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DIAGNOSTIC STATISTICS FOR THE ASSESSMENT AND CHARACTERIZATION OF COMPLEX TURBULENT FLOWS

J. R. Ristorcelli

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23681-0001

Diagnostic statistics for the assessment and characterization of complex turbulent flows

J. R. Ristorcelli [1]
Institute for Computer Applications in Science and Engineering
NASA Langley Research Center, Hampton, VA 23681

Abstract

A simple parameterization scheme for a complex turbulent flow using nondimensional parameters coming from the Reynolds stress equations is given. Definitions and brief descriptions of the physical significance of several nondimensional parameters that are used to characterize turbulence from the viewpoint of single-point turbulence closures are given. These nondimensional parameters reflect measures of 1) the spectral band width of the turbulence, 2) deviations from the ideal Kolmogorov behavior, 3) the relative magnitude, orientation, and temporal duration of the deformation to which the turbulence is subjected, 4) one and two-point measures of the large and small scale anisotropy of the turbulence and 5) inhomogeneity. This is an attempt to create a more systematic methodology for the diagnosis and classification of turbulent flows as well as in the development, validation and application of turbulence model strategies. The parameters serve also to indicate the adequacy of various assumptions made in single-point turbulence models and in suggesting the appropriate turbulence strategy for a particular complex flow. The compilation will be of interest to experimentalists and to those involved in either computing turbulent flows or whose interests lies in verifying the adequacy of the phenomenological beliefs used in turbulence closures.

1. Introduction

As turbulence models move into an area in which they are expected to calculate complex flows it becomes more relevant to better understand the physics of different classes of flows. A list of nondimensional diagnostic statistics and their physical interpretation and relevance to different turbulence strategies is the subject of this article. The nondimensional numbers that are used to understand laminar flows come from scaling the equations for the conservation of mass, momentum and energy. These nondimensional numbers reflect primarily molecular phenomena that are of secondary importance in turbulent flows. In turbulent flows there are several additional equations, the second-order moments equations [2], that describe the evolution of the turbulence. A nondimensionalization of these equations produces a very different and important set of dimensionless numbers essential for the characterization of a turbulent flow. These nondimensional numbers reflect properties of the flow, not the fluid; they describe the intensity, orientation, length and time scales of the turbulence and also the structure of the mean deformation straining the turbulence.

This article is an attempt to elucidate the parameters useful for 1) classifying a turbulent flow, and 2) understanding the assumptions underlying the different classes of turbulence closures. The parameters given come from, primarily, a single-point closure framework. As such they represent the application of simple ideas from kinetic theory to models for turbulence. This is, presently, the most fully developed, easily available and self consistent approach available for complex engineering flows. It is to be expected that there are a very substantial number of turbulent flows that do not fall into the class of flows subsumable by these simple kinetic theory metaphors. The parameters compiled here will serve to distinguish these different classes of flows.

In complex flows there is a scarcity of experimental data presented in a way that highlights the physics in a way most suitable for turbulence model development. This article attempts to provide the experimentalist (DNS, LES or laboratory), whose customer is sometimes the turbulence modeler, with a systematic delineation of the ideas and parameters that are used to understand, create and calculate turbulence closures. An appropriately documented experiment, from the viewpoint of a modeler, is an exceptionally valuable commodity not only for understanding the physics but also for assessing the plethora of models contending as accurate descriptions of the physics. This article is also an attempt to provide the turbulence model evaluator and user with a set of parameters that can be used to judge the adequacy of a particular turbulence model. The use of the parameters given here will help understand a flow from a more universal perspective in a way that brings out the physics common or different across several classes of flows.

The procedural viewpoint taken in this article is to describe briefly the ideal turbulence: the high Reynolds number, homogeneous, fully developed, statistically stationary, isotropic, Kolmogorov turbulence. Turbulent flows of engineering interest will deviate from the Kolmogorov ideal. A general turbulent flow can be expected to deviate from the ideal when there is 1) no extended spectral gap necessary for the small scale statistical equilibrium, 2) mean deformation, 3) nonstationarity, 4) anisotropy, and 5) inhomogeneity. The quantification, interpretation and implication of these departures from the ideal are the subject of this document. In addition, two general types of turbu-

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[2] In a purely incompressible mechanical turbulence - no heat transfer and no mass fluxes - these equations are the Reynolds stress equations

lent flows are distinguished: a slowly evolving nonlinear turbulence for which simple kinetic theory ideas are applicable and closures can be effected with local constitutive type relations and a rapidly evolving linear turbulence in which theories such as the Rapid Distortion Theory are applicable, Hunt (1973), Hunt and Carruthers (1992).

The organization of this article is now sketched.

In the next section the nomenclature and some basic equations are given. These equations form the basis of the system that will be used in subsequent developments. The third section contains a brief description of a Kolmogorov turbulence. Issues relating to the assessment of the small scale equilibrium assumptions and the importance of a spectral gap are also described in this third section. Subsequently deviations and measures of these deviations from the Kolmogorov archetype are described. The quantification of these departures from the ideal are the primary subject of the rest of the document.

The fourth section addresses the classification of the mean velocity field and thus the mean deformation to which the turbulence is subjected. This includes nonequilibrium effects as well as history effects.

Additional sections define several parameters useful in describing the turbulence - its intensity, orientation and coherence. This includes both large and small scale turbulence quantities. The main section of the article closes with various issues related to inhomogeneity and some higher-order moments of the turbulence.

The reader's attention is directed to the synopsis in the penultimate section; it provides a useful and short overview of the sometimes tedious earlier sections. It also concretizes many of these ideas with examples taken from two simple turbulent flows. Readers of early drafts of this manuscript have indicated the utility of reading the synopsis *before* attempting the body of the article.

In this article primary attention is directed to the incompressible turbulence problem. No attention is spent on fully empirical closures such as the zero (algebraic models) or one equation turbulence closures. The present exposition is directed towards two-equation turbulence models such as the $k - \varepsilon$ (or $k - \omega$) closures with Boussinesq or algebraic closures for the Reynolds stresses and also second-order closures in which an evolution equation for the Reynolds stresses is solved.

2. Nomenclature and equations

Upper case letters will denote (first-order) mean quantities and lower case letters will denote the fluctuating quantities. A comma followed by a subscript denotes differentiation with respect to the coordinate in the direction that the value of the subscript takes. Repeated subscripts denote summation over the values the indices take. Quantities with an asterisk denote the full field, mean and fluctuating: $u_i^* = U_i + u_i$. The averaging operation is indicated using the angle brackets, $< u_i u_j >$. The dependent variables are decomposed according to

$$u_i^* = U_i + u_i \qquad \text{where} < u_i >= 0 \qquad (1)$$

$$p^* = P + p \qquad \text{where} < p >= 0 \qquad (2)$$

Substituting the decomposition into the Navier-Stokes equations and applying the averaging oper-

ator $< >$ produces, for the mean momentum equations of an incompressible flow

$$\frac{D}{Dt} U_i + < u_i u_p >_{,p} = - P_{,i} + \nu U_{i,jj} . \tag{3}$$

Thus for high Reynolds number turbulence, except in the case for rapidly accelerating flows, the turbulence sets the changes in the mean momentum distribution. The pressure has been normalized by the density. A set of equations for the Reynolds stresses is derived by subtracting the mean equations for $U_i U_j$ from the equations for $u_i^* u_j^*$:

$$\frac{D}{Dt} < u_i u_j >= \quad - < u_i u_p > U_{j,p} - < u_j u_p > U_{i,p} + \Pi_{ij} - \varepsilon_{ij}$$
$$- [< pu_i > \delta_{pj} + < pu_j > \delta_{ip} + < u_i u_j u_p > + \nu < u_i u_j >_{,p}]_{,p} . \tag{4}$$

These are the exact and *only* equations describing the evolution of the Reynolds stresses. Any turbulence model is, to a greater or lesser degree, an approximation to the physics embodied in these equations. In flows for which an energy equation is required (where heat transfer or compressibility is important) additional equations are carried for the turbulent fluxes of the energy.

The form of the equations above reflects the following substitutions: the pressure-strain correlation is $\Pi_{ij} = < p (u_{i,j} + u_{j,i}) >$. The fluctuating pressure, p, involves an integral over the whole flow field and thus covariances with the pressure involve two-point statistics. It is usual to represent this unknown term as a function of local quantities and closure is achieved in terms of the local, single-point, turbulence quantities. Implicit in this is the assumption that the correlation length scale is the same in all directions. There are a variety of flow situations in which such an approximation produces adequate results; there are flow situations in which such an approximation is inadequate.

The contraction of the second-order equations produces the equation for the kinetic energy, $k = 1/2 < u_j u_j >$,

$$\frac{D}{Dt} k = - < u_j u_p > U_{j,p} - [< pu_p > + < u_j u_j u_p > + \nu k_{,p}]_{,p} - \varepsilon \tag{5}$$

without the complicating pressure-strain terms, Π_{ij} since $\Pi_{jj} = 0$. The term in the square brackets, is called the transport term. It is, in addition to the mean advection, responsible for non-local aspects of turbulence field. Both terms, the pressure transport and the turbulent transport - transport by the fluctuating pressure and velocity are important in inhomogeneous flows.

The first and last terms on the right hand side are responsible for the production, $P_k = - < u_j u_p > U_{j,p}$ and dissipation, $\varepsilon = \nu < u_{i,j} u_{i,j} >$, of the energy of the turbulence. The production terms, which in many flows are the most important terms, require *no* modeling in Reynolds stress closures. In two-equation closures the Reynolds stresses are algebraically related to the mean deformation and the production terms are no longer exact.

In the context of $k - \varepsilon$ and $k - \omega$ type closures the Reynolds stress term appearing in the production and in the mean momentum equations are typically of the form

$$< u_j u_p >= \frac{2}{3} k \delta_{pj} - \nu_t \, S_{pj} - \nu_1 \, \frac{k}{\varepsilon} \, [S_{pn} W_{nj} - S_{jn} W_{np}] - \nu_2 \, \frac{k}{\varepsilon} \, [S_{pn} S_{nj} - \frac{1}{3} S_{qn} S_{qn} \delta_{jp}]. \tag{6}$$

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The first two terms are the well known Boussinesq eddy viscosity approximation. The mean strain and rotation tensors are defined as $S_{ij} = \frac{1}{2}[U_{i,j} + U_{j,i}]$, $W_{ij} = \frac{1}{2}[U_{i,j} - U_{j,i}]$. In a simple planar shear flow, $U_{i,j} = U_{1,2}\,\delta_{i1}\delta_{2j}$ and $S^2 = W^2 = \frac{1}{2}U_{1,2}^2$. Terms quadratic in the mean rotation are not carried as they are inconsistent with results for the rotation of isotropic turbulence, Speziale (1987). The utility of these algebraic expressions varies from situation to situation depending on the importance of nonlocal effects in time and space. Consider that these are algebraic representations to the solutions of the nonlinear partial integro-differential equations for the $< u_i u_j >$. The various eddy viscosities can in general be a function of k, ε and some scalar measure of the strength of the mean strain or rotation.

Second-order closures constitute a class of turbulence models in which evolution equations are carried for the mean flow (mass, momentum, and energy) and for the turbulent fluxes of mass, momentum (Reynolds stresses), and energy. The idea here is to carry the first-order or mean equations, $\frac{D}{Dt}U_i$, exactly and model the unclosed terms in the second-order equations, $\frac{D}{Dt} < u_i u_j >$. The unclosed terms are modeled in terms of quantities for which equations are carried: the first and second-order quantities. Rather than modeling the Reynolds stresses directly (and thus the mean flow equations) one models higher-order effects in higher-order equations with (presumably) smaller error in the Reynolds stresses due to the lesser importance of the terms being modeled. The models for the unclosed terms are arrived at by requiring some sort of mathematical or physical consistency with known experimental or analytical results. They typically involve, to a greater or lesser degree, phenomenological arguments, handwaving, and empirical curve fitting. In a large number of flows the most important terms, in the dynamically significant portions of the flow, are the production terms which are carried exactly.

A discussion of the merits of different models in specific flow situations is not the purpose of this article; the reader interested in an overview of these issues is referred to Launder (1989) or Hanjalic (1994). The subject of this article is solely concerned with the diagnostic parameters necessary to characterize a complex flow. These parameters are relevant to understanding the physics and are also used to assess the adequacy of the underlying assumptions in different styles of closures.

The Reynolds stress or k equations are accompanied by an equation for either the dissipation, ε or a quantity sometimes called the specific dissipation, $\omega = \varepsilon/k$. The dissipation tensor is given by $\varepsilon_{ij} = 2\nu < u_{j,p}\,u_{i,p} >$. The dissipation of the turbulence energy, ε, is related to the homogeneous portion of the trace of the dissipation tensor. The trace of the dissipation tensor is

$$\varepsilon_{ii} = 2\,\nu\,[< \omega_k \omega_k > + < u_p u_q >_{,pq}]$$

and the dissipation of the kinetic energy of the turbulence, ε, is thus proportional to the enstrophy and given by $\varepsilon = \frac{1}{2}\varepsilon_{jj} = \nu < \omega\omega >$. Bradshaw and Perot (1993) have indicated, using low Reynolds number DNS, that for an incompressible turbulence, the enstrophy makes the most important contribution to the dissipation. In the above manipulations the identities $u_{i,p} = u_{p,i} - \epsilon_{pij}\omega_j$ where the vorticity, $\omega_i = \epsilon_{ijk}u_{j,k}$, have been used to express the dissipation in terms of the vorticity. Thus $< \omega_j\omega_j > = < u_{j,p}\,u_{j,p} > - < u_{j,p}\,u_{p,j} > = < u_{j,p}\,u_{j,p} > - < u_q u_p >_{,qp} + 2 < u_{q,q}\,u_p >_{,p} - < u_{q,q}\,u_{p,p} >$. where, $< u_{i,p}\,u_{p,i} > = < u_i u_p >_{,ip} - 2 < u_{i,i}\,u_p >_{,p} + < u_{i,i}\,u_{p,p} >$ has been used.

The modeled dissipation equation, with a gradient transport assumption for the turbulent transport,

is typically assumed to be of the form

$$\frac{D}{Dt}\,\varepsilon + [\nu_t \sigma_\varepsilon^{-1}\varepsilon_{,q}]_{,q} = -[c_{\varepsilon 1}P_k - c_{\varepsilon 2}\varepsilon]\,\varepsilon/k, \tag{7}$$

where $\nu_t = C_\mu\,k^2/\varepsilon$. The equation is arrived at by, more or less, phenomenological reasoning. There are some very important inadequacies with this equation; they are typically ignored or resolved using *ad hoc* corrections.

A few more definitions are necessary. The turbulent Reynolds number is given by $R_t = \tilde{u}\ell/\nu$ where $\tilde{u}$ is a characteristic magnitude of the velocity fluctuation, say $\tilde{u}^2 = \frac{2}{3}k$ and ℓ is a length scale characterizing the spatial correlation of the turbulence. The quantity ℓ is sometimes identified with an integral length scale of the turbulence the definition of which is given below. Using the high Reynolds number Kolmogorov scaling for a length scale, $\ell \sim \tilde{u}^3/\varepsilon$, with $\tilde{u} \sim (\frac{2k}{3})^{1/2}$, the following Reynolds number

$$R_t = \frac{4}{9}\frac{k^2}{\nu\varepsilon} \tag{8}$$

is obtained. This is not to be confused with the mean flow Reynolds number: $Re = U_\infty L/\nu$. The two are related by $R_t = (\tilde{u}/U_\infty)(\ell/L)Re$. The mean strain and mean rotation rates are sometimes made nondimensional with the "natural" time scale of the turbulence k/ε: $\hat{S} = (Sk/\varepsilon)$, $\hat{W} = (Wk/\varepsilon)$. Here S and W are suitable norms of the strain and rotation tensors $S = (S_{ij}S_{ij})^{1/2}$ and $W = (W_{ij}W_{ij})^{1/2}$. In a simple shear flow $S = W = \frac{1}{\sqrt{2}}U_{1,2}$.

3. Small scale equilibrium and isotropy

Most current engineering turbulence model developments assume a statistical equilibrium and thus a universal behavior of the small scales of a turbulent flow. These notions reflect an *ideal* turbulence with presumptions of statistical stationarity, homogeneity, and isotropy at very high Reynolds number, R_t, as envisioned in the seminal papers of Kolmogorov (1941a, 1941b) - a turbulence in a statistical equilibrium independent of initial conditions and boundary conditions. The consequences of these ideas are set forth in the next two subsections. Deviations from the Kolmogorov ideal turbulence, as occur in engineering flows, are then described.

A plausible energy spectrum as a function of wave number (inverse length scale) for the type turbulence assumed in the current developments in single-point turbulence closures is shown in Figure 1. In turbulence with a large range of scales there is expected to be a featureless portion of the spectrum (having a power law behavior) separating the large scales of the motion from the small scales. This is called the inertial subrange and for an ideal Kolmogorov turbulence has a $-5/3$ power law behavior.

The existence of an inertial subrange is considered evidence of a spectral gap separating and decorrelating the smallest scales of the motion from the largest making arguments for a statistical small scale equilibrium possible, with its implications of a universal small scale behavior, plausible.

The scales range from the large energy containing range, $\kappa\ell \sim 1$ down to the small dissipative scales of the motion, $\kappa\eta \sim 1$. The outer and inner length scales, ℓ and η, are called, respectively, the production and the Kolmogorov or dissipative scales of the motion.
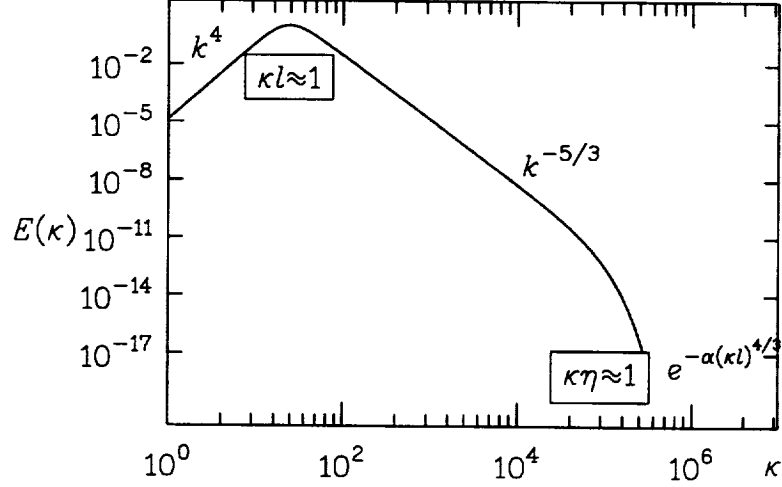
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Figure 1: Model spectrum for a fully developed turbulence

The phenomenological underpinning to most turbulence modeling strategies is the idea that the nonlinearity of the equations acts to cascade energy to the small scales scrambling information about the large scales. One consequence of this idea is that if the spatial and temporal scales of the energy containing and dissipative scales of the motion are disparate enough then one can argue for the plausibility of a statistical equilibrium of the small scales of the flow. The large scales of a turbulent motion are imagined to be effectively inviscid and characterized by length, velocity and frequency scales $[\ell, \tilde{u}, \tilde{u}/\ell]$. The smallest scales of the motion are essentially low Reynolds number viscous motions. The two scales of the motion are linked together by the cascade rate, ε. The dissipative processes occurring in the small viscous scales are related to the large scales of the flow by the fact that, in an equilibrium flow, it is equal to the cascade rate of energy from the large scales.

Dimensional analysis using the quantities $[\varepsilon, \nu]$ produces the following characteristic length, velocity, and time scales for the dissipative or Kolmogorov scales of the motion:

$$\eta = (\nu^3/\varepsilon)^{1/4}, \qquad v = (\nu\varepsilon)^{1/4}, \qquad \tau = (\nu/\varepsilon)^{1/2}. \tag{9}$$

The strain rate of the smallest scales of the motion, s, scale as τ^{-1} thus $s = (\varepsilon/\nu)^{1/2}$.

Dimensional reasoning, using the cascade rate and a characteristic scale for the velocity also produces a length scale. It is

$$\ell = \alpha(\frac{2k}{3})^{3/2}/\varepsilon. \tag{10}$$

The characteristic velocity has been taken to be $\tilde{u}^2 = \frac{2}{3}k$ and α is an order one quantity. In practice α has been found to be a flow dependent quantity, Sreenivasan (1995). This scaling is appropriate for high Reynolds number, *ideal* Kolmogorov turbulence.

The next few sections are related to an assessment of the separation of the large and small scales of the motion.

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Temporal and spatial spectral bandwidth

The ratio of the small to the large length scales η/ℓ determines the scale separation and is an important quantity in assessing the validity of the diverse modeling assumptions. The ratio of the small to large spatial scale, η/ℓ, is an indication of the bandwidth of the process. It can be related to the turbulent Reynolds number $R_t = \tilde{u}\ell/\nu$, using the definitions given above:

$$\frac{\eta}{\ell} = R_t^{-3/4} \tag{11}$$

where $\tilde{u}^2 = \frac{2}{3}k$ is related to the kinetic energy of the turbulence. The scaling $\ell \sim \tilde{u}^3/\varepsilon$ has been used. Similar arguments lead to a temporal bandwidth of the turbulence:

$$\frac{\tau}{T} = R_t^{-1/2} \tag{12}$$

where $T \sim \ell/\tilde{u}$. In general a turbulent Reynolds number, $R_t > 10^4$, at least, before a legitimate inertial subrange begins to appear. The fact that $Re_t^{3/4} >> 1$ and that $Re_t^{1/2} >> 1$ is an indication of the adequacy of the assumption of the statistical equilibrium of the small scales of the motion.

Small scale isotropy

Closely related to the extent of the bandwidth of the process is a useful approximation made regarding the isotropy of the dissipative scales. The cascade is assumed to scramble all directional preferences of the large scales so that the small scales are in a state of *statistical* isotropy. The spectral characteristics of such a process imply a broad and featureless spectrum as shown in Figure 1. The consequences of such an assumption are that Taylor (1935),

$$< u_{1,1}\, u_{1,1} > = \frac{1}{2} < u_{2,1}\, u_{2,1} > = \frac{1}{2} < u_{3,1}\, u_{3,1} > \tag{13}$$

and that

$$-\frac{1}{2} < u_{1,1}\, u_{1,1} > = < u_{1,2}\, u_{2,1} > = < u_{3,1}\, u_{1,3} > = < u_{3,2}\, u_{2,3} > . \tag{14}$$

The dissipation tensor can then be written in terms of one scalar $\varepsilon = \nu < \omega\omega > = 15\nu < u_{1,1}\, u_{1,1} >$, Batchelor (1953), Townsend (1976). (In point of fact the small scales will always be statistically anisotropic however small: this has been made very clear by a discussion of the physics by Lumley (1992) and shown, for turbulence in the presence of a mean shear, from a mathematical point of view in a clear and pithy development by Durbin and Speziale (1991). (Note that their argument is independent of R_t and thus the bandwidth of the process).)

Associated with the dissipation is another length scale, the Taylor microscale, λ, which allows $\varepsilon = 15\nu < u_1 u_1 > /\lambda^2$. It is related to the curvature at the origin of the two-point velocity correlation. From the zero crossing problem of stochastic processes it can be viewed as proportional to the number of maxima *per* unit time of the stochastic process. From its definition from integrals in wave space it is seen to be the energy weighted length scale of the turbulence being the second moment (with respect to κ) of the energy spectrum. The reader is directed to Tennekes and Lumley (1972) for further amplification of these ideas.

Spatial spectral bandwidth: shear flows

A ratio of length scales for turbulence in the presence of a mean shear is also possible. Using the

mixing length scaling the length scale at which the mean velocity gradients inputs energy into the spectral cascade is approximated by

$$\frac{\tilde{u}}{\ell} \sim S. \tag{15}$$

The dissipation scales are required to be much smaller than the scales at which energy is input into the turbulence $\eta/\ell \ll 1$:

$$\frac{\eta}{\ell} \sim \frac{Sk}{\varepsilon} R_t^{-3/4} \ll 1 \tag{16}$$

using $\frac{\tilde{u}}{S} \sim \ell$ and $\eta = (\nu^3/\varepsilon)^{1/4}$.

Temporal spectral bandwidth: shear flows

If the strain rates of the small scales of the flow are much larger (corresponding to very high frequency fluctuations) than the largest scales of the turbulence it can be argued that models based on the idea of a statistical small scale equilibrium will be useful. One requires that

$$\frac{s}{S} \gg 1. \tag{17}$$

This is similar to the detuning idea in the theory of linear oscillations. Using the enstrophy as a measure of the small scale strain rates, $< \omega^2 > \sim \varepsilon/\nu$ produces a stronger requirement than that for a spatial spectral gap

$$\frac{s}{S} \sim \frac{\varepsilon^{1/2}}{\nu^{1/2}S} = \frac{3}{2} R_t^{1/2} (\frac{Sk}{\varepsilon})^{-1} \gg 1 \tag{18}$$

thus, $\frac{3}{2} R_t^{1/2} \gg \frac{Sk}{\varepsilon}$, for the adequacy of a statistical independence of the dissipative scales of the turbulence. The scalings $\varepsilon \sim \tilde{u}^3/\ell$ and $S \sim \tilde{u}/\ell$ have been used.

The existence of a temporal and spatial spectral gap are an indication that the small dissipative scales of the flow adjust to the large scales of the motion in a way that is subsumable by a constitutive relationship. If this were not the case the small scales of the motion will have their own dynamics that will not be in equilibrium with the energy cascading from the larger scales of the motion. In this case the usual phenomenological dissipation equation will not be useful. In qualifying the relevance of DNS one should ascertain the magnitude of the above inequalities.

Small scale isotropy: shear flows

Similar arguments can be made wavenumber dependent using the Kolmogorov spectrum: $E(\kappa) \sim \varepsilon^{2/3}\kappa^{-5/3}$. Using the Kolmogorov spectrum and $s(\kappa) \sim (E(\kappa)\kappa^3)^{1/2}$ it is found that scales of the flow unaffected by the mean strain can be expected to have wavenumber κ satisfying

$$\kappa\ell \gg (\frac{2}{3}\frac{Sk}{\varepsilon})^{3/2} \tag{19}$$

Using $\frac{\kappa\eta}{\kappa\ell} = \frac{Sk}{\varepsilon} R_t^{-3/4}$ one finds $\kappa\eta R_t^{3/4} \gg (\frac{2}{3}\frac{Sk}{\varepsilon})^{3/2}$. For the dissipative scales $\kappa\eta = 1$ and thus

$$R_t \gg (\frac{2}{3}\frac{Sk}{\varepsilon})^2 \tag{20}$$

or in terms of the Taylor microscale $R_\lambda \gg \frac{2}{3}\frac{Sk}{\epsilon}$. Hunt and Carruthers (1990) have discussed the broadening of the turbulence spectrum in simple shear flows which will moderate these inequalities.

4. Characterization of the mean deformation

Deviations from the ideal Kolmogorov behavior are expected in the presence of mean deformations of the turbulence field. The deviations will be different depending on the type of mean deformation and thus the classification of the mean deformation is part of understanding the turbulence field: the energy containing eddies are strained primarily by the mean deformation and it is their history and orientation that is of primary importance to the development of the Reynolds stresses. The classification of the mean deformation is structural: the amount of strain versus rotation; and temporal - how long and how rapid is the strain.

There are, of course, regions of the flow in which the mean deformation plays a more minor role; in an aerodynamic context these occur at the periphery of turbulent regions and in separated regions where turbulent transport is most important. Turbulence models based on perturbations about a production-dissipation balance perform poorly in these regions.

Invariants of the mean deformation

The production is the contraction of the Reynolds stresses on the mean velocity gradient. The production can be written in terms of the mean strain as

$$P_k = - <u_i u_j> U_{i,j} = - <u_i u_j> S_{ij} \tag{21}$$

using the anti-symmetry of W_{ij}: rotation does not directly contribute to the production. Kinematically rotation continually rotates the vortex lines away from the direction of the principal strain axis - the most efficient direction for energy transfer between the mean and turbulence. This does not occur in an irrotational strain in which the alignment is maintained and the energy transfer to the turbulent motion is more efficient. A parameter that indicates the relative importance of the straining, S_{ij}, versus rotation, W_{ij}, components of the mean deformation is useful. This is typically done for the dissipation tensor, $\varepsilon_{ij} = \nu <u_{i,k} u_{j,k}>$ Tennekes and Lumley (1972). The trace of the mean flow equivalent is $U_{j,k} U_{j,k} = S_{jj}^2 + W_{jj}^2$. Following Hunt's (1992) very insightful suggestion one normalizes so that

$$Def = \frac{S_{ij}S_{ij} - W_{ij}W_{ij}}{S_{ij}S_{ij} + W_{ij}W_{ij}}. \tag{22}$$

Any mean deformation falls in the range $-1 < Def < 1$: $Def = -1$ corresponds to a pure (solid body) rotation, $Def = 0$ a pure shear, and $Def = 1$ a pure strain. More physically: $Def > 0$ indicates the mean deformation is splatting the eddies while $Def < 0$ indicates the eddies are being swirled.

Flows with a strong strain component, $Def \rightarrow 1$, appear to have a significant memory and local models for the Reynolds stresses (such as eddy viscosity formulations) lead to serious errors in *even the mean flow* predictions. A classic example of a failure of a local model for the Reynolds stresses for a strain dominated flow, is the $k - \varepsilon$ calculations for the pipe with a double bend connected by

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a diffuser as shown by Launder (1989). In these flows an eddy viscosity closure with its inadequate representation of the anisotropy of the normal stresses over-predicts the production terms. Second-order closures, in as much as they include substantial nonlocal effects, (the effects of advection on the Reynolds stresses are included) as well as exact production terms appear to be satisfactory except when the strain is very large, $Sk/\varepsilon \gg 1$.

For flows that are primarily shear flows, $Def \to 0$, local eddy viscosity type models perform substantially better. The continual rotation of the vorticity away from the direction of the principle strain is an additional decorrelating effect. The eddy viscosity formulations work reasonably well for simple flows, though they are still not able to predict the normal stresses. For this at least nonlinear models are required; Taulbee (1992) or Gatski and Speziale (1993) are nicely updated versions of Pope's (1975) rational formulation for equilibrium flows. A recent addition to this company is the *exact* solution of the nonlinear algebraic equations by Girimaji (1995). The tendency to a universal equilibrium state (nominally independent of initial conditions) appears to be a good approximation in simple shear flows an eddy turnover or so past inception. The fact that $\frac{<uv>}{k} \to const.$ in wide variety of shear flows is an important fact; this has been recognized and exploited with success by Speziale *et al.* (1990a). This does not appear to be the case, as already mentioned for flows with a strong strain component, $Def > 0$, or for rapidly distorted flows that occur in high lift situations in which nonlinear effects are less important than linear distortion effects and the flows dependence on its history precludes a local model. Hunt and Carruthers (1992) suggest that this is the case because of the appearance of statistical eigenfunctions in pure shear flows and associate this fact with the success of universal or equilibrium type theories as embodied in local eddy viscosity closures.

For primarily rotational flows, $Def \to -1$, fewer things are known. Rotation, in isotropic homogeneous flows, does not contribute to the generation of the turbulence energy though it does substantially modify the anisotropy of two-point statistics; in bounded systems rotation affects the anisotropy of the Reynolds stresses and the production is directly affected. Rapid system rotation appears to align the vorticity of the large scales actively suppressing the stretching and tilting of vorticity necessary for the cascade, Ristorcelli (1995). The energy exchange between the normal stress aligned with the rotation and those in the plane perpendicular to the rotation is reduced. In these cases, in theory at least, more general rapid strain models (see Ristorcelli *et al.* (1995), Kassinos and Reynolds (1995)) constitute two such valiant attempts for equilibrium and non-equilibrium flows.

A "natural" time scale of the turbulence: k/ε

While not directly related to the mean deformation it is necessary for subsequent developments in this section to introduce a time scale of the turbulence. The strain rate, a time scale characteristic of the mean flow, no sense unless compared to a time scale characteristic of nonlinear mechanisms. The ratio of the turbulent kinetic energy to the dissipation rate, k/ε, is often interpreted as the eddy turnover time. It characterizes the natural time scale of the turbulence and may be thought of as the amount of time it takes for a nonlinear process to decorrelate the fluctuating field. It is, when properly scaled with a characteristic velocity difference and length scale, an order one quantity in strongly turbulent regions where the spectral cascade moves energy to the small scales rapidly. Near the boundaries of laminar/turbulent regions k/ε becomes large indicating a weaker

less dissipative turbulence.

Total strain parameter

For small amounts of strain or over short periods of time turbulence behaves as an elastic medium with perfect memory of its initial conditions. Its behavior is then a function of the total strain. The accumulated deformation felt by a fluid particle, called the total strain, is often used to unify measurements across a number of different flows, Townsend (1976), Sreenivasan (1985). This quantity appears quite naturally in the linear rapid distortion theory for a homogeneous flows; the mathematics is only tractable in the Fourier domain on a grid stretching and rotating with the mean deformation, Pearson (1959). If the total strain is large nonlinear effects have had substantial time to act and the turbulence is thought to approach some sort of universal structure, independent of initial conditions. In such a case the turbulence is thought to act like a viscous medium and stresses are proportional to strain rate and not total strain.

The point is that the total strain can be used to indicate what sort of mathematical procedure is most adequate for a specific flow. If x_s is the distance along a mean streamline and U_s is the velocity along that streamline then the total strain can be understood as $\alpha_T = \frac{x_s}{U_s} U_{1,2}$ for constant $U_{1,2}$. More generally the total strain can be defined

$$\alpha_{ij} = \int^t U_{i,j} \, dt'. \tag{23}$$

The exponential of this function is also used. The total strain is used to assess the equilibrium nature of a flow. In general if $\alpha > 4$, Sreenivasan (1985), the turbulence has had enough time to adjust to the imposed deformation. If this is the case then usual single-point structural equilibrium models can be used to model the flows evolution. For shear and strain flows Sreenivasan (1985) and Townsend (1976) use the definitions

$$\alpha_T = \int^t U_{1,2} \, dt', \qquad \alpha_{T\beta} = \int^t U_{\beta,\beta} \, dt' \tag{24}$$

following a mean fluid particle and β refers to the principle rate of strain. These quantities can be related to the nondimensional time used in the linear rapid distortion theories; $\beta = \alpha_T t$. Townsend (1976) reports good agreement between RDT and experiments for small β. For large β the inherent nonlinearity of turbulence causes departures from the rapid distortion theory results, Townsend (1976). The reader is referred to Hunt (1992, 1978, 1973) and in particular to the exceptional Hunt and Carruthers (1990) for more recent interpretations.

Relative strain parameter: Sk/ε

The strain rate normalized by the time scale of the turbulence, Sk/ε, is sometimes called the relative strain parameter. It has many interpretations; as has already been seen, from one point of view, it can be related to the ratio of mean deformation to the self straining by the turbulent eddies. Its utility can be seen in the nondimensionalization of the second-order moment equations when re-expressed in terms of the anisotropy tensor (to be defined shortly). The details and their implications in the context of a specific class of turbulence closures are outlined clearly in Speziale et al. (1990).

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One can use the total strain α_T to understand Sk/ε. In which case it can be taken as an indication of the total strain an eddy experiences over the time that it is correlated. The total strain and the relative strain are indicators of *how long and how hard* the turbulence has been strained by the mean deformation.

Rapidly changing turbulence

Hunt (1992) has very clearly divided turbulent flows into two simple categories which he names, very appropriately, rapidly changing turbulence (RCT), $T_I^{-1}k/\varepsilon \gg 1$, and slowly changing turbulence (SCT), $T_I^{-1}k/\varepsilon < 1$. In a rapidly changing turbulence there is a strong applied mean deformation and the relative movement of two fluid particles is primarily due to the mean distortion and not the turbulence. The rapid distortion theory for turbulence is a linear procedure; its solution structure is a substantial improvement over hydrodynamic (linear) stability theory which uses the same equations. Hunt (1973), Townsend (1976), and Hunt and Carruthers (1990) provide suitable resumes of the theory.

The fact that an imposed time scale is much smaller than the turbulence time scale $T_I^{-1}k/\varepsilon \gg 1$ means that the turbulence is highly correlated with its initial state and short term memory assumptions invoked in constitutive arguments are not valid. As noted below a large production rate will create turbulence rapidly more effectively decorrelating the turbulence. This suggests that if $T_I S = (T_I \varepsilon/k)(Sk/\varepsilon) \gg 1$ then the turbulence is rapidly decorrelated from its initial state and constitutive arguments made for the unclosed terms may still be relevant.

The usual understanding of RDT is as a small time expansion, $t \ll k/\varepsilon$, for the response of turbulence to a rapid change in deformation. As S and k/ε are not independent parameters the inherent nonlinearity of the turbulence will adjust Sk/ε to a more modest value for which the usual single-point closures will be successful. There are however a number of important flows in which the turbulence never has time for the nonlinear adjustment to take place. These flows that current form of single-point closures are unable to legitimately predict.

To summarize the three mean flow parameters, total strain α_{ij}, relative strain, Sk/ε and relative rate of change of strain $S^{-1}\overset{\circ}{S}k/\varepsilon$ loosely speaking *how long, how hard and how fast* the strain deforms the turbulence are important quantities for the characterization of a turbulent flow.

Imposed time scales and memory

Many of the arguments that lead to constitutive relationships for unclosed terms in the moment equations involve arguments about the relative time and length scales of imposed strains, geometries, boundary conditions and initial conditions. For a turbulence with short term memory and limited awareness Lumley (1967, 1970) has discussed the details under which constitutive relationships are tenable. At issue here is the application of equilibrium type turbulence closures in which linear relaxation arguments (with time scale k/ε) are used to model various unknown terms in situations where things are happening on a time scale fast with respect to k/ε. To see these issues more clearly it is useful to define an imposed time scale, T_I, and compare it to the time scale of the turbulence, k/ε. A slow distortion will correspond to $T_I^{-1}k/\varepsilon < 1$.

The imposed time scales may be associated with time varying mean deformations as might occur

in unsteady motions or in the vortex shedding from a bluff body. In the case of vortex shedding from a bluff body the imposed time scale can be related to a Strouhal number: $T_I^{-1} = St\ U_o/D$ and the current forms of the Reynolds stress models can be considered relevant tools if,

$$St\ \frac{Sk}{\varepsilon}\ \frac{\ell}{D}\ \frac{U_\infty}{\tilde{u}} < 1. \tag{25}$$

With $St \simeq 0.2$ and $\tilde{u}/U_\infty \sim 0.3$ as it is in separated flows, this is an order one quantity and the current form of relaxational turbulence models should yield useful results. Which is to say that unsteady fluctuations occurring, in separation and shedding, can be treated within the context of a quasi-static approximation.

Alternatively one can think of the imposed time scale as the time it takes for a fluid particle to traverse a region of rapidly changing mean deformation of length L at average speed U_0 then $T_I = L/U_0$. A simple situation, easily abstractable to other flows, might be the case of flow in a pipe with sudden changes in shape over a length L. Such a configuration might occur in the isolator portion of a ram jet or in the curved flow around a high lift element. If the percentage change in the mean distortion is large, $\frac{\ell \nabla S}{S} > 1$, over times short with respect to the the decorrelation time

$$\frac{U_0}{L}\frac{k}{\varepsilon} >> 1 \tag{26}$$

the flow may be characterized as a rapidly changing turbulence, Hunt (1992). An imposed time scale might be more unambiguously related to the percentage rate of change of the deformation following a fluid element $T_I^{-1} = \overset{\circ}{S}\ S^{-1}$. In such flows the nonlinear processes are not effective in erasing the turbulence memory and modeling assumptions made to close unknown terms invoking localness assumptions in time and space are questionable. The $k - \varepsilon$ and second-order closures, as presently constructed, involve assumptions that are not consistent with such rapid changes. The only potentially saving grace in these flows, as Hunt (1992) so acutely observes, is that the turbulence has very little time to influence the flow so that a poor turbulence model (presumably) will have little effect on the flow.

Extra strain parameters: curvature

The turbulence in most flows of engineering interest are subject to more complex strains than the simple thin shear which forms the basis of most metaphors for turbulent flows. In general there are additional strains which, though small with respect to the main production mechanism (shear) have very sizable effect on the flow. Bradshaw (1988, 1975, 1981)) give surveys of these issues and a comprehensive list of references. These extra strains, in a 2D aerodynamic context, occur in flows with streamwise curvature or flows with streamwise accelerations. A useful parameter that is a measure of the extra strain is the ratio of the curvature to mean shear. Holloway and Tavoularis (1992) in their study of the effects of curvature in a homogeneous shear use

$$stb = \frac{U/R}{dU/dn} \tag{27}$$

where R is local radius of curvature and dU/dn is the local cross-stream derivative of the mean velocity. This parameter is obtained from the expression for the production in streamline coordinates.

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The symbol stb is used in a generic sense to indicate that the quantity is a stability parameter: $stb = 0$ corresponds to the usual simple shear flow; $stb = 1$ to rigid body rotation and $stb = -1$ to an irrotational curvature. In general, when $stb > 0$ the turbulence production by the mean shear is suppressed with a tendency to relaminarization taking place for $stb > 0.5$. When $stb < 0$ the flow is destabilized and curvature enhances the turbulence production over and above that due to the mean shear. Reflection on the definition of stb indicates it can be interpreted as a measure of departure from a simple shear.

Extra strain parameters: pressure gradient

In flows with pressure gradients additional production terms in the Reynolds stress equations, due to streamwise velocity gradients, contribute to the increase or decrease of the turbulence. The parameter of interest, easily obtainable from the production in the Reynolds stress equations,

$$P_k = - <u_i u_j> U_{i,j} = - [<u_1 u_2> U_{1,2} + <u_1 u_1> U_{1,1} + <u_2 u_2> U_{2,2}] \qquad (28)$$

is the ratio of the production by the streamwise velocity gradient, $<u_1 u_1> U_{1,1}$, to that due to the crossstream gradient $<u_1 u_2> U_{1,2}$. Restating this in streamline coordinates and using continuity produces

$$stb = \frac{<uu> - <vv>}{<uv>} \frac{U_{,s}}{U_{,n}} = \frac{b_{11} - b_{22}}{b_{12}} \frac{U_{,s}}{U_{,n}}. \qquad (29)$$

Note that the anisotropy of the turbulence determines whether the mean deformation is stabilizing or destabilizing. Flows in which combinations of these effects, as might occur in aerodynamic situations, are studied in Nakayama (1987). Nakayama also investigates the adequacy of eddy viscosity assumptions in his flow; he shows, as one might expect, that they are inconsistent with experimental data.

There are many extra strain parameters: in general they are found from a ratio of specific components of the production tensor in the second-order equations. The examples chosen here reflect situations which might be seen in multi-element airfoils including the high lift genre. In three dimensional situations, $ie.$ skewed boundary layers, there are additional extra strains of importance. Bradshaw (1975) has discussed the effects of the extra strain rates in simple shear flows as well as their impact on the thin shear layer assumptions often invoked. Bradshaw (1981) has devised a very useful classification scheme using the ratio of the extra strain, e, to the primary shear, dU/dy, as a nondimensional parameter: 1) simple shear layers: $stb < 0.001$, 2) thin shear layers: $0.001 < stb < 0.01$, 3) fairly thin shear layers: $0.01 < stb < 0.1$, 4) strongly distorted flows: $stb > 0.1$. For example for the curved flow $stb = e/dU/dy$. Extra strains begin to have some very important effects, Bradshaw (1981), once $stb > 0.001$.

5. Parameters from the second-moment equations

A few of the parameters that are found in the Reynolds stress equations are useful in understanding the nature of a turbulent flow. In this section the production to dissipation ratio is highlighted; it is useful as a measure of the equilibrium nature of the flow as well as a measure of the suitability of various localness assumptions. Also defined and described are three measures of anisotropy: that of the large scales (Reynolds stresses), the small scales (the dissipation), and the two-point length scale anisotropy.

Classification of the Reynolds stresses: anisotropy

The Reynolds stress tensor, a collection of six independent quantities is often replaced with the anisotropy tensor, b_{ij}, and the kinetic energy, $k = \frac{1}{2}(<uu> + <vv> + <ww>)$, where

$$b_{ij} = \frac{<u_i u_j>}{2k} - \frac{1}{3}\delta_{ij}. \tag{30}$$

This very insightful decomposition distinguishes variations in Reynolds stresses due to changes in energy, k, and those due to changes in structure, b_{ij}. The Reynolds stress anisotropy tensor is essentially a normalized Reynolds stress and represents deviations from an isotropic state, b_{ij}, of the large scales of the turbulence. Many model developments are done using expansions about an isotropic state, which is to say in powers of the anisotropy tensor. Thus the level of anisotropy is important in assessing the adequacy of using such models in a particular flow. There are other measures of anisotropy used in some experimental work but they do not possess the proper tensor characteristics. The only rational way to specify the anisotropy of the Reynolds stresses is by using a quantity that transforms as a tensor. Note the diagonal terms have the following very useful bounds: $-\frac{1}{3} < b_{\alpha\alpha} < \frac{2}{3}$ (no sum on Greek indices); $b_{\alpha\alpha} = -1/3$ indicates no energy in one component of the energy of the turbulence; $b_{\alpha\alpha} = 2/3$ indicates all the kinetic energy of the turbulence resides in one component of the energy.

In the usual simple shear flows the only turbulent stress of importance is the $<uv>$ shear stress which is reflected in b_{12}. Allusion has been made to the utility of the approximation $\frac{<uv>}{k} \sim const.$; this reflects the preservation of structure in simple shear flows and is used in the development of Algebraic Stress Models where $\frac{D}{Dt}b_{ij} = 0$ is assumed.

The anisotropy tensor is a very useful indicator of the flow state of the large scales of the turbulence. Consider the following characterization using the two scalar invariants associated with the tensor. The two invariants of the anisotropy tensor are defined as

$$II = -\frac{1}{2}b_{ij}b_{ij}, \quad III = \frac{1}{3}b_{ip}b_{pj}b_{ji} \tag{31}$$

For a two-dimensional flow the definitions are easily expanded to yield $II = -\frac{1}{2}((b_{11})^2 + (b_{22})^2 + (b_{33})^2 + 2(b_{12})^2)$ and $III = \frac{1}{3}((b_{11})^3 + (b_{22})^3 + (b_{33})^3 + 3b_{11}(b_{12})^2 + 3b_{22}(b_{12})^2)$. Note the following bounds: $0 < -II < \frac{1}{3}, -\frac{1}{108} < III < \frac{2}{27}$.

Any experimental measurement (or numerical result) that does not fall within these bounds is wrong (which may or may not be important). Using these invariants, II, III, a simple 2D picture indicating the state of the turbulence to be drawn. The figure, see Figure 2, has come to be called the Lumley triangle, Kassinos and Reynolds (1995), after Lumley (1978) in which it first appeared.

A few points about the triangle are worth mentioning. The origin at $(0,0)$ corresponds to an isotropic turbulence. The uppermost straight line corresponds to, what is sometimes called a two dimensional turbulence, in the sense that one of the eigenvalues of the Reynolds stress tensor is zero. The graph of this line is given by $0 = 1 + 9II + 27III$. For that reason the quantity $F = 1 + 9II + 27III$, where $0 \leq F \leq 1$ is used to characterize the departure from a two dimensional
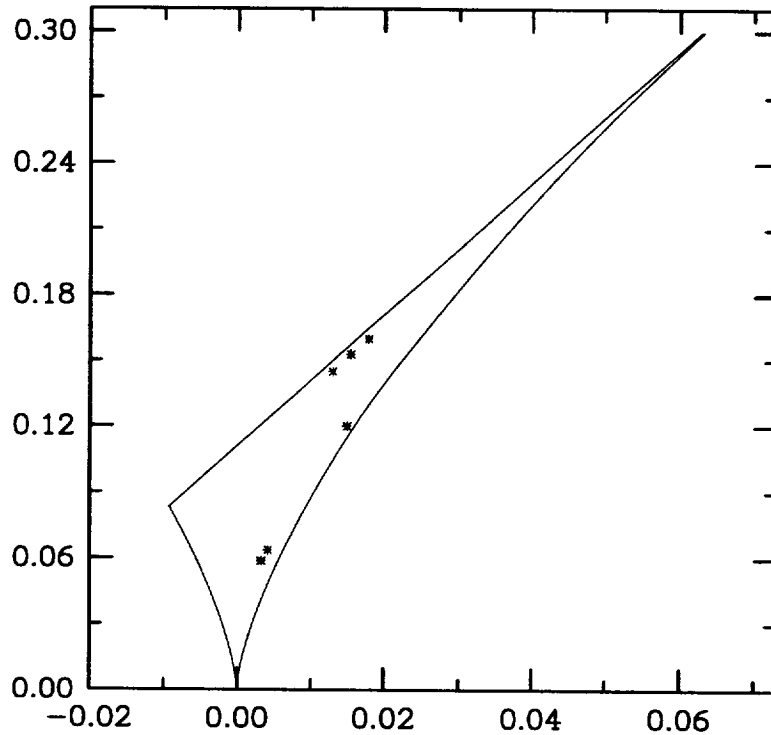
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Figure 2: Anisotropy invariant map, Lumley (1978): $-II$ vs. III

state (by which the disappearance of one of the normal stresses in principal axes is meant). For an isotropic turbulence $F = 1$ while for a two component Reynolds stress turbulence $F = 0$.

The upper right vertex of the triangle is sometimes called the one dimensional state and corresponds to a turbulence constituted of parallel vortex sheets. The upper left vertex of the triangle corresponds to a turbulence with two equal eigenvalues; a configuration that will lead to this portion of triangle is a flow state comprised of series of parallel line vortices. The two curved boundaries correspond to axisymmetric states of the turbulence. The right boundary corresponds to a state in which one normal stress is larger than the other two (which are equal). It has equation $III = 2(-II/3)^{3/2}$ and is sometimes called a cigar turbulence. The left boundary corresponds to a state in which one normal stress is smaller than the other two (which are equal). It has equation $III = -2(-II/3)^{3/2}$ and is sometimes called a pancake turbulence. The vertical line $III = 0$ corresponds to a flow state in which one eigenvalue is the average of the other two. III is a measure of the asymmetry of the distribution of the eigenvalues about the middle eigenvalue. These are all statements about the relative size of the normal components of the Reynolds stresses (in principal axes) not about the shape of eddies though such information can be inferred.

For an equilibrium homogeneous shear $II_\infty \simeq -0.0587, III_\infty \simeq 0.0032, F_\infty \simeq 0.5736$. A similar value is found for the log layer region; both of these are given by the two adjacent points in the central regions of the triangle. The third point reflects the value the invariants attain in the $15 < y^+ < 30$ region. In boundary layer flows, F varies from 1 in the outermost isotropic portions of the flow and $F \rightarrow 0$ in the inner layers, $y^+ < 10$ of the boundary layer. Some interesting figures along these lines are given by Antonia et al. (1994a). F can also be expressed in terms of the

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Reynolds stresses: $F = (R_{jj}^3 - 3R_{jj}R_{jj}^2 + 2R_{jj}^3)/6$ where $R_{ij} = < u_i u_j > / < u_p u_p >$.

Equilibrium Values	2DMFI Model	SL Model	FLT Model	SSG Model	Experimental Data
b_{11}^∞	0.209	0.202	0.208	0.219	0.203
b_{12}^∞	-0.155	-0.080	-0.146	-0.164	-0.156
b_{22}^∞	-0.148	-0.195	-0.144	-0.146	-0.143
b_{33}^∞	-0.061	0.007	-0.064	-0.073	-0.06
$(P_k/\varepsilon)_\infty$	1.88	3.42	1.99	1.88	1.73
$(SK/\varepsilon)_\infty$	6.08	21.35	6.84	5.76	5.54

As a benchmark in comparing turbulent flows to the equilibrium homogeneous shear flow (or initializing computations) a table taken from Ristorcelli et al. (1995) in which several different models are compared to the homogeneous shear flow (DNS and laboratory) is given. The equilibrium log layer has very similar values for the anisotropy while $P_k/\varepsilon = 1.0$ and $Sk/\varepsilon \simeq 3.1$.

A few examples of how the anisotropy plays a role in complex turbulent flows are worth considering.

In flows with strong streamwise accelerations (pressure gradient driven flows), the anisotropy of the normal stresses creates an important additional production term for the kinetic energy of the turbulence. The anisotropy of the normal turbulence stresses provides additional mechanisms for the production of turbulence

$$\frac{D}{Dt} k = - < u_j u_p > U_{j,p} + \ldots - \varepsilon \tag{32}$$

$$= - [< u_1 u_2 > U_{1,2} + < u_1 u_1 > U_{1,1} + < u_2 u_2 > U_{2,2}] + \ldots - \varepsilon \tag{33}$$

which can be rewritten in terms of the anisotropy tensor

$$k^{-1} \frac{D}{Dt} k = - b_{12} U_{1,2} - [b_{11} - b_{22}] U_{1,1} + \ldots - \varepsilon \tag{34}$$

presuming a two-dimensional mean and using continuity $U_{1,1} = -U_{2,2}$. The anisotropy of the normal stresses, $b_{22} - b_{33}$ will also give rise to secondary circulations (mean streamwise vorticity) in sheared flows in complex geometries as can be seen by inspection of the streamwise mean vorticity equation

$$\frac{D}{Dt} \Omega_1 = \Omega_j U_{1,j} + [k (b_{22} - b_{33})]_{,23} - [(kb_{23})_{,22} - (kb_{23})_{,33}] \ldots \tag{35}$$

Similar effects are seen in wall jets; see Launder and Rodi (1983) for a very similar physical discussion. Additional effects of anisotropy in complex mean flows can be found in Bradshaw (1987) and almost all the Bradshaw references given in the bibliography. The inadequacy of the linear eddy viscosity models in predicting the inequality of the normal stresses required to produce observed secondary circulations is well known and has been nicely treated by Speziale (1987).

Also in the streamwise vorticity, Ω_1, equation is the anisotropy of the spatial gradients of the spanwise turbulent shear stress, $b_{23} k = < u_2 u_3 >$. In three-dimensional boundary layers, eg. flows

with spanwise pressure gradients, this becomes a more important source of secondary circulations. In flows with strong streamwise variations the anisotropy of the normal stresses is an additional source of the *primary*, Ω_3, (spanwise) vorticity.

Production to dissipation

The ratio of production to dissipation, P_k/ε, is used to characterize the localness of the turbulent flow. The production is related to

$$P_k = - < u_i u_p > U_{i,p} = -2 < u_i u_p > S_{ip} = -2\, k b_{ij} S_{ip} \qquad (36)$$

is the rate of growth of the kinetic energy, $k = \frac{1}{2} < u_j u_j >$, due to the Reynolds stresses working on the mean strain. In the equilibrium log layer $P_k/\varepsilon = 1$ while in an equilibrium homogeneous shear $P_k/\varepsilon \simeq 2$. In a wake flow $P_k/\varepsilon < 1$.

In the equilibrium log layer of a flat plate in which $P_k/\varepsilon = 1$ one can, with only nominal handwaving, show that the Reynolds stress is described by an eddy viscosity. It is this nominal and singular success and its similarity to thin simple free shear layers that has spawned an engineering tool whose foundation is no more universal than a flat plate. Part of the reason for this success is that the locally generated turbulence dominates the local structure. It is this fact along with the inherent decorrelating effects of turbulence that suggests the possibility of a universal behavior determined by local quantities. It is, in part, for this reason that the local algebraic eddy viscosity closures have performed so well in the simple flows whose primary mean flow is, like the flat plate, a simple shear.

Flows with nominal production like the wake are noticed to have a stronger dependence on the initial conditions and these effects are not accounted for in local eddy viscosity type closures. This reflects the well known fact that turbulence does not exhibit universal behavior at low Reynolds number.

The anisotropy of the dissipation

The anisotropy of the dissipation is an important quantity - it sets the levels of dissipation of the individual Reynolds stresses and thus modifies the anisotropy of the Reynolds stresses. The anisotropy of the dissipation tensor represents deviations from an isotropic state of the small scales of the turbulence. They are a measure of the departure from an ideal Kolmogorov turbulence. Reynolds stress anisotropies, mean deformations, and low Reynolds number effects, are all associated with small scale anisotropies. Similar to the anisotropy tensor for the Reynolds stresses one can define an anisotropy tensor for the dissipation:

$$d_{ij} = \frac{\varepsilon_{ij}}{2\varepsilon} - \frac{1}{3}\delta_{ij} \qquad (37)$$

where $\varepsilon_{ij} = 2\,\nu < u_{i,k} u_{j,k} > = 2\varepsilon(\frac{1}{3}\delta_{ij} + d_{ij})$. Similar ideas describing b_{ij} apply here. For the dissipation to be isotropic $d_{ij} = 0$. The dissipation is not isotropic and this fact has been recognized for many years, Townsend (1954). The anisotropy of the dissipation is sometimes measured, Tavoularis and Karnik (1989), Antonia *et al.* (1994b). In a simple shear, Tavoularis and Karnik (1989) have indicated that $d_{11} \simeq 0.15$, $d_{22} \simeq= -0.05$, $d_{33} \simeq -0.09$ and $d_{12} = -0.14$. In general

they find that the dissipation of the small scales can be approximated by $d_{ij} = 0.85 b_{ij}$ - not reason enough for assuming the dissipation isotropic as is assumed in some turbulence closures.

A definition of anisotropy using the vorticity is also sometimes used, Lee *et al.* (1990). If one is interested in comparing the anisotropy of the small and large scales such a definition of the anisotropy (based on a vector) is misleading. Consider for a moment the fact that a one dimensional vorticity field is associated with a two dimensional velocity field, Lumley (1984).

<u>Correlation coefficients</u>

Note that the bounds on the various quantities, b_{ij}, II, III, F, formed from the Reynolds stresses can be used to verify experimental measurements. The bounds can also be used to design turbulence models that do not predict flows outside these bounds. This is a feature sometimes incorporated in turbulence models and can be very useful from the viewpoint of computational stability in complex flows. The bounds on the invariants of the anisotropy tensor are very closely related to statistical inequalities that come from the Cauchy-Schwartz inequality. From the simplest point of view, these quantities are the correlation coefficients and are required to be bounded, in magnitude, by unity: for example

$$-1 \leq \rho_{\alpha\beta} = \frac{< u_\alpha u_\beta >}{< u_\alpha u_\alpha >^{1/2} < u_\beta u_\beta >^{1/2}} \leq 1. \tag{38}$$

Like b_{12} the quantity ρ_{12} in simple shear flows usually varies much less than the Reynolds stresses and the energy indicating a relatively slow evolution of structure. Thus the correlation as well as the anisotropy tensor are measures of turbulence structure and reflect, to a certain degree, the coherence and stability of turbulent eddies. A sterling example of this phenomena is the decaying turbulence: the correlation coefficient decays algebraically while the energy decays nearly exponentially, Narashima and Sreenivasan (1979).

As an indication of the structural changes a small change in the mean deformation can have, consider the boundary layer with nominal streamwise curvature: ρ_{12}, usually about 0.6 throughout a boundary layer, now has a zero and changes sign throughout the outer portions of the layer. Such a change in sign without a simultaneous change in sign of the mean velocity gradient indicates that the Reynolds stresses begin to decrease rather than increase the energy of the turbulence.

The turbulence intensity

It is customary to measure and report turbulence intensities as a reflection of the importance of turbulent processes. The turbulence intensity is defined as the square root of the different Reynolds stresses normalized by the local mean velocity;

$$\frac{< uu >^{1/2}}{U}, \qquad \frac{< uv >^{1/2}}{U}. \tag{39}$$

In many flows the turbulence intensity is quite small, a few *per cent*. For many of these flows simple turbulence closures do well in simple flows. However there are many flows in which intensities are quite large. In separated flows turbulence intensities can be quite large, even 30% and simple models are no longer adequate (which does not mean they can't be tuned to get the right answers). In such flows there is also the possibility of important transfer of energy from the turbulence to the

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mean flow, a mechanism not accounted for in eddy viscosity closures. Stagnation regions are also places in which energy transfer due to turbulent transport is more important than advection and simple ideas built into turbulence models assuming the predominance of a rapid mean flow are no longer adequate.

Measures of two-point (length scale) anisotropy

In the single-point closure methods the flow, at any one point in the flow, is assumed to be characterized by one length scale. There are in fact several length scales in a given turbulent flow as can readily be defined from the two-point correlations of the velocity:

$$L_{\alpha\beta,k} < u_\alpha u_\beta > = \int < u_\alpha(\mathbf{x}) u_\beta(\mathbf{x} + \Delta x_k) > dx_k. \tag{40}$$

One then assumes that all the other length scales scale with this length and constructs models that are parameterizable by this one scale. Depending on the flow situation this may or may not be a suitable assumption. The most commonly used length scales are the longitudinal integral scale

$$L_{11,1} < u_1 u_1 > = \int < u_1(\mathbf{x}) u_1(\mathbf{x} + \Delta x) > dx. \tag{41}$$

and transverse length scale

$$L_{11,2} < u_1 u_1 > = \int < u_1(\mathbf{x}) u_1(\mathbf{x} + \Delta y) > dy. \tag{42}$$

Typically $L_{11,1} \simeq 2L_{11,2}$. Measures of the two-point anisotropy are useful in assessing the adequacy of the modeling assumptions involving a single "isotropic" length scale (which is taken to be the Kolmogorov length scale). A good measure for the two-point anisotropy would be a comparison of quantities such as $L_{11,2}$ and $L_{22,1}$. The knowledge of the integral length scales in two different directions, $L_{11,2}$ and $L_{22,1}$ draws attention to the adequacy of the single length scale assumptions used in single-point closures and delineate the flows in which such effects are important. Free stream turbulence effects are often characterized using the integral length scales and the comparison of the different length scales will give a very useful specification of the anisotropy of the free stream turbulence. The recognition of the anisotropy of the correlation lengths in rapid distortion problems is something that is indirectly recognized in the structure function methodology proposed by Reynolds; Kassinos and Reynolds (1995) is a summary of the current view of this method.

A "natural" length scale of the turbulence: ℓ

It is possible to define several different length scales. There is the mixing length defined using the eddy viscosity approximation

$$< uv > = -\nu_t U_{1,2} \simeq -(\frac{2}{3}k)^{1/2} \ell_m U_{1,2} \tag{43}$$

thus

$$\ell_m = < uv > /((\frac{2}{3}k)^{1/2} U_{1,2}). \tag{44}$$

There is, as discussed, the two-point correlation length scale,

$$L_{ij,k} < u_i u_j > = \int < u_i(\mathbf{x}) u_j(\mathbf{x} + \Delta x_k) > dx. \tag{45}$$

Lee *et al.* (1990) have quite nicely shown, for the rapid homogeneous shear, that the most appropriate scaling parameter is the length scale that comes from the Kolmogorov scaling $\ell = \alpha \tilde{u}^3/\varepsilon$. This scaling is appropriate for a high Reynolds number, *ideal* Kolmogorov turbulence but is a very robust scaling. It is sometimes identified with an integral length scale such as the longitudinal correlation length, $\ell = L_{11,1}$. See Sreenivasans (1995) summary for a number of simple incompressible shear flows. The constant of proportionality does not however appear to be a universal constant. Sreenivasan (1995) has assessed the accuracy of this expression in several canonical simple shear flows. For homogeneous shear flows the data indicates $\alpha \sim 1 - 2$. For the log layer or wake flows $\alpha \sim 4$. For flows with smaller microscale Reynolds numbers Sreenivasan (1984, 1994) shows that $\alpha \sim R_\lambda^{-1}$.

6. Homogeneity, transport and higher order moments

Many turbulence model developments, because of the possibility of exact mathematical results, use a quasi-homogeneous assumption. For developments involving the assumption of quasi-homogeneity one presumes the turbulence to be fine-grained and one can identify a fine-graining parameter, say $\epsilon_\ell = \ell/L$ a ratio of correlation scale to a geometrical length characteristic of the inhomogeneity. Developments are then thought of as a series expansion in ϵ_ℓ the leading order term being the homogeneous result. This is essentially an invocation of kinetic theory ideas about the ratio of length scales of the turbulence to the mean flow. In this way model development captures the leading order terms for quantities requiring closure. What can one expect of a turbulence model that does not capture the proper behavior in the simplest of flows ? One should insist that a turbulence model be consistent with results from homogeneous turbulence. This is an important issue that has been exploited profitably by Speziale *et al.* (1990), and in the context of Algebraic Stress Models by Taulbee (1992), Gatski and Speziale (1993), and most recently in a new development by Girimaji (1995).

A variety of statistics related to diverse issues concerning homogeneity are given. In general one is interested in the homogeneity of the mean as well as that of the turbulence. One assesses the homogeneity on the scale of the turbulence length scale, ℓ. In inhomogeneous flows issues regarding how the turbulence transports itself, not accounted for in most simple closures for the Reynolds stresses are also important. Two additional statistics the skewness and the kurtosis indicate much about processes, such as entrainment, occurring at the edges of turbulent regions.

Homogeneity of the mean deformation

As has been seen the mean deformation field plays an important role in determining the turbulence. The Kolmogorov ideal is homogeneous and most turbulence model developments are made assuming homogeneity. A natural measure of inhomogeneity will be the inhomogeneity of the mean deformation. One measure of the inhomogeneity of the mean deformation is the length scale invented by Prandtl in his application to mixing length models near inflectional points: $L_{\nabla U} = U_{1,2}/U_{1,22}$ which is readily generalizable to multi-dimensional flows as $L_S^{-1} = \frac{\nabla S}{S}$. In which case a measure of the inhomogeneity of the mean deformation is

$$\frac{\ell}{L_S} = \ell \frac{\nabla S}{S} < 1. \tag{46}$$

The assumption that ∇U is uniform over an integral scale of the turbulence is called the quasi-

homogeneous assumption and is invoked in constitutive arguments using results from homogeneous turbulence to close unknown terms. It is related to the limited awareness assumption mentioned earlier. This measure of inhomogeneity has problems in regions of the flows were $S = 0$ as occurs at points of symmetry; one can then use $\nabla\nabla S/\nabla S$.

Homogeneity of the turbulence field

In the single-point closures mixing length hypotheses are often used to account for the effects of turbulent transport. The mixing length hypotheses will do very well if the inhomogeneity and the anisotropy of the field is small or the turbulent transport is not important as might be in regions of the flow dominated by mean advection and production. Similarly,

$$\frac{\ell}{L_k} = \ell\frac{\nabla k}{k} < 1. \tag{47}$$

There are several situations in which the quasi-homogeneous mixing length type assumptions are inadequate, ie. when $\ell/L_k > 1$. These are now considered. These situations arise when there is a mixing of turbulence with different origins. For example, when large scale free stream turbulence produced by a turbine blade or an aircraft component interacts with a downstream turbulent shear layer. Pressure fluctuations in the downstream will produce turbulence stresses of opposite sign to those predicted by an eddy viscosity model. This is one of many mechanisms of countergradient diffusion, ie. diffusion up the gradient which is in effect a negative viscosity in the context of a mixing length approximation. Similarly in merging or asymmetric shear layers in which the mean shear varies substantially over a distance ℓ, ie. $\ell/L_{\nabla U} > 1$, similar countergradient transport can be seen. The Reynolds stress will not be proportional to the shear nor have the sign as might be predicted by an eddy viscosity model. For additional examples the reader is referred to Launder (1989) or Hunt (1992).

Turbulent transport

Turbulent transport of the mean flow momentum is done by the Reynolds stresses. In the way mean velocity gradients are a production mechanism for the Reynolds stresses, Reynolds stress gradients are production mechanisms for the turbulent transport of Reynolds stresses. Thus an assessment of the inhomogeneity of k according to the previous section is also an assessment of the importance of turbulent transport.

In $k - \varepsilon$ or second-order closures one is interested in the turbulent transport of the kinetic energy and of the Reynolds stresses. These are the third-order moments of the velocity field, $< u_i u_j u_k >$, and appear in the k and $< u_i u_j >$ equations in engineering flows as in Section 2.

For flows in which the mean advection is dominant, $< u_i u_j u_k >$ are typically not so important and simple mixing length models seem to be adequate. For flows in which the mean advection is small, say at the edges of wakes, shear layers and jets, behind bluff bodies or in the stagnation regions associated with separation turbulent transport is expected to be important. In these flows it is not unusual to see turbulence intensities on the order of 0.3. Loosely speaking whenever $\mathcal{P}/\varepsilon < 1$ transport or mean advection will be important and locally generated turbulence will not dominate the local structure. The importance of turbulent transport, gradients of the third-order moments can be estimated from experiment by comparison of its derivatives to the mean flow advection:

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$< u_i u_j u_p >_{,p} vs. U_p < u_i u_j >_{,p}$. Its size relative to production is an equivalent measure of its importance. In the case of the energy equation

$$< u_p k >_{,p} \qquad vs. \qquad U_p k_{,p} . \tag{48}$$

There are a variety of models available for the turbulent transport. The simplest one is an isotropized eddy viscosity transport model and has the general form

$$< u_j u_j u_k > = - \nu_t \left[< u_i u_j >_{,k} + < u_i u_k >_{,j} + < u_j u_k >_{,i} \right]. \tag{49}$$

Large scale skewness

An interesting statistic that indicates much about the underlying probability density function and the aspects of turbulence structure and entrainment as well as transport is the large scale skewness. The skewness is the third-order moment of the velocity field. It is defined as

$$S(u_\alpha) = < u_\alpha^3 > / < u_\alpha^2 >^{3/2}; \tag{50}$$

in a temporal sense it represents the predominance of fluctuations above (positive) or below (negative) the local mean. As such it is related to the *asymmetry* of the probability density function of the velocity fluctuations. It is a sensitive indicator of changes in the large scale structure. In a spatial sense it represents the turbulent transport in the positive or negative α direction. It is a statement about the direction of entrainment. For a Gaussian pdf $S_\alpha = 0$. From a more pragmatic modeling point of view this quantity can be used to assess the validity of models for the turbulent transport so essential in regions of the flow where production terms are not dominant.

Large scale flatness

Another statistic indicating facts about the underlying probability density function and aspects of turbulence structure is the flatness (kurtosis). It is the fourth-order moment of the velocity field:

$$K(u_\alpha) = < u_\alpha^4 > / < u_\alpha^2 >^2 . \tag{51}$$

The flatness is an indication of the occurrence of fluctuations far from the mean: it is an indicator of the relative frequency of rare events. It has been related to the intermittency of the flow, $I_\alpha = 3/K_\alpha$. Intermittency is defined as the fraction of time the flow is turbulent versus laminar and is important in the mixing regions at the edge of a turbulent region. For a Gaussian pdf $S_\alpha = 0$ and $K_\alpha = 3$. Much use is made of the Gaussian value of the kurtosis, $K_\alpha = 3$, in model developments. The fact of the matter is that, because of intermittency, in the near wall region and in the laminar/turbulent interfacial regions the kurtosis can be as high as 10-40. Changes in these quantities as function of additional strains or pressure gradients on the turbulence are indicative of a change in the physics of turbulence processes at the periphery of flows.

7. Miscellaneous additional ideas and statistics

A few miscellaneous ideas not already accounted for which appear necessary from the viewpoint of completeness are given.

The turbulent Reynolds number and the eddy viscosity metaphor

In the two-equation family of turbulence models substantial use of an eddy viscosity idea is made:

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typically defined as $\nu_t = c_\mu k^2/\varepsilon$ with $c_\mu \simeq 0.09$. The ratio of a turbulent eddy viscosity to the molecular viscosity appears in the turbulent Reynolds number, $R_t = \tilde{u}\ell/\nu$, when the definitions $\tilde{u}^2 = \frac{2}{3}k$ and $\varepsilon^2 = \alpha(\frac{2}{3}k)^3/\ell^2$ are used: $R_t = \frac{4}{9}\frac{k^2}{\nu\varepsilon}$ with $\alpha = 1$ thus $\nu_t/\nu = c_\mu R_t$. R_t is significant not only as an indication of the bandwidth of the process but also indicates, in a mixing layer argument context, the importance of turbulent versus molecular transport.

The Boussinesq eddy viscosity formulation is $<u_i u_j> = \frac{2}{3}k\,\delta_{ij} - \nu_t[U_{i,j} + U_{j,i}]$. For a simple shear flow

$$<uv> = -\nu_t U_{1,2}. \tag{52}$$

It is well known that, in only nominally complex flows, an eddy viscosity formulation does not predict the Reynolds stresses correctly. This does not mean that one cannot predict many mean quantities of interest using an eddy viscosity formulation. (I believe it was Gauss who said given 12 free constants he could construct an elephant (the quote is from Lumley (1978)).

As has been mentioned, in the equilibrium log-layer of a flat plate, where $P_k/\varepsilon = 1$, it can be argued that the one important Reynolds shear stress can be described by an eddy viscosity. It is this singular success and its similarity to thin simple free shear layers in which production closely balances dissipation in the dynamically significant portions of the flow that suggests the possibility of an eddy viscosity. An argument no more universal than a flat plate.

It is easy to convince oneself of the inadequacy of an eddy viscosity hypothesis by measuring an eddy viscosity from $\nu_t = <uv>/U_{1,2}$ and comparing it to $\nu_t = c_\mu k^2/\varepsilon$. One will find that zeros of $<uv>$ and $U_{1,2}$ are rarely coincident in any flow that has some form of asymmetry. The consequence of this is that the eddy viscosity defined as $\nu_t = -<uv>/U_{1,2}$ varies between zero and infinity in a substantially small portion of the flow. While ν_t is not useful as a predictor of the Reynolds stresses in more complex flows it is useful in its nondimensional form, $c_\mu R_t$, as a relative measure of the turbulent mixing.

The small scale skewness

This was mentioned earlier so the exposition will be brief. In equilibrium theories of turbulence, for homogeneous isotropic turbulence the skewness, $<u_{1,1}^3>/<u_{1,1}^2>^{3/2}$, is thought to be a universal constant. (It is in fact slightly dependent on the Reynolds number.) Some researchers take its attaining a value of $-\frac{1}{2}$ as an indication of the state of development of the cascade toward some sort of equilibrium form. The skewness is the production mechanism for the dissipation. In highly complex flows very little is known about the skewness. More details can be found in Tavoularis *et al.* (1978).

Estimating ε

Physically the dissipation - the rate at which *viscous* forces dissipate the energy of the turbulence - is often understood, in current models, as rate at which *inviscid* nonlinear processes cascade energy to the smaller scales of the motion. It is this fact that is exploited in the arguments used in many current turbulence models. It is a simple idea with several very important and even elegant ramifications. The most important of which is that it allows one to ignore the details of the small scale motions of the fluid. It is for this reason that so much time was spent on the questions of the

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existence of a universal small scale equilibrium. The cascade rate and the dissipation rate are not equal in nonequilibrium situations.

It is usual to estimate the dissipation from the time trace of the velocity field and Taylor's hypothesis: in which case $\varepsilon = 15\nu < u_{1,1}\,u_{1,1} >= 15\nu < u_{,t}\,u_{,t} > /U^2$ using the assumption of local isotropy. Measurement of the dissipation in some complex flow situations may be difficult. A recent review of various methods of measuring vorticity is given in Wallace and Foss (1995).

Alternatively, it is possible, assuming local isotropy, to obtain an estimate for the dissipation from the trace of the Reynolds stress equations. For an incompressible flow

$$\frac{D}{Dt} k = - < u_j u_p > U_{j,p} - [< pu_k > + 1/2 < u_p u_p u_j > - Re^{-1} k_{,j}]_{,j} - \varepsilon. \tag{53}$$

In high Reynolds number flows some neglect the pressure flux in which case

$$\frac{D}{Dt} k = - < u_j u_p > U_{j,p} - 1/2 < u_j u_j u_p >_{,p} - \varepsilon. \tag{54}$$

Sometimes the pressure flux is estimated using Lumleys (1978) $< pu_k >= -\frac{1}{5} < u_k q^2 >$. Note that the pressure has been normalized by the density. Thus one only need measure the production, the transport and the advection. A measurement of the large scale quantities can be used to produce an estimate for the dissipation. In general graphs of these four quantities are called energy budgets and are very useful for assessing the physics of any particular flow. A good sampling of energy budgets is given by Rodi in Launder (1975). Tennekes and Lumley (1972) give some examples also.

It should be stressed that whatever measurement is being done at the very least an energy budget, using the k equation should be made. The utility of relying on the governing equations, as obvious as it seems, still seems to require amplification and demonstration, George (1990).

8. Synopsis, summary and sample

An attempt to draw all the key parameters into one short concise and cohesive summary with examples is made. As the objective is to highlight physics all the parameters are nondimensional. To make the relevance of the nondimensional parameters more concrete they are given for two simple flows: the wake and the plane jet. These are exceedingly simple flows and the interesting phenomena associated with a complex engineering flow are not present: the present purpose however is to illustrate the utility of the nondimensional parameters to describe the flow. Quantitative accuracy and model assessment are *not* the present subject. The object is to indicate how these parameters are used.

The data in the accompanying figures comes from a Reynolds stress calculation for the plane jet and the plane wake. The plane wake calculation was done using the standard Launder, Reece, and Rodi (LRR) rapid strain model with a Daly and Harlow turbulent transport model. The plane wake calculation was done using the pressure strain model by Speziale *et al.* (1990a) with a Daly and Harlow turbulent transport model.

The horizontal axis in the figures is the cross-stream coordinate normalized by the "half width"; at $y = 1$ the axial velocity is half its maximum.

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The apparent singular behavior of some quantities at the edge of the jet, $y > 2$, is a result of a combination of issues associated with the imposition of boundary conditions, numerical differentiation, and low Reynolds number modeling issues and the inadequate modeling of the physics of the far regions. For the round jet simulation this in the outer regions of the jet which are not dynamically significant and this is not an important shortcoming. This may not be the case in complex jet flows where entrainment and mixing with the ambient are the quantities required.

Small scale equilibrium criteria	Parameter	Limit
Spatial/temporal spectral gap	$R_t = \frac{4}{9} k^2/(\nu\varepsilon)$	$R_t^{1/2} >> 1$
Temporal spectral gap - shear	$(\varepsilon/\nu)^{1/2}/S$	$\frac{3}{2} R_t^{3/2} >> Sk/\varepsilon$

Bandwidth: The $R_t = \frac{4}{9}\frac{k^2}{\nu\varepsilon}$ is useful for indicating the bandwidth of the spectrum of the fluctuations. This is crucial to understanding the adequacy of spectral gap assumptions and the associated assumptions regarding the statistical equilibrium of the small scales. The turbulent Reynolds is loosely related to the mean flow Reynolds number by: $R_t = \frac{\tilde{u}}{U_\infty}\frac{\ell}{L} Re$ where $Re = U_\infty L/\nu$.
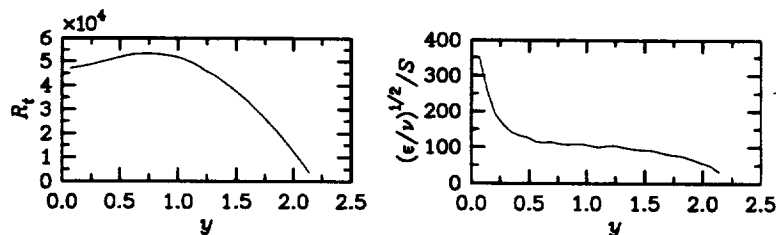

Figure 3: Statistics from a Reynolds stress calculation for the plane jet: spectral gap parameters.

As is seen in Figure 3, $R_t \sim 10^4$ vindicating assumptions regarding a separation of scales throughout most of the jet. At the periphery of the jet the turbulent Reynolds number becomes small, $R_t \sim 50$ and many assumptions used in turbulence modeling are no longer tenable. The saving grace here is that in many applications this region of the flow is not dynamically significant. This is seen by the very small k in these regions. Unfortunately there are many flows in which the mixing across the outer portions of the flow are important. The current turbulence models have problems in these regions, as will become evident in the behavior of k and ε at the periphery of the flows.

To give a counter-example to the very nice spectral gap that appears in the jet flow (implied by the large turbulent Reynolds number) the same quantities are shown for the wake flow. The values are much smaller and the assumption of a spectral gap and the implied isotropic small scale equilibrium is not acceptable. The point is that the independence of the small scales of the motion from the large scales is simply not adequate when the strain rate of the small scales is the same as that of the large scales. One might describe this flow as only weakly turbulent.

Production to dissipation: The ratio of production to dissipation gives a measure of the importance of nonlocal effects; it indicates whether the local turbulence determines the local structure.
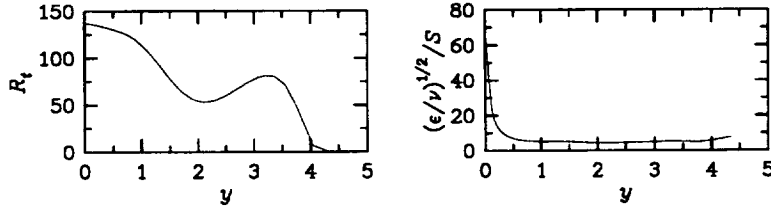
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Figure 4: Statistics from Reynolds stress calculation for the wake: spectral gap parameters.

In the figure for the jet flow the production to dissipation is seen to deviate from $P_k/\varepsilon \simeq 1$ in the central portions of the jet indicating that most of the energy of the turbulence is imported from other regions.
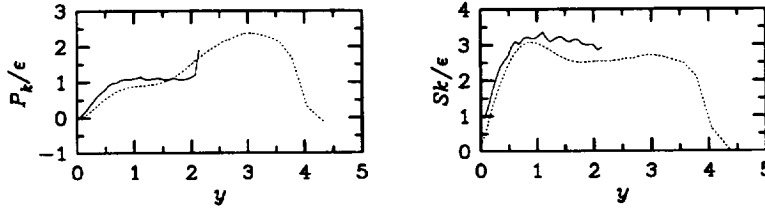

Figure 5: Production to dissipation and relative time scales for the jet (solid line) and the wake.

Over a large portion of the jet flow , $0.5 < y < 2$ - the most energetic portions of the flow - $P_k/\varepsilon \simeq 1$ indicative of a local equilibrium and thus a flow in which most of the turbulence is generated *and* dissipated locally. This suggests that the local turbulent shear stress can be determined by local quantities. In such a simple shear flow a Bousinesq eddy viscosity formulation will do fine.

Contrast this behavior to the wake flow in which production is about 20% less than dissipation over most of the energetic portion of the wake. The wake is known to have a much larger dependence on the initial conditions and this is consistent with the fact that turbulence produced is not dissipated locally. P_k/ε is a measure suggesting the need of a differential closure and more refined turbulent transport models. An issue of some importance in complex flows.

In the outer jet regions the ratio drifts upwards in an unrealistic fashion reflecting the lack of development of closures and boundary conditions for the dynamically insignificant portions of simple jet flows.

A list of parameters relevant to the assessment of the nonequilibrium nature of the flow, $\alpha_{ij}, Sk/\varepsilon$, and $T_I^{-1}k/\varepsilon$ - a characterization of the mean deformation along the lines of how long, how hard and how fast - will be relevant to many high speed flows with rapid changes in the streamwise direction. The point here is the more important nonlocal effects in time or space the less adequate a local closure (eddy viscosity or algebraic stress model). The accompanying chart indicates values

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for which the deformation of the turbulence may be conceived as an equilibrium process.

Importance of nonequilibrium effects	Parameter	Limit
Production to dissipation	$P_k/\varepsilon = 2\,(k/\varepsilon)\,b_{ij}S_{ij}$	~ 1
Total strain	α_T	$\alpha_T > 4$
Relative strain parameter	Sk/ε	$Sk/\varepsilon \sim 5$
Imposed time scales	$S^{-1}(k/\varepsilon)D/Dt\,S$	~ 1
Relative time scale	$T_I S$	> 1

Total strain: The length of time the strain has been acting on the turbulence is

$$\alpha_{ij} = \int^t S_{ij}dt' \qquad (55)$$

where the differential is understood as following a mean fluid particle. The longer the strain has been acting the more adequate the single-point structural equilibrium turbulence models, such as the algebraic Reynolds stress models.

The relative strain rate: Sk/ε, can be interpreted either as the total amount of shear an eddy experiences during its lifetime; as the magnitude of the deformation by the mean *versus* the deformation by the turbulence. For an equilibrium homogeneous shear flow $Sk/\varepsilon \simeq 6$; in the figure $Sk/\varepsilon \sim 0$ near the axis due to symmetry of the mean flow; off axis it approaches a modest $Sk/\varepsilon \sim 3$. The flow is therefore self consistent, within the class of flows for which turbulence models such as this one, are useful.

Mean strain change rate: A closely related quantity in flows with imposed time scales would be $S^{-1}(k/\varepsilon)D/DtS$ and an indicator of the adequacy of the localness assumptions made in modeling assumptions. The larger the number the more questionable the local approximation. This is moderated by the strength of the strain which through its effect on the production of turbulence will scramble the memory of the turbulence. A number characterizing this effect might be $T_I S$ or equally well thought of as the total strain.

Natural time and length scales of the turbulence: As was mentioned one typically interprets, *modulo* a coefficient of proportionality, k/ε as an eddy decorrelation time (in units made nondimensional by the maximum velocity difference and the half width). The figure indicates its minimum in the inner regions of the jet indicating a rapid cascade of energy to smaller scales. It drifts to higher values at the edges of the jet indicating a weaker cascade and a weaker (more intermittent with higher flatness) turbulence.

Also shown in the same figure is a natural length scale, sometimes called the dissipation length scale, for its definition from Kolmogorov scaling, $\ell = \alpha(\frac{2}{3}k)^{3/2}/\varepsilon$. It has been normalized by the flow half width. It is seen, in both the flows, that ℓ scales very nicely with the half width of the flow. The constant of proportionality has been taken to be unity, $\alpha = 1$.
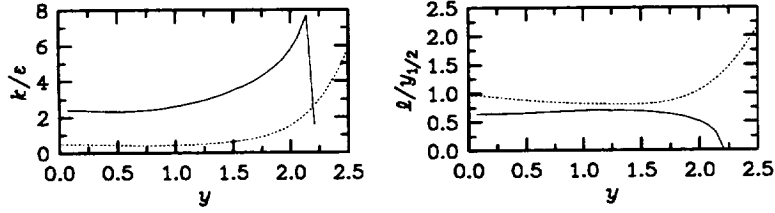
Figure 6: Statistics from a Reynolds stress model calculation for the jet (solid line) and wake: turbulence time and length scales.

Further characterization of the deformation	Parameter	Limit
Mean velocity gradients	Def	$-1 < Def < 1$
Extra strains	stb	

Mean deformation: Of substantial importance is the type of deformation the turbulence is undergoing. Many of the nonequilibrium parameters given above also describe the mean deformation. The deformation parameter, as normalized by Hunt (1992) is very useful:

$$Def = \frac{S_{ij}S_{ij} - W_{ij}W_{ij}}{S_{ij}S_{ij} + W_{ij}W_{ij}} \tag{56}$$

which varies between $-1 \leq Def \leq 1$ which corresponds to a pure rotation through a pure strain. As jet and wake flows are, primarily, pure shear flow $Def \sim 0$ and is not shown.

Extra strains: Diverse strains and combinations thereof can have a stabilizing or destabilizing effect on the turbulence in different parts of the flow. For two-dimensional mean flows of aerodynamic interest are the curvature and the streamwise acceleration parameters.

$$stb = \frac{U/R}{dU/dn} \qquad\qquad stb = \frac{b_{11} - b_{22}}{b_{12}} \frac{U_{,s}}{U_{,n}} \tag{57}$$

Reynolds stress anisotropy: Further information about the turbulence can be found from the anisotropy tensor,

$$b_{ij} = \frac{< u_i u_j >}{2k} - \frac{1}{3}\delta_{ij} \tag{58}$$

and its invariants, II, III. The bounds on these quantities are given above and a presentation in the form of an anisotropy invariant map, or Lumley triangle, is useful. The components b_{11} and b_{12} of b_{ij} are given in the figure. Also shown is the correlation ρ_{12} and two invariants of the anisotropy tensor, II and F. From these three figures it is seen that the turbulence is more isotropic in the central regions of the jet. For sake of comparison results from the homogeneous shear, with $P_k/\varepsilon \simeq 2$, indicate $b_{11} \simeq 0.2$ and $b_{12} \simeq -0.16$. (Note that the difference in coordinate system accounts for the difference in sign.) At the periphery of the jet, $y > 2.25$, though not shown, the calculation returns values for these quantities that are outside of the allowed range for reasons already indicated.
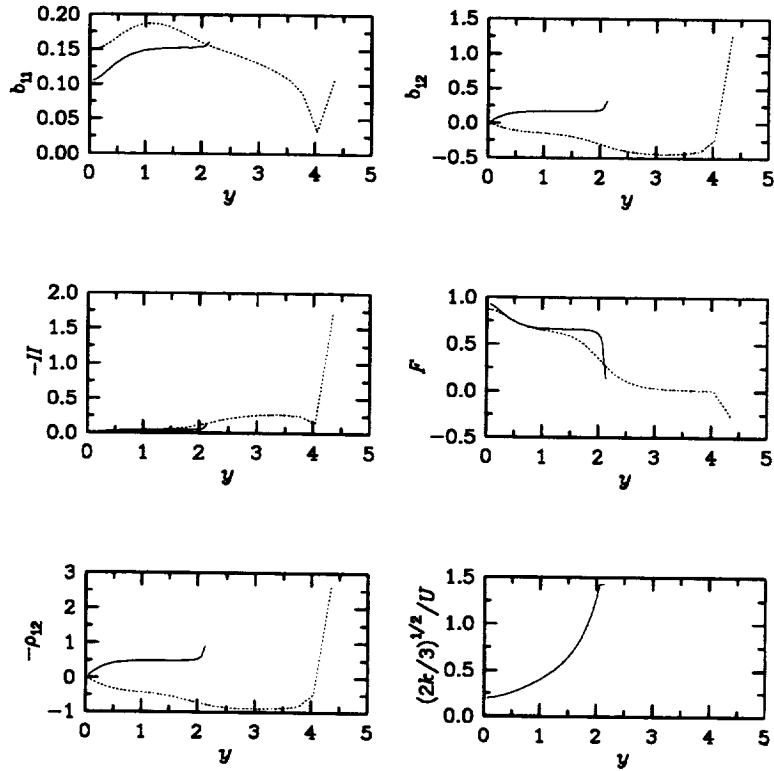
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Figure 7: Statistics from a Reynolds stress model calculation for the jet and wake.

Correlation coefficient: In simple flows this is typically an uninteresting quantity; typically about $0.5 - 0.8$ or so. It is however something that must be bounded in magnitude by one; $\rho_{12} = < uv > /(< uu >< vv >)^{1/2}$ it is useful as an indication of the reliability of the calculation. In more complex flows it becomes a more important quantity zeroes of this quantity indicating changes in the stability of the flow.

Characterization of the turbulence	Parameter	Limit
Reynolds stress anisotropy	$II = -\frac{1}{2}b_{ij}b_{ij}$	$0 \leq -II \leq \frac{1}{3}$
Turbulence intensity	$< uu >^{1/2}/U$	$\ll 1$
Dissipation anisotropy	$II_e = -\frac{1}{2}\varepsilon_{ij}\varepsilon_{ij}$	$0 \leq -II \leq \frac{1}{3}$
Correlation coefficient	ρ_{12}	$-1 \leq \rho_{12} \leq 1$
Two-point anisotropy	$L_{\alpha\alpha,\beta}$	$L_{11,1}$ vs. $L_{22,2}$

Turbulence intensity: From the figure it is seen that the turbulence intensity, $(\frac{2}{3}k)^{1/2}/U$, where U is the local streamwise mean, becomes large at the periphery of the jet indicating the relative importance of turbulent transport over mean advection. In regions such as this or in separated flows, where turbulence fluctuations are much higher than the local mean flow, accurate turbulent transport models for more complex flows are expected to be important if mixing between different

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streams is to be sought. The intensity of the wake is so small that it does not show on the plot.

Inhomogeneity and transport	Parameter	Limit
The mean deformation	$\ell\frac{\nabla S}{S}$	< 1
The turbulence field	$\ell\frac{\nabla k}{k}$	< 1
Relative length scales	ℓ/L	< 1
Turbulent vs. molecular transport	$R_t = \frac{4}{9}\frac{k^2}{\nu\varepsilon}$	$c_\mu R_t >> 1$

Homogeneity of mean and turbulence fields : The mean deformation field is inhomogeneous with respect to the turbulence length scale. In the inner regions this is due to the fact that S has a zero on the symmetry axis. In the regions where production is important where the mean shear is highest, (near $y \sim 1$), the inhomogeneity is modestly consistent with the $P_k/\varepsilon \simeq 1$ seen above. The inhomogeneity increases again in the outer portions of the jet.

Measures of the inhomogeneity of the turbulence field are also shown. From the figure it is seen that the gradients of the kinetic energy are getting large at the periphery of the jet where turbulent transport is important. Recall that gradients in Reynolds stresses are the production mechanisms for the third-order moments.
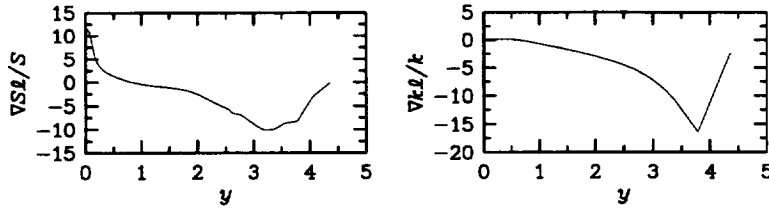

Figure 8: Homogeneity statistics from a Reynolds stress model calculation for the plane jet.

A gloss on the high-lift aerofoil scenario: As a further concretization of these ideas a thought experiment in a complex flow situation might be considered. Consider the high lift three-element aerofoil. If the upstream flow is turbulent, a free stream turbulence problem, the upstream fluctuations need to be characterized by their one and two point anisotropies. Free stream turbulence has a strong effect on transition, skin friction and the Reynolds stresses in the outer portions of the boundary layer. In the slat region between the first two elements there are strong curvature and strong streamwise accelerations the effects of these deformations on the turbulence will be a strong function of the anisotropy of the turbulence. The flow in this region is in all likelihood a rapidly changing one as can be verified by the calculation of quantities such as P_k/ε, Sk/ε, $T_I^{-1}k/\varepsilon$ or α_T. The Reynolds stresses in such a flow require treatment by RDT; on the other hand it may well be that the turbulence does not have enough time to alter the mean flow so that the effects of an inadequate turbulence model are mitigated. It should be kept in mind that the flow in the slat drastically alters the initial conditions on the more slowly evolving turbulence downstream, regions in which the turbulence is expected to play a more important role in the mean flow development.

Further downstream above the main element the effects of turbulent transport in the mixing of the wake flow from the slat region with the growing boundary layer on the main element are important.

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Squire (1989) provides a survey of these issues. This is also the case in the region above the third element and wherever the outer portions of shear layers (in which production is less important than transport) come together. In the reversed and separated flows off the rear element the turbulence intensity is very high and transport is expected to play an important role in their development. In these regions and also at high angles of attack near the separation streamline Reynolds stresses are typically of the opposite sign as predicted by eddy viscosity models.

9. Conclusions

Kolmogorov (1941) suggested the possibility and the circumstances under which a turbulent flow might behave in a universal fashion. The circumstances under which such an ideal turbulence might occur include high turbulent Reynolds number, isotropy, temporal and spatial stationarity. Such as turbulence characterized by $[k, \varepsilon, \nu]$, has been the archetypal idea underlying most turbulence model closures. Complex flows, the subject of this article, do not in general conform to this ideal. Real world effects cause substantial deviation from the universal behavior associated with this ideal. A general turbulent flow can be expected to deviate from the ideal when there is 1) no extended spectral gap necessary for the small scale statistical equilibrium, 2) a mean deformation, 3) nonstationarity, 4) anisotropy, 5) inhomogeneity, or 6) poorly correlated. Nonetheless the engineering utility and adequacy of such a viewpoint has been both very successful and also very misleading. This article has systematically outlined, in the incompressible aerodynamic context, measures of departure from the ideal. This is in an effort to make possible a more realistic appraisal and qualification of the different strategies for the computation of flows influenced by turbulence.

The single-point closure methods such as the two-equation or second-moment closures while seemingly complicated really simplify to a few very simple ideas that are easily described by a few simple parameters. The small number of parameters, all of which come from the measurements of $< u_i u_j >$, $< u_i u_j u_k >$, $U_{i,j}$, R_t, and ε is not to be understood as an indication of the simplicity of the subject as much as it is taken to be an indication of the simple minded (and potentially inadequate) metaphors from kinetic theory applied to the problem of turbulence. A shorter list of variables $< u_i u_j >$, $U_{i,j}$, and ε form the core of a relatively complete classification of a turbulent flow; the relevant parameters are shown in the accompanying table.

It is hoped that this article will be understood in the following ways:

1) A consolidation and quantification of a number of ideas and their associated parameters with which to understand and classify complex turbulent flows. It is hoped that this article will serve as a guide in compiling more physically meaningful data and a more consistent presentation of numerical and experimental data. If experimental data is compiled in the nondimensional fashion recommended physical insight into the nature of the turbulence is more readily apparent.

2) Related to these issues is the possibility of assessing the underlying assumptions of a particular turbulence model and thus to understand what sort of strategy will produce adequate or inadequate results in a specific flow.

3) It is hoped that this document for those not familiar with this field will help sort out relevant concepts and issues and thus be more able to evaluate the scientific merit and engineering adequacy

of a particular methodology for a particular turbulent flow. This is also an effort to avoid the inevitable disillusionment with the blind application of the current form of turbulent models to flow situations which violate the premises on which the closures are based. This seems to be particularly relevant at this time as more become discontent with the unpredictable performance of the simple turbulence models in more complex flows.

It seems appropriate to close with a paraphrase from Hunt (1992). Hunt (1992) has reflected that the fact that turbulence models work in situations in which they have no business working could be a subject of research all by itself.

Basic nondimensional parameters	Parameter
Production to dissipation	P_k/ε
Relative strain parameter	Sk/ε
Total strain parameter	$\alpha_{ij} = \int S_{ij} dt$
Extra strain parameters	stb
Imposed time scales	$S^{-1}(k/\varepsilon)D/Dt\ S$
Turbulence intensity	$< uu >^{1/2}/U$
Anisotropy of Reynolds stresses	b_{ij}, II, III
Turbulent Reynolds number	$Re_t = \frac{4}{9}k^2/\nu\varepsilon$
Temporal bandwidth	$(\varepsilon/\nu)^{1/2}/S$
Anisotropy of the dissipation	d_{ij}, II_d, III_d

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J. R. Ristorcelli

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13. ABSTRACT (Maximum 200 words)
A simple parameterization scheme for a complex turbulent flow using nondimensional parameters coming from the Reynolds stress equations is given. Definitions and brief descriptions of the physical significance of several nondimensional parameters that are used to characterize turbulence from the viewpoint of single-point turbulence closures are given. These nondimensional parameters reflect measures of 1) the spectral band width of the turbulence, 2) deviations from the ideal Kolmogorov behavior, 3) the relative magnitude, orientation, and temporal duration of the deformation to which the turbulence is subjected, 4) one and two-point measures of the large and small scale anisotropy of the turbulence and 5) inhomogeneity. This is an attempt to create a more systematic methodology for the diagnosis and classification of turbulent flows as well as in the development, validation and application of turbulence model strategies. The parameters serve also to indicate the adequacy of various assumptions made in single-point turbulence models and in suggesting the appropriate turbulence strategy for a particular complex flow. The compilation will be of interest to experimentalists and to those involved in either computing turbulent flows or whose interests lies in verifying the adequacy of the phenomenological beliefs used in turbulence closures.

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