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STATISTICAL OPTIMIZATION OF SHEAROGRAPHY INSPECTIONS

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1. INTRODUCTION

The space industry has developed many composite materials that have been designed to have high durability in proportion to their weights. Many of these materials have a likelihood for flaws that is higher than in traditional metals. There are also material coverings (such as paint) that develop flaws that may adversely affect the performance of the system in which they are used. Therefore, there is a need to monitor the soundness of composite structures. To meet this monitoring need, many nondestructive evaluation (NDE) systems have been developed. An NDE system is designed to detect material flaws and make flaw measurements without destroying the inspected item. Also, the detection operation is expected to be performed in a rapid manner in a field or production environment.

Within the last few years, several video-based optical NDE methodologies have been introduced. Some of the most recent of these methodologies are shearography, holography, thermography, and video image correlation. A detailed description of these may be found in Chu et al. (1985), Hung (1982), and Russell and Sutton (1989).

This research focuses on a performance evaluation of shearography equipment. Users of this equipment realize that performance is a function of a set of control variables as well as a set of noise variables. However, there is a shortage of research that characterizes this relationship in model form. Hence, this project has four major objectives:

(1) Identify the control and noise variables that are most likely to influence shearography performance.

(2) Define a model that connects equipment output to control and noise variables.

(3) Determine the control and noise variables and their interactions that have significant influence on shearograph performance.

(4) Identify the setting for control variables that will optimize equipment performance. At this setting, compute a probability of detection (POD) curve.

Due to the necessary shortness of this research period, objective (1) is achieved through engineering judgment. The remainder of this paper addresses objectives (2) through (4).

In the following discussion, y is considered to be an output from an NDE inspection system. Output y is assumed to be a random variable with some probability distribution. Depending on the system, y is either continuous (eddy current is an example) or binary (shearography is an example). Berens and Hovey (1984) described an a-hat analysis method for a continuous y and a hit/miss methodology for systems that produce binary output. For completeness, this report discusses a setup where the continuous y is expressed as a function of control and noise variables. A similar setup is discussed for binary y along with a prescription for achieving the above-mentioned objectives.

2. CONTINUOUS OUTPUT VARIABLE

Assume that y is the continuous output of an NDE inspection system with a normal probability distribution of mean μ. Further assume that control variables x_1, x_2, ..., x_k and noncontrollable noise variables $z_1, z_2, ..., z_m$ have an influence on output y. For compactness, let control vector $\underline{X} = (x_1, x_2, ..., x_k)'$ and noise vector $\underline{Z} = (z_1, z_2, ..., z_m)'$. A general equation that connects output y to $\underline{X}$ and $\underline{Z}$ is

$$y = \beta_0 + \underline{X}'\underline{\beta} + \underline{Z}'\underline{\delta} + \underline{X}'\Lambda\underline{Z} + \varepsilon \ , \tag{2.1}$$

where $\underline{\beta}$ is a general parameter of vector coefficients of control variables, $\underline{\delta}$ is a coefficient vector of noise variables, Λ is a matrix which contains the coefficients of the interactions between noise and control variables, and ε is a random lack of fit component.

Model (2.1) generates two response surfaces that are of benefit to NDE performance evaluation. They are system output mean which is given by

$$E[y] = \mu = \beta_0 + \underline{X}'\underline{\beta} \ , \tag{2.2}$$

and system output variance

$$\mathrm{Var}[y] = \sigma^2 = [\underline{\delta}'+\underline{X}'\Lambda] \ V \ [\underline{\delta}'+\underline{X}'\Lambda]' \ , \tag{2.3}$$

where V is the variance-covariance matrix for noise variables in $\underline{Z}$. Observed data on y, $\underline{X}$, and $\underline{Z}$ may be used to estimate parameter values β_0, $\underline{\beta}$, $\underline{\delta}$, and Λ. Prior observations on z (perhaps field data) are used to estimate V. Once parameters are estimated, hypotheses tests may be performed to determine if control and/or noise variables have an influence on NDE performance. In particular, NDE evaluators are interested in variable interactions.

The reader may recognize components of the Taguchi methodology in this presentation. Actually, model (2.1) remedies some of the shortcomings in the Taguchi method. In fact, unlike Taguchi, model (2.1) provides for interaction analysis and allows a complete analysis of output behavior with respect to changes in control variables. Myers et al. (1992) give a description of model (2.1) and details its connection to the Taguchi method. In this paper, we apply model (2.1) to the evaluation of NDE equipment performance.

<u>Application Example</u>

To illustrate the usefulness of model (2.1) to NDE performance evaluation, we use an example constructed by Myers et al. (1992). For our case, we pretend that there are two control (x_1, x_2) and two noise (z_1, z_2) variables for which (2.1), (2.2), and (2.3) become (2.1)′, (2.2)′, and (2.3)′, respectively.

$$\begin{aligned} Y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \delta_1 z_1 + \delta_2 z_2 + \lambda_{11}x_1z_1 + \lambda_{12}x_1z_2 \\ + \lambda_{21}x_2z_1 + \lambda_{22}x_2z_2 + \varepsilon \end{aligned} \ , \tag{2.1$'$}$$

$$E[Y] = \beta_0 + x_1\beta_1 + x_2\beta_2 \ , \tag{2.2$'$}$$

$$
\begin{aligned}
Var(Y) = &\sum_{j=1}^{2} \delta_j \sigma_j^2 + 2\,X_1(\delta_1\lambda_{11}\sigma_1 + \delta_2\lambda_{12}\sigma_2^2) + 2\,X_2(\delta_1\lambda_{21}\sigma_1^2 + \delta_2\lambda_{22}\sigma_{22}^2) \\
&+ \sum_{j=1}^{2}\sum_{i=1}^{2} X_j^2\,\lambda_{ji}^2\sigma_i^2 + 2\,X_1X_2(\lambda_{11}\lambda_{21}\sigma_1^2 + \lambda_{12}\lambda_{22}\sigma_2^2) + \sigma_\varepsilon^2
\end{aligned}
\quad . \qquad (2.3)'
$$

In (2.2)′ and (2.3)′, $\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$, $\underline{\beta} = (\beta_1, \beta_2)'$, $\underline{\delta} = (\delta_1, \delta_2)'$, $\text{var}(\underline{Z}) = V = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$, and $\text{Var}(\varepsilon) = \sigma_\varepsilon^2$.

For the variables used to construct (2.1)′ through (2.3)′, if a statistical analysis shows that $\beta_2 = 0$ while all other parameters in (2.1)′ are nonzero, then (2.2)′ and (2.3)′ show that an adjustment on control variable x_2 affects Var[Y] and has no effect on mean E[Y]. Additional comments about this situation appear in the data analysis section.

3. BINARY OUTPUT VARIABLE

In many NDE systems, the output is 1 or 0 where 1 represents a flaw detection and 0 denotes a nondetection. This output variable y is said to be binary or hit/miss. For each single flaw inspection, it is assumed that p is the probability of flaw detection and thus y has a Bernoulli probability mass function $f(y) = p^y(1-p)^{y-1}$. The mean of y is given by

$$E[y] = \mu = p \ . \qquad (3.1)$$

It is desirable to devise a setup for output y that is similar to (2.1). One difficulty, among many, is that the variance $\text{Var}(y) = p(1-p)$ which is not constant over flaw size and model (2.1) requires a constant variance. However, the difficulties posed by the binary nature of y are remedied by using (3.1) to express mean μ (i.e., p) as a function of control variables $\underline{X}$ and noise variables $\underline{Z}$. Such a remedy is offered by Beren and Hovey (1984) where they express p as a function of one variable (crack size). They also listed seven candidate relationships between crack size and probability of detection. Of their seven, this research uses their recommendation that the logistic relationship best describes the connection between p and crack size a. This research is also the first that we could find where p is expressed as a function of control vector $\underline{X}$ and noise vector $\underline{Z}$ for NDE equipment.

For variables a and p, as defined above, the logistic relationship is

$$p = \frac{e^{\beta_0+\beta_1 a}}{1+e^{\beta_0+\beta_1 a}} \ , \qquad (3.2)$$

where β_0, β_1 are constants. A transformation of (3.2) gives

$$\ln\left(\frac{p}{1-p}\right) = \beta_0+\beta_1 a \ . \qquad (3.3)$$

Cox (1970) showed that when $\bar{p} = \frac{s}{N}$, where s represents the number of successes (flow detections) out of N trials (flaws) at size a, then $\ln\left(\frac{p}{1-p}\right)$ has an approximate normal distribution provided p is replaced by $\bar{p}$ and N is large.

Since $\ln\left(\frac{p}{1-p}\right)$ is near normal, we may let

$$\ln\left(\frac{p}{1-p}\right) = \beta_0 + \underline{X}'\underline{\beta} + \underline{Z}'\underline{\delta} + \underline{X}'\Lambda\underline{Z} + \varepsilon \ , \tag{3.4}$$

and (3.4) is similar to (2.1) and will be used for a statistical evaluation of shearograph NDE equipment. In fact, the setup in (3.4) permits parameter estimation, statistical tests for model terms, identification of variable settings which will optimize variance in $\ln\left(\frac{p}{1-p}\right)$ and for construction of POD curves.

4. EXPERIMENTAL DESIGN MODEL

The engineers on this project initially identified 13 control and 4 noise variables. This number of variables generates a design that is too large for this research time period. Hence, using engineering judgment, their initial set was reduced to a short list which consisted of 3 control and 3 noise variables. On the short list, variables field of view (FOV), shear, and heat were classified as control while flaw size, flaw orientation, and lighting were treated as noise. Variables FOV, heat, lighting, and orientation contained 2 levels, flaw size and shear contained 3. The 6 variable combinations produced a total of 144 experimental design settings (cells). The experimental design model is

$$\ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta X + P + Q + R + S + T + (\text{interactions}) + \varepsilon \ , \tag{4.1}$$

where P, Q, R are control, S, T are noise such that $E[S] = E[T] = 0$ and X is flaw size where $E[X] = a$, $\mathrm{Var}(X) = \lambda_{11}$. That is, X is a variable with both a fixed but uncontrollable component and a noise component.

5. DATA COLLECTION AND ANALYSIS

To conduct experiments at the 144 settings for all 6 variables, a specimen panel was designed so that it contained flaws that are similar to those that the shearograph equipment would evaluate in an applied situation. The panel contained flows of 3 sizes, 2 orientations, and 2 of each size-orientation combination. Thus, the panel contained a total of 12 flaws. This single panel was inspected once by a single operator at each of all possible combination settings for variables heat, FOV, shear, and light. This gives 2 observations for each of the 144 cells. Thus, the experiment consisted of 288 observations.

Two observations per cell is too low to completely analyze the effects of all variables in a six factorial design. In fact, the literature on this subject suggests about 25 observations per cell. However, we judged that this preliminary study would give excellent insight on shearograph performance even with the low cell counts provided we used an alternative to the full six-factorial design. Thus, we chose to analyze the collected data by using a two-factorial experiment to analyze each combination in the set of all possible two-variable subset combinations. The analysis is repeated for all possible three-variable subset combinations. Each analysis included a test of significance of at most two-variable interactions and a test for main affects. Of all the possible two-variable and three-variable analyses, Table 1 shows what variables showed at least one single significant main affect (row 1) or was involved in at least one significant interaction [redacted] or less.

Table 1

	Size	Shear	Heat	Orient	FOV	Light
1	X	X		X		
2	X	X	X	X		

Since FOV and light are not checked in Table 1, we deleted them and performed a qualitative analysis with size (3,6,9), shear (0,2,4), heat (5,10), and orientation (–1,1) as independent variables. The significant terms from this analysis are as shown in equation (5.1).

$$\ln\left(\frac{p_i}{1-p_i}\right) = 0.6337\,(a_i{*}h) + 0.8498\,(a_i{*}s) - 4.09\,(r) + 1.861\,(a_i{*}r) + 0.2268\,(h{*}r) \qquad (5.1)$$