$$\theta = arccos \left(\frac{\vec{\alpha} \cdot \vec{\beta'}}{|\vec{\alpha}||\vec{\beta'}|} \right) \tag{8}$$

Comparing with the nonlinear method of utilizing the nonlinear dependence of refractive on pulse intensity suggested by Hasegawa and Tappert, the present one has three features as follows: For the first, the optical bright solitary wave can be achieved in the normal (positive) GVD region. This feature can greatly extend the range of optical wavelength for realizing transmission of the bright solitary-wave. It is unnecessary to search for special light source, of which the wavelength lies in the range of anomalous GVD for optical guide materials. For the second, it may achieve a condition of "zero-dispersion", in which a solitary wave of arbitrarily small amplitude may propagate with no dependence on its pulse width. While the pulse amplitude A_0 is proportional to the inverse of pulse half-width τ for the nonlinear refractive index case. This implies that the pulse intensity will increases rapidly with the decrease of pulse half-width (to the second order). Therefore, in realizing ultra-high bit-rate optical soliton communications, it will finally meet the limit set by the damage threshold of optical guide materials and other nonlinear effects. This difficulty may be overcome easily in our case as one may achieve ultra-high bit-rate transmission of pulses in optical soliton communication systems, in which the pulse half-width is narrow enough while the intensity still keeps at a low level. Besides above mentioned, it may conveniently utilize all of the advantages of linear techniques (e.g. wavelength division multiplex)in the future optical soliton communication systems. However, it should be noted that this solitary wave is homogeneous. What influence on the optical communication is it? It should be considered in the next work.

4 Conclusion

In conclusion, We have obtained an optical envelope optical solitary wave solution in (3+1)-dimensional dispersive linear wave equation. It is of the following features:

1) It is only in the normal (positive) dispersion range that there exists the solitary wave solution described by (3) in a dispersive linear medium.

2) The optical envelope solitary wave solution represents an inhomogeneous wave.

3) It may achieve a condition of "zero-dispersion", in which a solitary wave of arbitrarily small amplitude may be propagated with no dependence on its pulse width.

Acknowledgments

References

[1] A. Hasegawa and F. Tappert, Appl. Phys. Lett. **23**, 142 (1973); **23**, 171 (1973).

[2] L.F. Mollenauer, R.H. Stolen, and G.P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).

[3] A. Hasegawa, *optical Solitons in Fibers* (Springer-Verlag, Berlin, 1989).

[4] R.Y. Chiao, E. Garmire, and C.H. Townes, Phys. Rev. Lett. **13**, 479(1964).

[5] E.B. Treacy, IEEE J. Quantum Electron. **5**, 454 (1969).

[6] S.A. Akhmanov, A.P. Sukhorukov and A.S. Chirkin, Sov. Phys. JETP **28**,748 (1969).

[7] P. Emplit, J.P. Hamaide, F. Reynaud, C. Froehly, and A. Barthelemy, Opt. Commun. **62**, 374 (1987).

[8] A.M. Weiner, J.P. Heritage, R.J. Hawkins, R.N. Thurston, E.M. Kirschner, D.E. Leaird, and W.J. Tomlinson, Phys. Rev. Lett. **61**, 2445 (1988).

[9] S. Maneuf and F. Reynaud, Opt. Commun. **66**, 325 (1988).

[10] J.S. Aitchison, A.M. Weiner, Y. Silberberg, M.K. Oliver, J.L. Jackel, D.E. Leaird, E.M. Vogel, and P.W.E. Smith, Opt. Lett. **15**, 471 (1990).

[11] D.R. Andersen, D.E. Hooton, G.A. Swartzlander, Jr., and A.E. Kaplan, Opt. Lett. **15**, 783 (1990).

[12] G.A. Swartzlander, Jr., D.R. Andersen, J.J. Regan, H. Yin, and A.E.Kaplan, Phys. Rev. Lett. **66**, 1583 (1991).

[13] G.R. Alan, S.K. Skinner, D.R. Andersen, and A.L. Smirl, Opt. Lett. **16**, 156 (1991).

[14] Y. Silberberg, Opt. Lett. **15**, 1282 (1990).

[15] M. Jain and N. Tzoar, J. Appl. Phys. **49**, 4649 (1978).

[16] E.W. Laedke and K.H. Spatschek, Phys. Rev. Lett. **52**, 279 (1984).

[17] K. Hayata and M. Koshiba, Phys. Rev. Lett. **71**, 3275 (1993); 72, 178(E) (1994).

[18] Z.H. Li and G.S. Zhou, Chinese Science Bulletin **40**, 119 (1995).

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The fourth International Conference on Squeezed States and Uncertainty Relations was held at Shanxi University, Taiyuan, Shanxi, China, on June 5 - 8, 1995. This conference was jointly organized by Shanxi University, the University of Maryland (U.S.A.), and the Lebedev Physical Institute (Russia). The first meeting of this series was called the Workshop on Squeezed States and Uncertainty Relations, and was held in 1991 at College Park, Maryland. The second and third meetings in this series were hosted in 1992 by the Lebedev Institute in Moscow, and in 1993 by the University of Maryland Baltimore County, respectively.
The scientific purpose of this series was initially to discuss squeezed states of light, but in recent years, the scope is becoming broad enough to include studies of uncertainty relations and squeeze transformations in all branches of physics, including, of course, quantum optics and foundations of quantum mechanics. Quantum optics will continue playing the pivotal role in the future, but the future meetings will include all branches of physics where squeeze transformations are basic transformation. This transition took place at the fourth meeting of this series held at Shanxi University in 1995.
The fifth meeting in this series will be held in Budapest (Hungary) in 1997, and the principal organizer will be Jozsef Janszky of the Laboratory of Crystal Physics, P.O. Box 132, H-1052. Budapest, Hungary.

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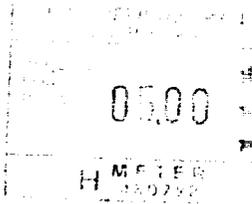
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