1995

NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM

MARSHALL SPACE FLIGHT CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

RELATIVITY EFFECTS FOR SPACE-BASED COHERENT LIDAR EXPERIMENTS

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INTRODUCTION

An effort was initiated last year[1] in the Astrionics Laboratory at Marshall Space Flight Center to examine and incorporate, if necessary, the effects of relativity in the design of space-based lidar systems. A space-based lidar system, named AEOLUS, is under development at Marshall Space Flight Center and it will be used to accurately measure atmospheric wind profiles. Effects of relativity were also observed in the performance of space-based systems, for example in case of global positioning systems[2], and corrections were incorporated into the design of instruments. During the last summer, the effects of special relativity on the design of space-based lidar systems were studied in detail, by analyzing the problem of laser scattering off a fixed target when the source and a co-located receiver are moving on a spacecraft. Since the proposed lidar system uses a coherent detection systems, errors even in the order of a few microradians must be corrected for to achieve a good signal-to-noise ratio. Previous analysis assumed that the ground is flat and and the spacecraft is moving parallel to the ground, and developed analytical expressions for the location, direction and Doppler shift of the returning radiation. Because of the assumptions used in that analysis, only special relativity effects were involved. In this report, that analysis is extended to include general relativity[3] and calculate its effects on the design.

METHOD OF ANALYSIS AND PATH GEOMETRY

Laser Atmospheric Wind Sounder systems were described in detail elsewhere[4]. Figure 1. shows the path geometry. Initially earth is assumed to be spherical and a satellite is moving with an angular velocity of ω radians/sec at a height H above the ground. For convenience, it is assumed that the satellite is in the equatorial orbit. Our goal is to find out first the direction of a light ray from a moving source (satellite) to the stationary ground, as observed from the ground and the paths taken by the retro-reflected and diffuse rays from the ground towards the satellite as observed by a receiver adjacent to the transmitter in the moving coordinate system.

Figure 1. also shows two coordinate systems. K system is a fixed coordinate system (coordinates are x, y, z and t). K_1 system describes the satellite motion and is moving with respect to K at an angular velocity of ω radians/sec (the coordinates are x_1, y_1 and z_1). Both have a common center and an angular separation ϕ_i between them at the starting time. In the analysis, all the coordinates are converted into spherical coordinates (r, θ, ϕ) where θ is the angle For convenience, another coordinate system K', centered on the satellite and rotating synchronously with K_1, is shown (the coordinates are x', y' and z') and used in the analysis.

A correct approach to analyze this problem is to consider Kerr metric which is valid in the presence of gravitational field due to spinning mass and analyze the problem of the bend of the path of a photon under this metric. A general solution to this problem is not available except for the case of large gravitational fields such as those near black holes. In addition, consideration of Schwartzchild metric (mass not spinning but stationary), shows that the correction term in the metric for space and time scales is in the order of 10^{-9}. Hence a different approach is used. First zero mass

corrections that are valid for rotational geometry were calculated following the method Ashworth and Davies[5]. Then gravitational corrections were added linearly. Such methods were used in case of GPS and corrections appear to be acceptable[2].

Also if A and B are two points in a rotating system, the distance between them can be used by using a radar technique (A sends a signal to B and B returns it and the distance is given by multiplying half the round trip time with the light velocity) or by finding the shortest distance between them. In the literature, both techniques are used and it appears that the former method leads to Lorentz-like transformations.

In order to set up proper coordinate transformations, consider in the rotating system two orbits, separated by a distance dR in the equatorial plane and consider two points A and B in it (Figure 2.). The system is rotating about the origin O with an angular velocity ω (measured in the fixed system K). A laser transmitter is located at a distance of R from the origin of the K_1 system (at the center of K$'$) and a ray is sent at time t_1 by the transmitter A when it is at A_1 towards an observer B when B is at B_1 on the ground. When the ray reaches ground, A is A_2 and B is at B_2. When B sends the ray back along the retro-reflection (or in an arbitrary) direction, assume A is at A_3 and B is B_3. The corresponding angles and geometry are shown in Figure 2.

The corresponding coordinate transformations between the inertial and rotating coordinates are given by

$$r_1 = r \ [1-R^2 \sin^2(\theta)\omega^2/c^2] \qquad r = r_1 \ [1+R_1^2\sin^2(\theta_1)\omega_1^2/c^2] \tag{1}$$

$$\cos(\theta_1) = \frac{\cos(\theta)}{[1-R^2\sin^2(\theta)\omega^2/c^2]} \qquad \cos(\theta) = \frac{\cos(\theta_1)}{[1+R_1^2\sin^2(\theta_1)\omega_1^2/c^2]}$$

$$\Phi_1 = \frac{\Phi-\omega \ t}{1-R^2\omega^2/c^2} \qquad \Phi = \frac{\Phi_1+\omega_1 \ t_1}{1+R_1^2\omega_1^2/c^2}$$

In addition,

$$\omega_1 = \frac{\omega}{1-R^2\omega^2/c^2} \qquad \omega = \frac{\omega_1}{1+R_1^2\omega_1^2/c^2} \tag{2}$$

and

$$[1-R^2\omega^2/c^2] \ [1+R_1^2\omega_1^2/c^2] = 1$$

Using the above transformations, the direction and location of at the point of incidence on the earth of a light ray, transmitted in the direction (θ_{1t}, ϕ_{1t}), is calculated in both coordinate systems. Then the ray is retro-reflected or sent along an arbitrary direction in the fixed coordinate system of the earth and the corresponding direction and location of return are calculated in the satellite coordinate system. The results are then corrected by adding the term $1+\chi/c^2$ for the differential space-time metric. The results will be detailed in a separate report.

GENERAL RESULTS

If ϕ_{1t} is the azimuth angle of the transmitted ray and ϕ_{1ret} is that of the returning radiation after retro-reflection at the ground, then

$$\Delta\phi = \phi_{1ret} - \phi_{1t} = \omega \, T_{ret} \left[1 + R^2\omega^2/C^2+..... \right] \tag{3}$$

where all parameters are measured in the fixed coordinate system and T_{ret} is the round trip time. The corresponding change in the azimuthal angle at the receiver is calculated by multiplying the above change with $(R_e+h)/h$ where R_e is the radius of the earth and h is the satellite altitude (Figure 3.). Similar calculations can also done for nadir angle.

$$\cos(\theta_{1rec}) = \cos(\theta_{1t}) \left[1-\frac{1}{2}\frac{R_1^2 \, \omega_1^2\sin^2(\theta_{1t})}{C^2} + \frac{1}{2}\frac{R^2\omega^2\sin^2(\theta)}{C^2} + ... \right] \tag{4}$$

After estimating the angle difference, the result is multiplied by $(R_e+h)/h$ to get the nadir angle change at the transmitter/receiver location. Another simple approach is to estimate the arc lengths and calculate as follows:

$$\tan(\Delta\theta) = \Delta\theta = \frac{\omega \, T_{ret} \, (R_e+h) \, \sin(\theta_{1t}) \, \cos(\phi_{1t})}{h} \tag{5}$$

The relative Doppler shift in frequency is given by

$$f'_0-f_0 = \frac{1}{2}\frac{R^2\omega^2}{C^2} + \frac{G \, M}{C^2}\left[\frac{1}{R_e+h}-\frac{1}{R} \right] \tag{6}$$

where the first term is due to special relativity and the second term is due to gravity. It will be shown that the relativistic changes in both the coordinate angles are small compared with the lag angles.

ADDING THE GRAVITATIONAL EFFECTS

The gravitional effects can be incorporated, by recognizing the that Schwartzchild metric is given by

$$ds^2 = (1+2\chi/c^2) \, c^2dt^2 - (1-2\chi/c^2)(dr^2+r^2d\theta^2+r^2\sin^2(\theta^2)d\phi^2 \tag{7}$$

This modifies all the results in the above equations, by modifying the terms $R^2\omega^2/c^2$ as $R^2\omega^2/c^2 + \chi/c^2$. for the forward path and by $R^2\omega^2/c^2 - \chi/c^2$ for the return path. The results are modified accordingly.

RESULTS FOR THE PROPOSED WIND SOUNDER

AEOLUS has a proposed altitude of 350 Kilometers, an angular velocity of 7704.3 meters/sec, nadir angle of 30^0, azimuth angle of 45^0 and wavelength of 2 μm. We assume that the earth has a radius (R_e) of 6432 Kilometers. For this data, in the case of retro-reflection, the change in the azimuth angle is 74 micro radians and that in nadir is 27 μ radians. For higher velocities (v/c >= 0.1), the corrections could be substantial. For LAWS, backscatter may be reasonably acceptable. In case of diffuse

scattering off the ground or off aerosols, for any transmission angle, there exists a return direction, for which no compensation is required at the receiver. For AEOLUS data, this direction did not differ greatly from the retro-reflection direction (36 μ radians in azimuth and 14 μ radians for nadir). There are Doppler shifts due to the satellite motion and relativity. It is found that the Doppler shift due to the general relativity leads to an error of at best 1% in wind sending.

RECOMMENDATIONS

First, the analysis has to be redone, taking into consideration the gravity effect by adding the term $\frac{2\chi}{c^2}$ to the term $(1-R^2\omega^2/C^2)$ in several coordinate transformations. The above results use radar measurements. To use geodesic lines, we have to expand Kerr metric and solve the path of a photon under this field. Non-sphericity of the earth should also be accounted for.

SUMMARY

In summary, we derived several analytical results based on the theory of general relativity, useful for space-based lidar experiments. We applied our results to the proposed AEOLUS system and found out that the current design recommendations are within acceptable tolerances. The results agree, where required, with those, derived last year. Some important next steps that will improve the analysis and provide better design rules have been identified.

ACKNOWLEDGEMENTS

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Z, Z_1

Z'