

HYBRID GRID TECHNIQUES FOR PROPULSION APPLICATIONS

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ABSTRACT

During the past decade computational simulation of fluid flow for propulsion applications has progressed significantly, and many notable successes have been reported in the literature. However, the generation of a high quality mesh for such problems has often been reported as a pacing item. Hence, much effort has been expended to speed this portion of the simulation process. Several approaches have evolved for grid generation. Two of the most common are structured multi-block, and unstructured based procedures. Structured grids tend to be computationally efficient, and high aspect ratio cells necessary for efficiently resolving viscous layers. Structured multi-block grids may or may not exhibit grid line continuity across the block interface. This relaxation of the continuity constraint at the interface is intended to ease the grid generation process, which is still time consuming. Flow solvers supporting non-contiguous interfaces require specialized interpolation procedures which may not ensure conservation at the interface. Unstructured or generalized indexing data structures offer greater flexibility, but require explicit connectivity information and are not easy to generate for three-dimensional configurations. In addition unstructured mesh based schemes tend to be less efficient and it is difficult to resolve viscous layers. Recently, hybrid or generalized element solution and grid generation techniques have been developed with the objective of combining the attractive features of both structured and unstructured techniques. In the present work recently developed procedures for hybrid grid generation and flow simulation are critically evaluated, and compared to existing structured and unstructured procedures in terms of accuracy and computational requirements.

In the present grid generation procedure multi-body configurations are decomposed into a number of simple geometric entities. A structured grid generator is first employed to construct a high quality grid around the body with appropriate packing. One grid must be designated as a main grid and enclose the solid surfaces of all other component grids. Upon completion these structured grids are converted to the hybrid grid data structure format. Based upon an input normal distance from the surface, holes are cut in the main grid for each component grid. Overlapping and hole cells are deleted from the hybrid grid data structure. Delaunay triangulation is then used to construct cells to fill the void between the cut main grid and the truncated component grid. Upon completion of this procedure the hybrid grid is written in a format useable by the flow solver.

The non-dimensionalized Euler equations in integral form provide the mathematical formulation for this scheme. The discretized flow domain is represented by a set of non overlapping polygons and the cell averaged variables are stored at each cell center. Each individual cell is treated as its own control volume. The numerical flux at the cell edge is calculated using Roe's approximate Riemann solver. An assumed linear distribution in each cell is employed to reconstruct the edge values, which results in a second order discretization. The flux limiting procedure of Barth is used to suppress spurious oscillations near discontinuities. An implicit pseudo-time integration procedure using the Generalized Minimum RESidual (GMRES) method for solving the sparse matrix system is employed. The results has been varified with the standard benchmark results.

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Hybrid Grid Techniques For Propulsion Applications

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**ENGINEERING TO
RESEARCH CENTER**

**COMPUTATIONAL
FIELD SIMULATION**

COMPLEX GEOMETRY / COMPLEX PHYSICS





OUTLINE

- MOTIVATION
- GRID GENERATION APPROACH
- DATA STRUCTURE
- FLOW SOLVER
- RESULTS
- CONCLUSIONS



MOTIVATION

UNSTRUCTURED GRIDS

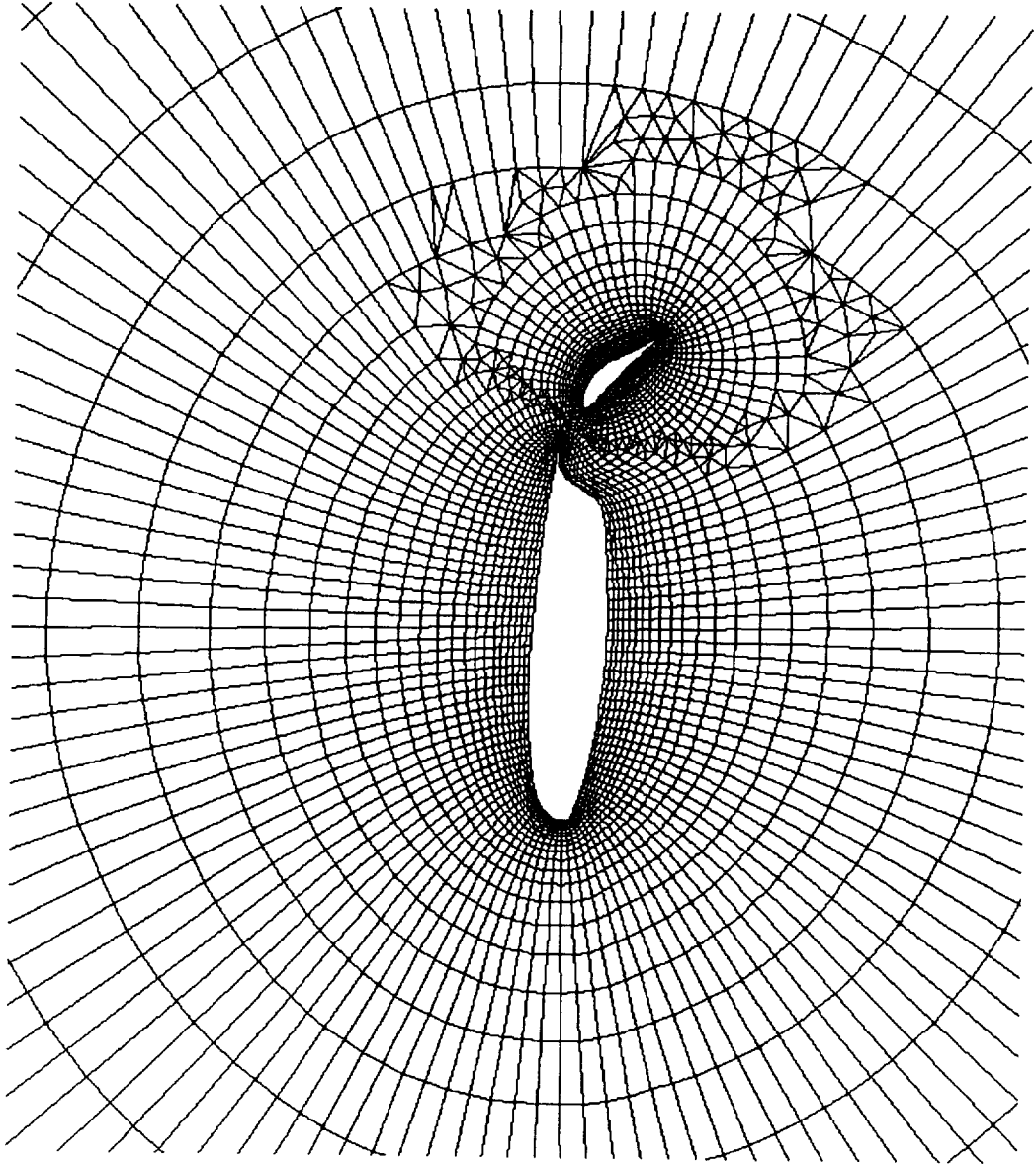
- GREATER FLEXIBILITY IN HANDLING COMPLEX CONFIGURATIONS
- EASE OF GRID ADAPTION
- DIFFICULT TO MAKE HIGHLY STRETCHED VISCOUS GRIDS
- DIFFICULT TO RESOLVE CONVECTIVE AND VISCOUS FLUXES FOR HIGH REYNOLDS NUMBERS
- TURBULENCE MODELLING IS DIFFICULT

HYBRID GRIDS

- COMBINING ADVANTAGES OF STRUCTURED AND UNSTRUCTURED GRIDS
- GRID GENERATION TIME CAN BE REDUCED



HYBRID GRID





APPROACH

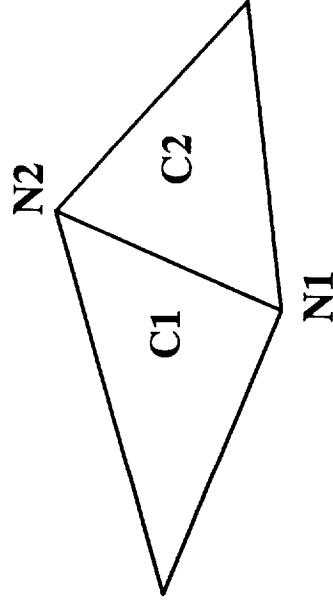
GRID GENERATION

- DECOMPOSE COMPLEX BODIES INTO SIMPLE ENTITIES
- GENERATE STRUCTURED GRIDS FOR THESE GEOMETRIC ENTITIES USING STANDARD PACKAGES
- CUT HOLES IN THE MAIN GRID WHERE THE COMPONENTS OVERLAP
- CONNECT COMPONENT GRIDS USING UNSTRUCTURED GRIDS BY DELAUNAY TRIANGULATION OR OTHER METHODS
- AVOIDED INTERPOLATIONS OF CONSERVED VARIABLES BETWEEN THE COMPONENT AND MAIN GRIDS AS IN CAMERA GRIDS



DATA STRUCTURE

- EDGE BASED DATA STRUCTURE



- EDGE(K,1) = FIRST NODE (N1)
- EDGE(K,2) = SECOND NODE (N2)
- EDGE(K,3) = CELL ON LEFT (C1)
- EDGE(K,4) = CELL ON RIGHT (C2) or BOUNDARY CONDITION
- ADVANTAGE : ANY ARBITRARY POLYGONS CAN BE HANDLED



CELL AREA

$$Area = \oint_{\partial\Omega} x \, dy = \sum_{edges} x_e \, dy$$

Loop over the edge

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N1 = EDGE ( I, 1 )
N2 = EDGE ( I, 2 )
C1 = EDGE ( I, 3 )
C2 = EDGE ( I, 4 )
dy = Y ( N2 ) - Y ( N1 )
XE = ( X ( N2 ) + X ( N1 ) ) * 0.5
AREA(C1) = AREA(C1) + XE * dy
AREA(C2) = AREA(C2) - XE * dy

```

endloop



GOVERNING EQUATIONS

$$\frac{\partial}{\partial t} \oint_{\Omega} Q \, dA + \oint_{\partial\Omega} \underline{F}(Q) \cdot \underline{n} \, ds = 0$$

Where

$$\underline{F} = f \, i + g \, j \quad \underline{n} = n_x \, i + n_y \, j$$

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix} \quad f = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(E + p) \end{bmatrix} \quad g = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 \\ v(E + p) \end{bmatrix}$$

$$p = (\gamma - 1) \left[E - \rho \left[\frac{u^2 + v^2}{2} \right] \right]$$

Non Dimensionalization w.r.t freestream conditions



FINITE VOLUME DISCRETIZATION

$$A_i \frac{\partial Q_i}{\partial t} = - \oint_{\partial\Omega} \underline{F}(Q_i) \cdot \underline{n} \, ds = - \sum_{j=1}^k \underline{F}_{ij} \cdot \underline{n}_j \, ds_j$$

Where k Number of sides of the polygon

i Cell number

j Edge number

SUMMATION OF FLUXES

Loop Over the edges

CALCULATE F_{ij}

FLUX (C1) = FLUX(C1) + F_{ij}

FLUX (C2) = FLUX(C2) - F_{ij}

Endloop

APPROXIMATE RIEMANN SOLVER (ROE)



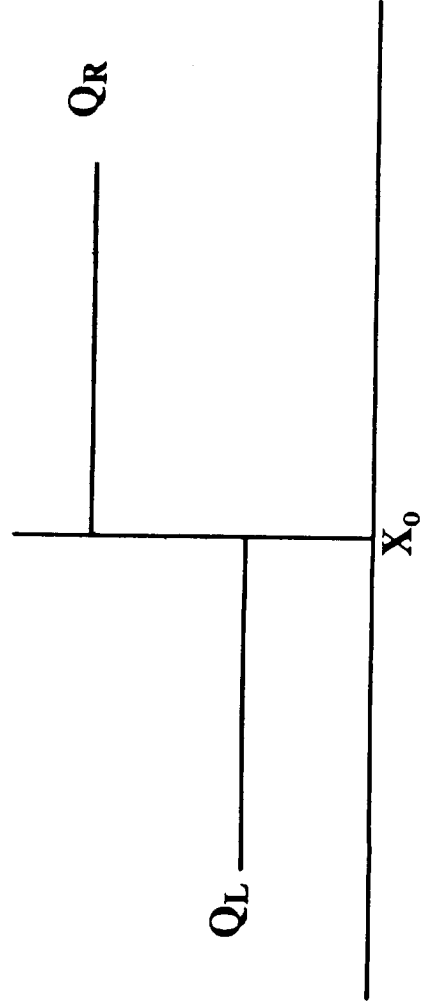
$$F_{ij} = \frac{1}{2} [F(Q_R) + F(Q_L) - | \bar{A} | (Q_R - Q_L)]$$

where

$$| \bar{A} | (Q_R - Q_L) = | \Delta F_{1,2} | + | \Delta F_3 | + | \Delta F_4 |$$

Corresponding to 3 different eigen values

$$\lambda_{1,2} = u \quad \text{and} \quad \lambda_{3,4} = u \pm c$$





**EXPLICIT SCHEME
(RUNGE-KUTTA TIME INTEGRATION)**

$$Q^{(0)} = Q^{(N)}$$

$$Q^{(1)} = Q^{(0)} + \alpha_1 \frac{\Delta t}{A} R(Q^{(0)})$$

$$Q^{(2)} = Q^{(0)} + \alpha_2 \frac{\Delta t}{A} R(Q^{(1)})$$

$$Q^{(3)} = Q^{(0)} + \alpha_3 \frac{\Delta t}{A} R(Q^{(2)})$$

$$Q^{(3)} = Q^{(0)} + \alpha_4 \frac{\Delta t}{A} R(Q^{(3)})$$

$$Q^{(N+1)} = Q^{(4)}$$

$$\alpha_1 = 0.0833 \quad \alpha_2 = 0.2069 \quad \alpha_3 = 0.4265 \quad \alpha_4 = 1.0$$

Where $R = - \sum_{edges} F_{ij} \cdot \underline{n} \, ds$



IMPLICIT SCHEME

$$F_{ij}^{n+1} = \frac{1}{2} (F(Q_R^{n+1}) + F(Q_L^{n+1})) - |\bar{A}| \cdot (Q_R^{n+1} - Q_L^{n+1})$$

LINEARIZATION

$$F_{ij}^{n+1} = F_{ij}^n + \left[\frac{\partial F_{ij}}{\partial Q_R} \right]^n \Delta Q_R + \left[\frac{\partial F_{ij}}{\partial Q_L} \right]^n \Delta Q_L$$

APPROXIMATE ANALYTIC JACOBIANS

NUMERICAL JACOBIANS



APPROXIMATE ANALYTIC JACOBIANS

$$F_{ij}^{n+1} = \frac{1}{2} (F(Q_R^{n+1}) + F(Q_L^{n+1})) - | \bar{A} | \cdot (Q_R^{n+1} - Q_L^{n+1})$$

$$D_i \Delta Q_i + \sum_{j=1}^k (U_{n(t)} \Delta Q_{n(t)}) = R^n$$

$$\text{Where } D_i = \frac{V_i}{\Delta t} I + \frac{1}{2} \sum_{j=1}^k (H_i + | \bar{H} |_{\frac{i+n(t)}{2}})$$

$$U_{n(t)} = \frac{1}{2} (H_{n(t)} - | \bar{H} |_{\frac{i+n(t)}{2}})$$

$$A N_x + B N_y = H, \quad | \bar{A} | N_x + | \bar{B} | N_y = | \bar{H} |$$

$$A = \frac{\partial f}{\partial Q}, \quad \& \quad B = \frac{\partial g}{\partial Q}$$



NUMERICAL FLUX JACOBIAN

$$A_{ij}(Q) = \frac{F_i(Q + h e_j) - F_i(Q)}{h}$$

Where e_j , j^{th} unit vector

$h = (\text{machine zero})^{1/2}$

$$j^{\text{th}} \text{ Column of } F'(Q) = \frac{F(Q + h e_j) - F(Q)}{h}$$



HIGHER ORDER SCHEME

$$Q(x, y) = Q(x_i, y_i) + \nabla Q(x_i, y_i) \cdot \Delta \bar{r}$$

Where

$$\Delta \bar{r} = (x - x_i) \bar{i} + (y - y_i) \bar{j}$$

Using Green's Theorem

$$\nabla(Q) = \frac{1}{V_i} \oint_{\partial\Omega} Q \bar{n} ds$$

$$Q_{n_i} = \frac{\sum_{j=1}^{nk} \frac{Q_{c_j}}{r_j}}{\sum_{j=1}^{nk} \frac{1}{r_j}}$$



LIMITER (Barth)

$$Q(x, y) = Q(x_i, y_i) + \phi_i \nabla Q(x_i, y_i) \cdot \Delta \underline{r}$$

$$Q_i^{\min} = \min (Q_{ci}, Q_{adj})$$

$$Q_i^{\max} = \max (Q_{ci}, Q_{adj})$$

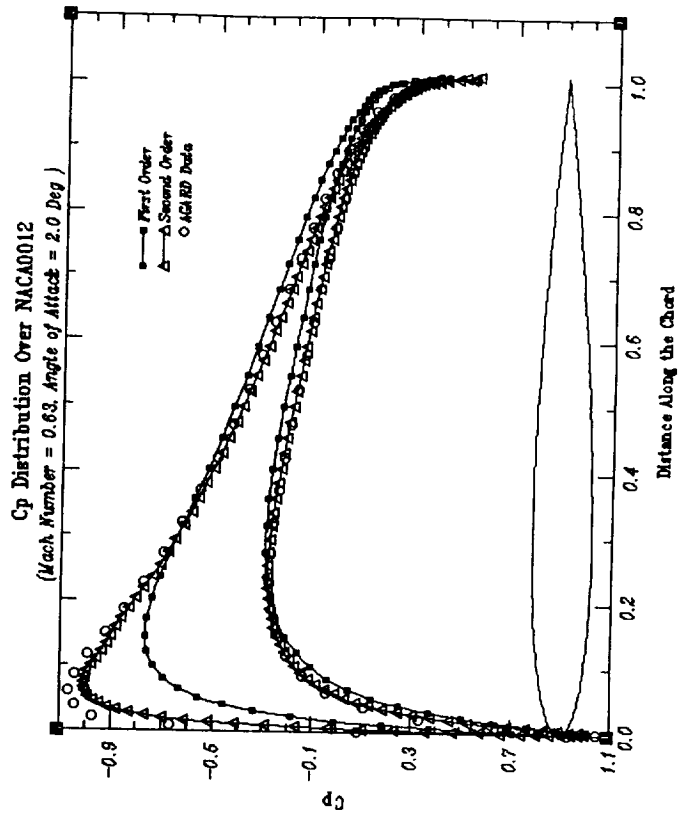
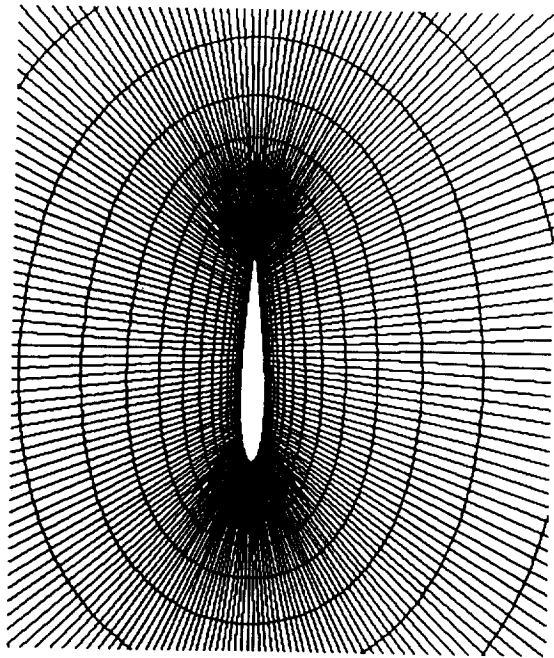
then $Q_i^{\min} \leq Q(x, y) \leq Q_i^{\max}$

$$\phi_{ij} = \begin{cases} \min \left[1, \frac{Q_i^{\max} - Q_{ij}}{Q_i - Q_{ij}} \right] & \text{if } Q_i - Q_{ij} > 0 \\ \min \left[1, \frac{Q_i^{\min} - Q_{ij}}{Q_i - Q_{ij}} \right] & \text{if } Q_i - Q_{ij} < 0 \\ 1 & \text{if } Q_i - Q_{ij} = 0 \end{cases}$$

and $\phi_i = \min(\phi_{ij's})$

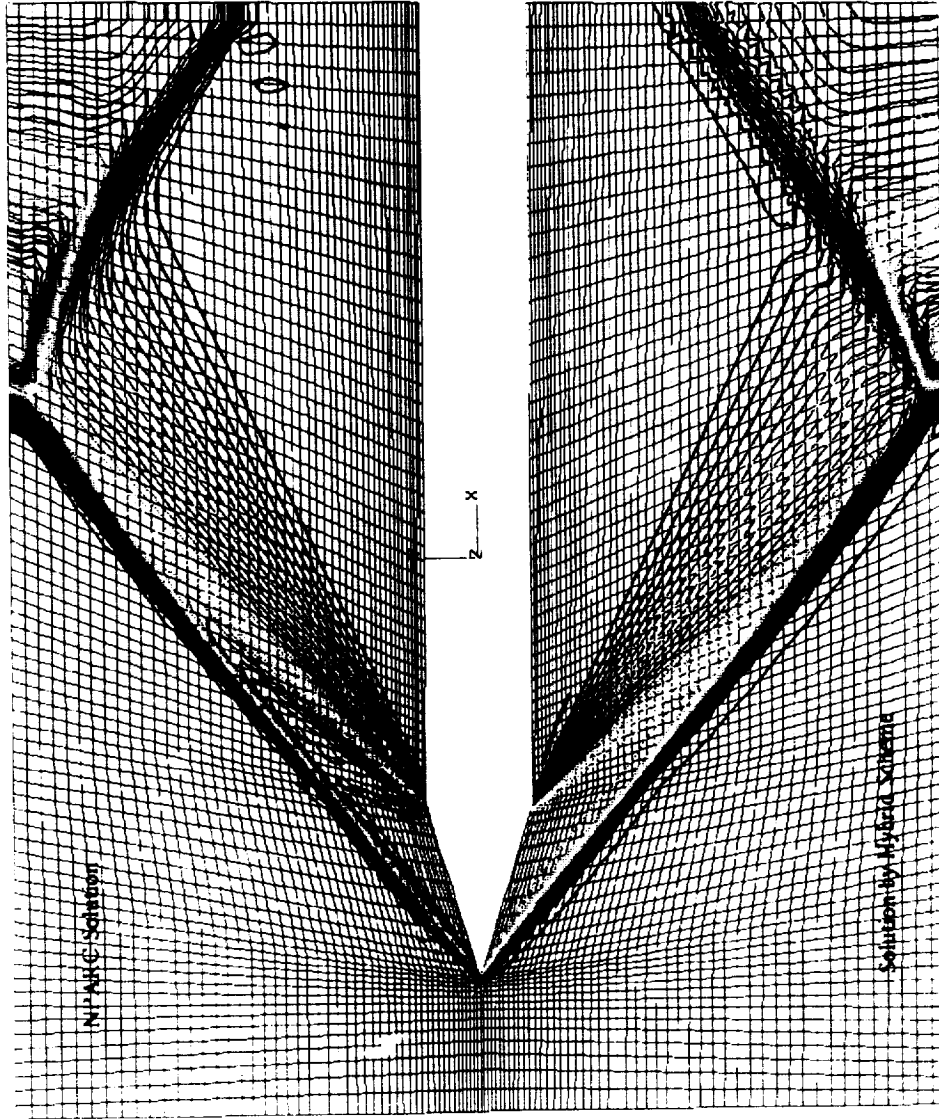


VARIIFICATION OF THE RESULTS



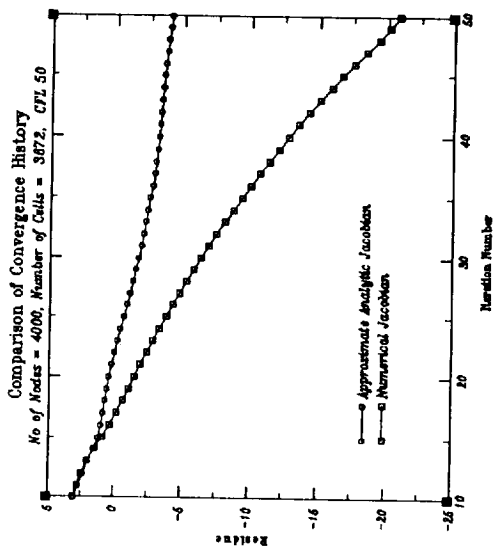
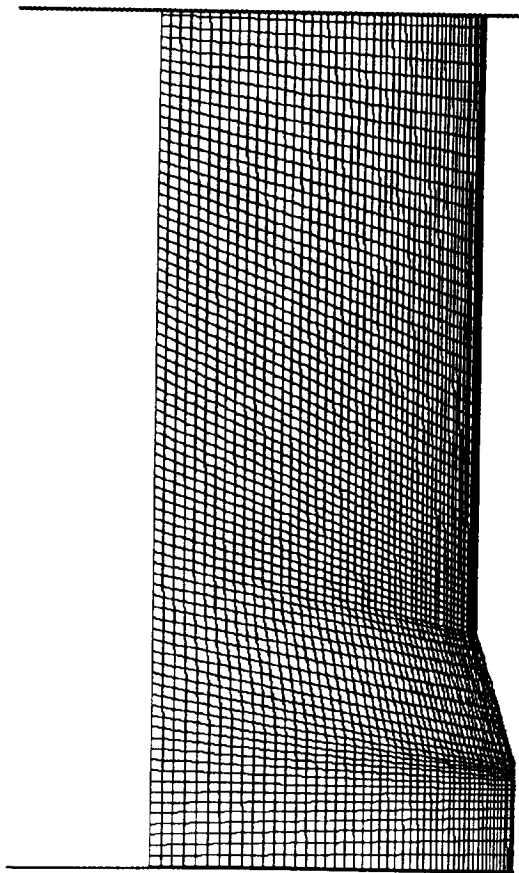
Comparison with NPARC Solution

Pressure Contours (Mach No - 2.5)



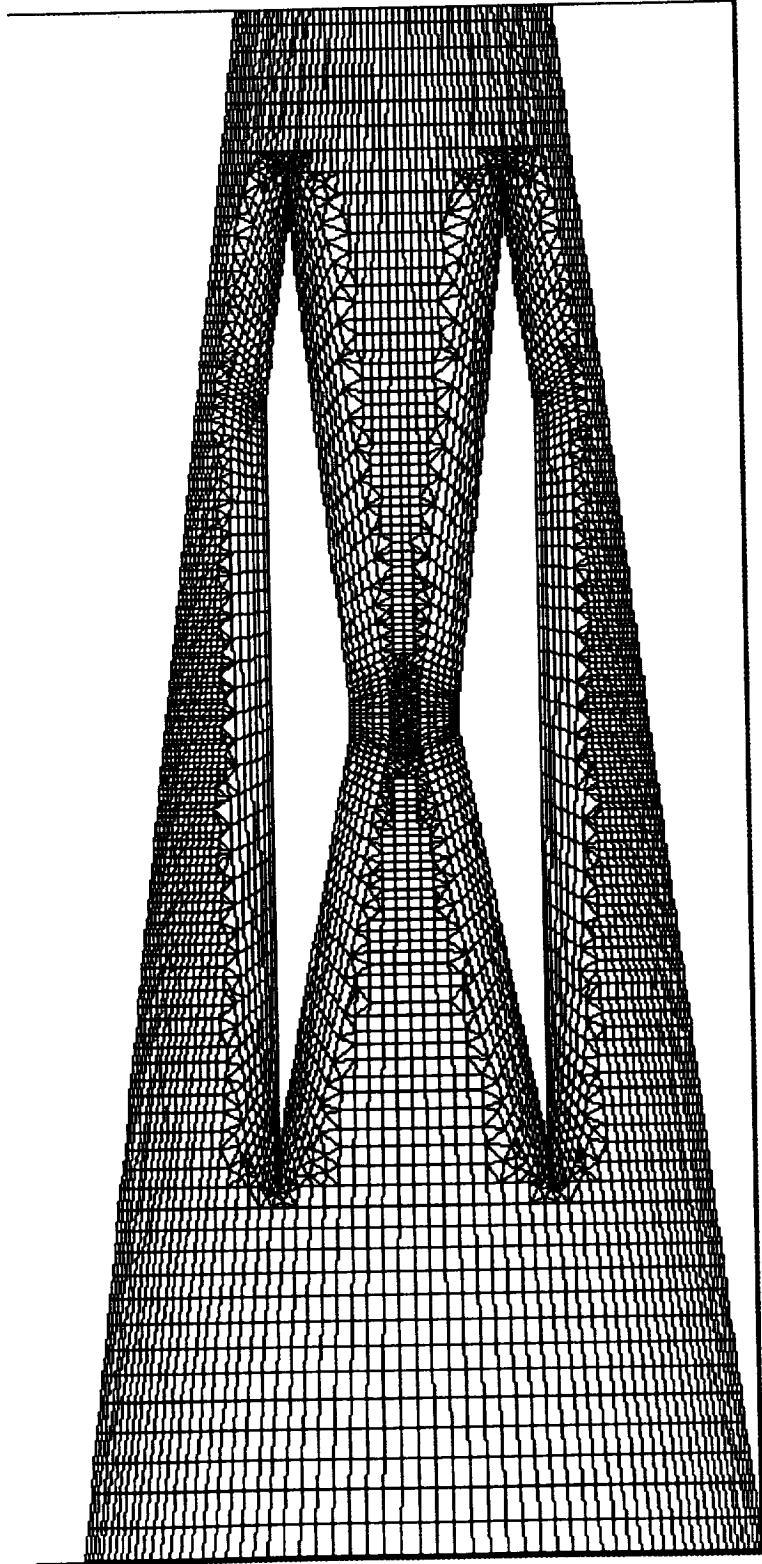


COMPARISON OF CONVERGENCE

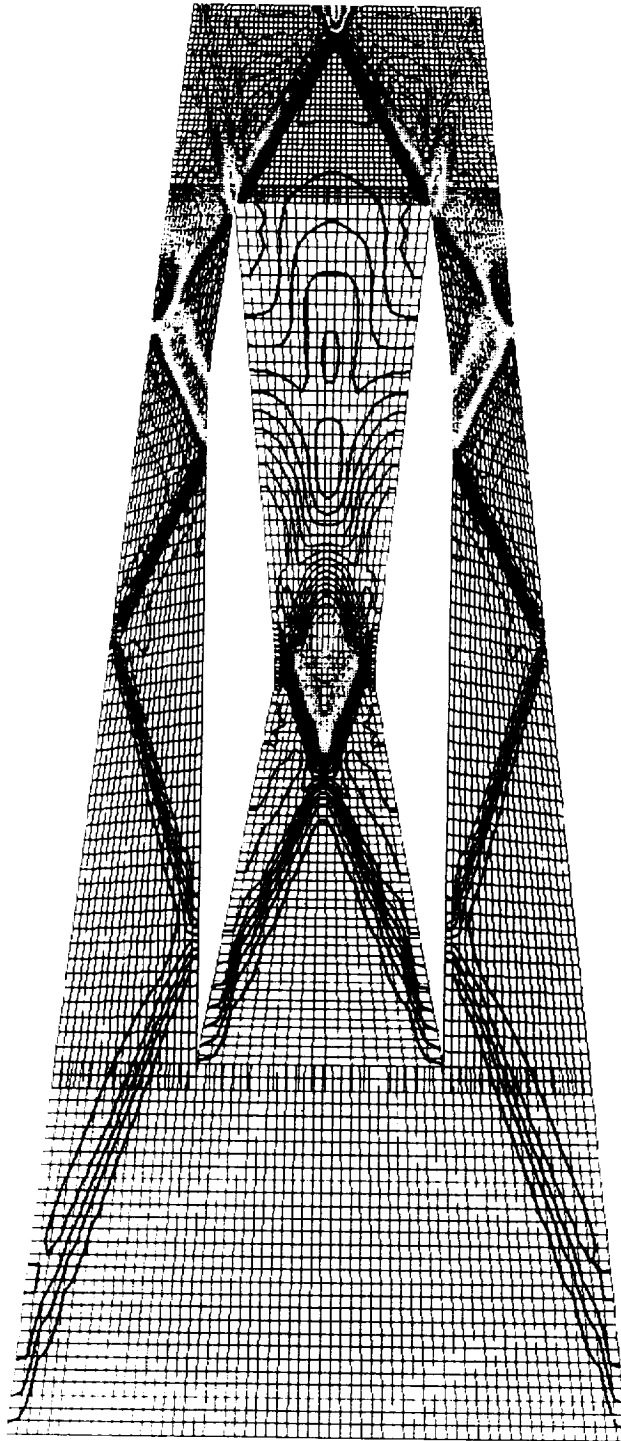




SCRAMJET INLET GEOMETRY



Scramjet Inlet Like Geometry (NPARC)



Pressure (M=2.5)



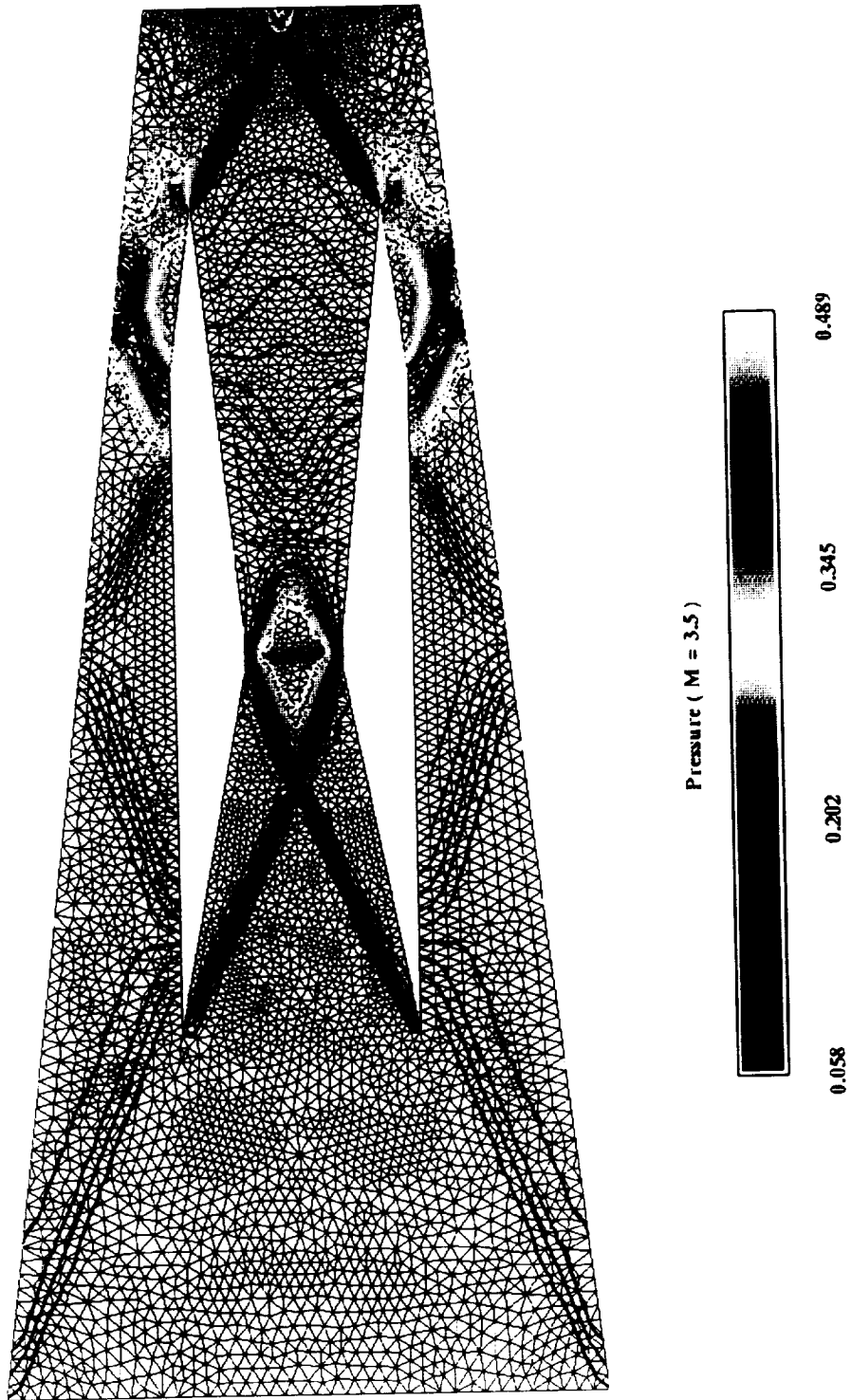
0.654

2.734

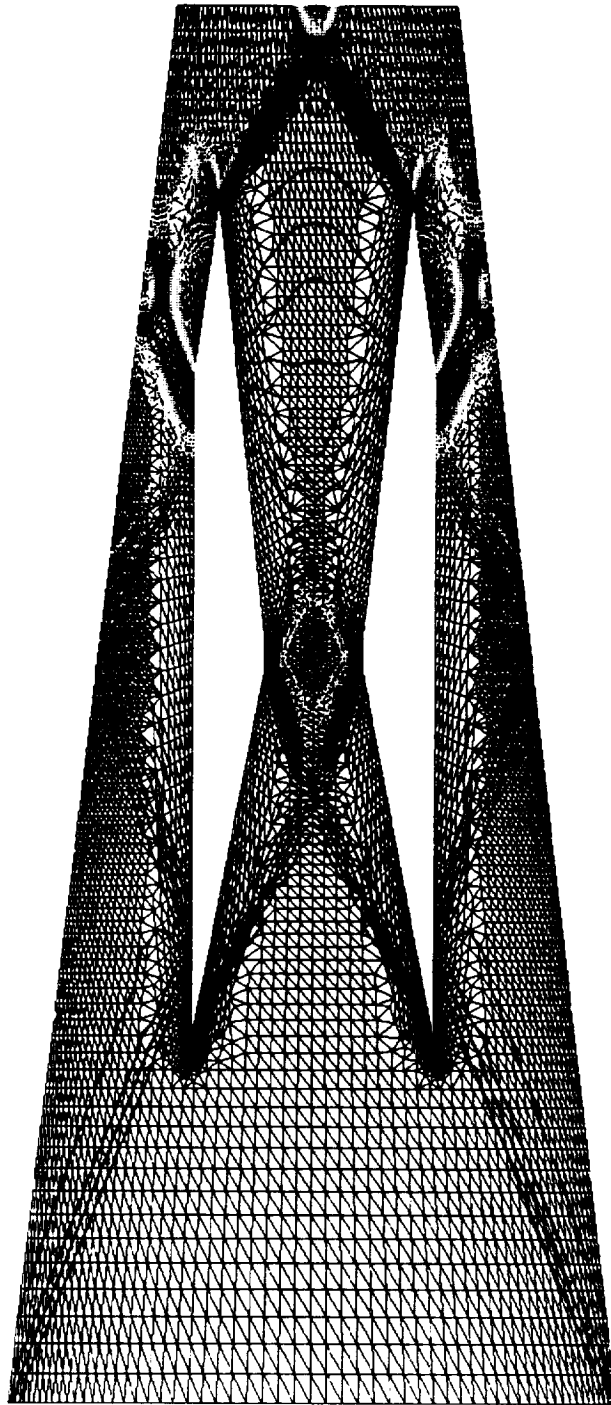
4.815

6.895

Pressure Distribution in Scramjet Inlet Like Geometry



Scramjet Inlet Like Geometry (HYBRID)

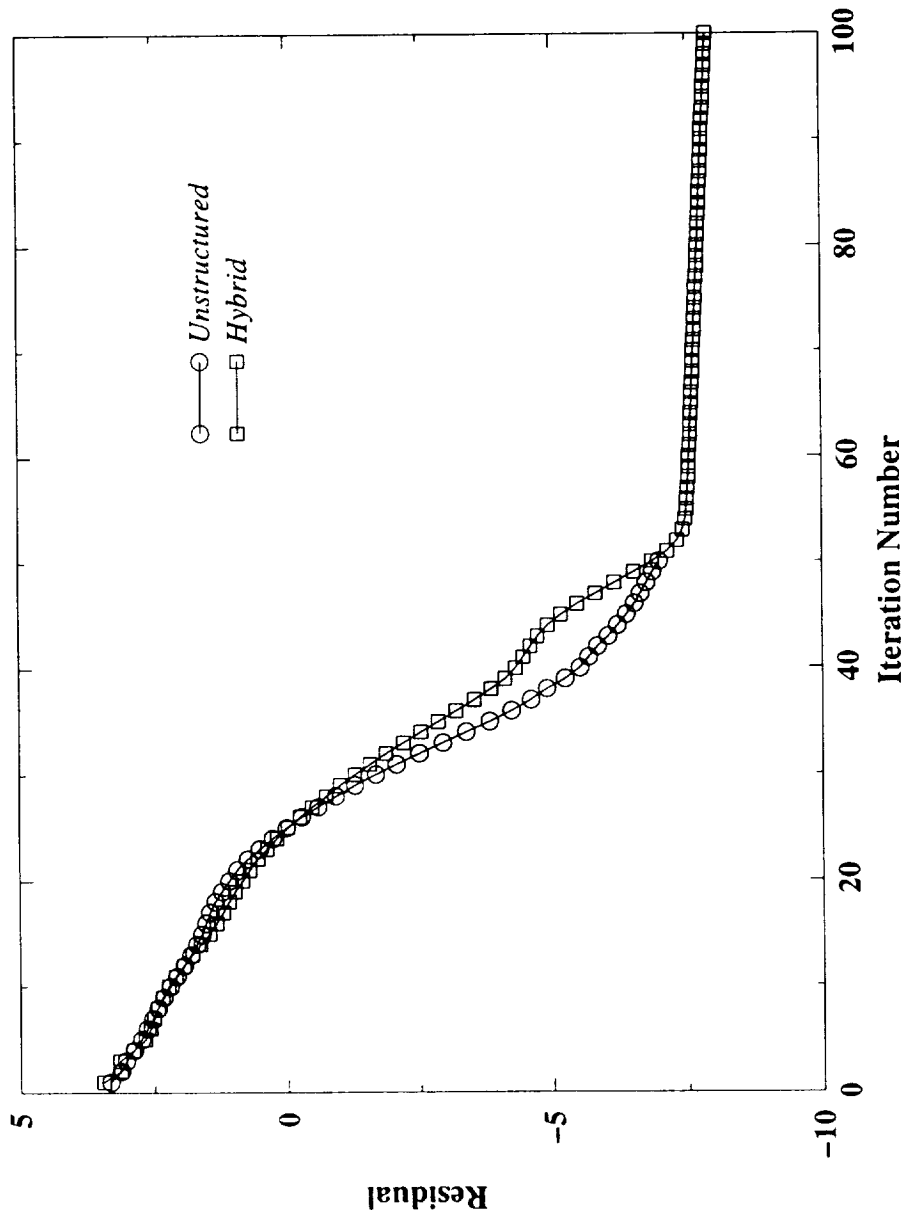


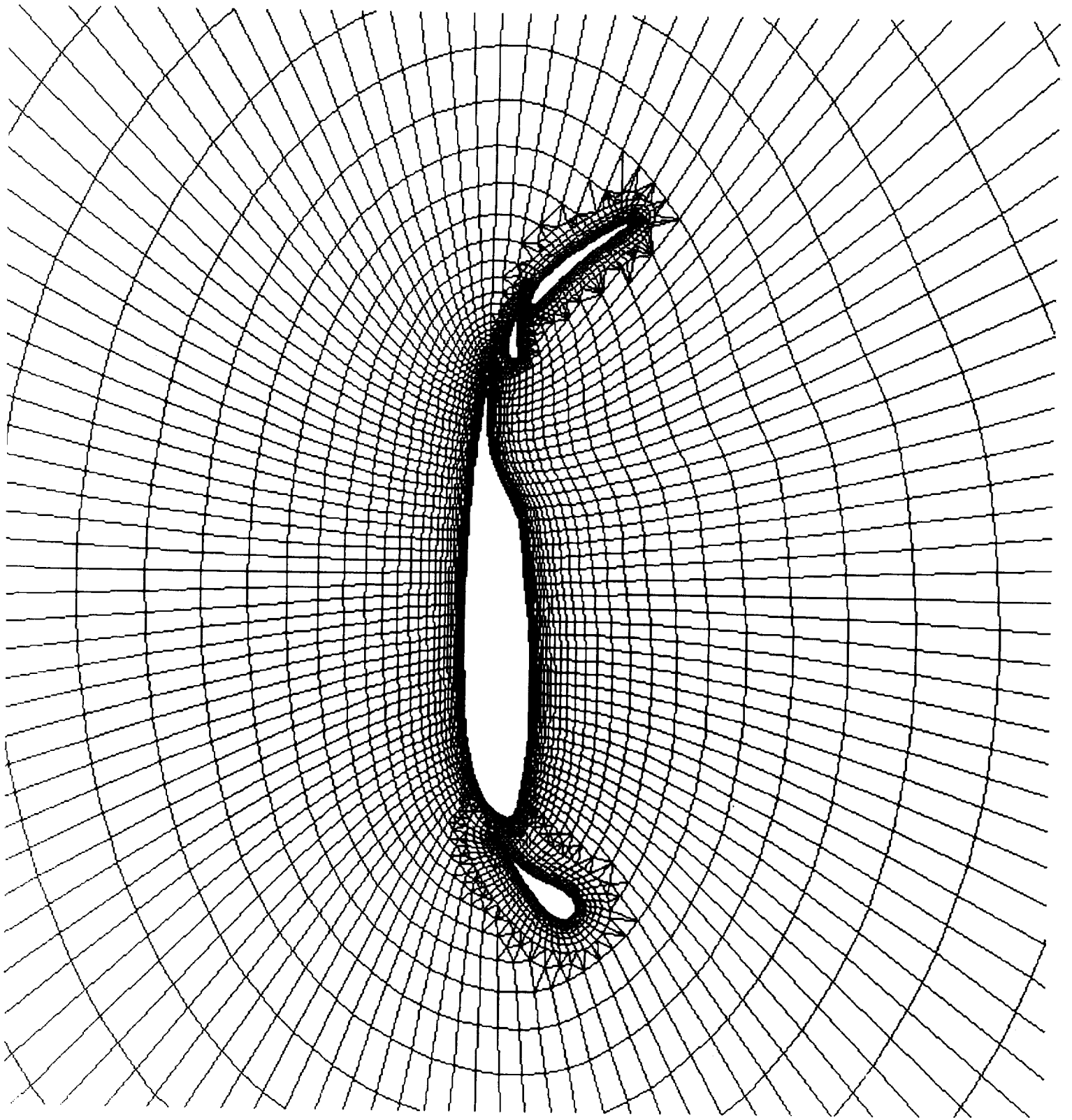
Pressure (M = 2.5)

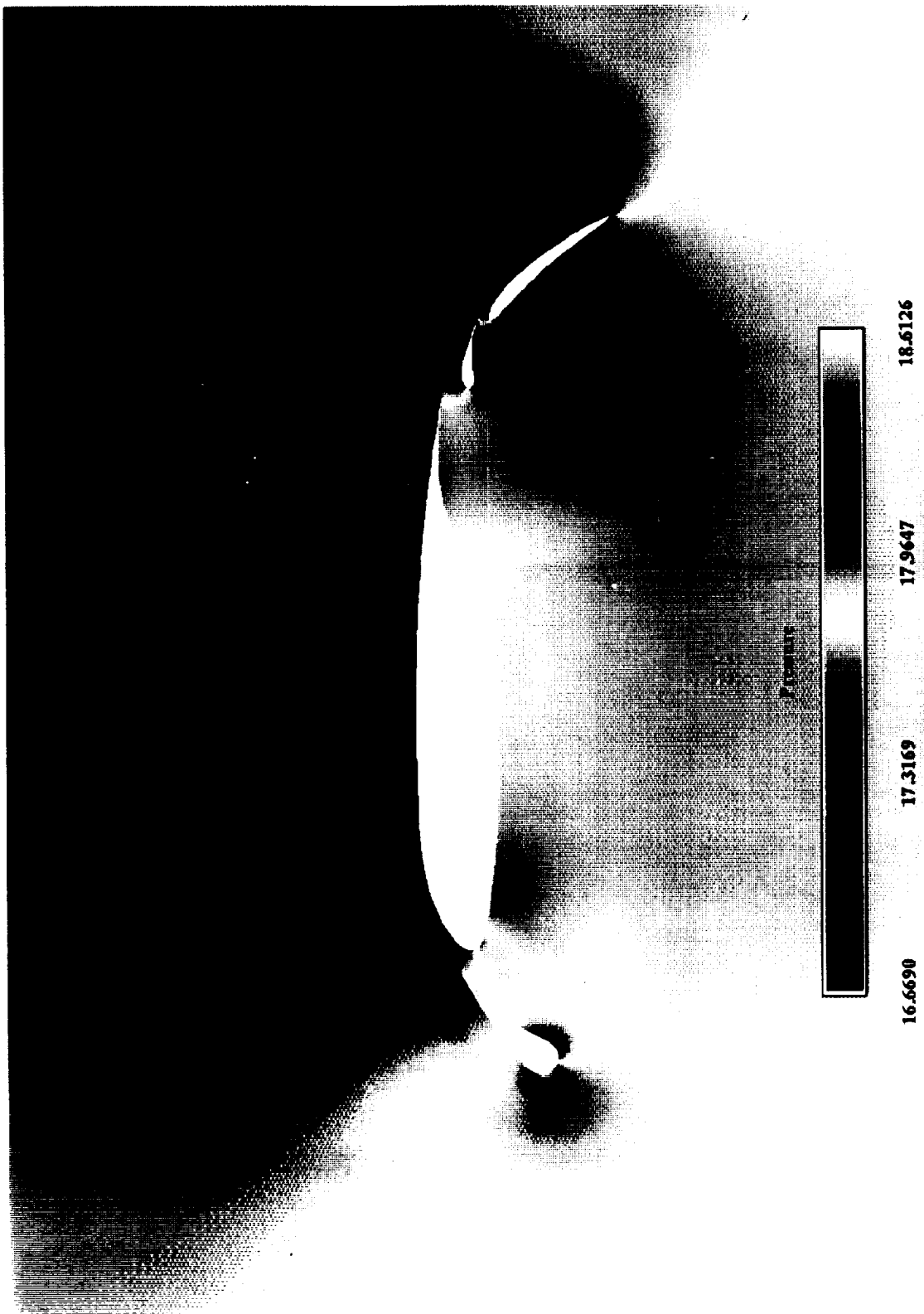


Convergence History

$CFL = 25$







Mach Number 0.201, Alpha = 0.0, CFL = 1.5



CONCLUSION

- DEVELOPED AND VARIFIED 2-D HYBRID FLOW SIMULATION SYSTEM.
- NUMRICAL FLUX JACOBIANS GIVES BETTER CONVERGENCE
- TREATED STRUCTURED AND UNSTRUCTURED GRIDS IN HYBRID SYSTEM AS A SINGLE BLOCK

FUTURE WORK

- DEVELOP A 3-D NAVIER-STOKES FLOW SIMULATION SYSTEM USING HYBRID GRIDS
- TEST THE SYSTEM FOR INTERNAL AND EXTERNAL FLOW PROBLEMS

