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A Simplified Pattern Match Algorithm for Star Identification

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Abstract

A true pattern matching star algorithm similar in concept to the Van Bezooijen[1] algorithm is implemented using an iterative approach. This approach allows for a more compact and simple implementation which can be easily adapted to be either an all-sky, no a priori algorithm or a follow on to a direct match algorithm to distinguish between ambiguous matches. Some simple analysis is shown to indicate the likelihood of mis-identifications. The performance of the algorithm for the all-sky, no a priori situation is detailed assuming the SKYMAP star catalog describes the true sky. The impact of errors and omissions in the SKYMAP catalog on performance are investigated. In addition, differing levels of noise in the star observations are assumed and results shown. The implications for possible implementation on-board spacecraft are discussed.

I. Introduction

The simplest approach to star identification is the "direct match" method. In this method, a star is considered to be identified if only one reference star is within a given angle of the observed star (transformed to an inertial frame by use of the estimated spacecraft attitude) and within a pre-determined tolerance of the observed light intensity.

With the advent of multi-observation star sensors, the possibility now exists to replace the direct match method of identifying stars onboard spacecraft with a pattern recognition system. However, existing pattern matching algorithms are not designed in a way which would allow maximum use of data from any onboard coarse attitude sensors. The concept of the algorithm described in this paper is to use a pattern match approach to distinguish the true match from a set of potential matches for each observed star. This allows a direct match approach to be used to create the initial set of potential matches for each observation. In this way, the attitude determined from the coarse sensors, along with an estimate of the coarse attitude accuracy, can be used to determine a small set of potential matches for input to the pattern match algorithm. The algorithm is then more efficient than a traditional pattern match for ground systems and can be considered for onboard systems as the memory requirements are greatly reduced. For ease of reference, this algorithm will be known as the "hybrid" star identification method.

Other analysis presented here attempts to provide some practical guidelines in the use of pattern match algorithms. Various parameters such as observation noise and the number of stars identified have an influence on the likelihood of mis-identification of observed stars. Analysis is presented to determine the probability of incorrect identification for the simplest pattern, the 3 star case, and to show the influence due to pattern geometry.

Finally, the hybrid star identification algorithm will be applied to the "all-sky" case, where no attitude information is available. This is not a realistic case for most spacecraft which should have at least a knowledge of the Sun direction, and is especially not a reasonable approach if both the Sun and the magnetic field vector for the Earth are available (as for any low-earth orbiting spacecraft). However, although the hybrid algorithm does not give any efficiencies over other pattern matching algorithms in the all-sky case, the results for several situations (3, 4, and 5 stars observed with differing noise levels) illustrate the likelihood of mis-identification.

II. The Algorithm

The hybrid algorithm first uses a direct match algorithm which matches the stars in the reference catalog to the observations, choosing all stars within the (user input) angular and intensity tolerances as potential candidates for identification. These candidates are then input to the pattern matching portion of the algorithm. An estimate of the current attitude is needed for transforming the observations to the reference frame of the star catalog. This estimate can be derived from coarse sensors or based on previous star measurements propagated using gyro measurements.

The pattern matching part of the hybrid algorithm uses a pairwise matching approach similar to the Bezooijen approach. For each potential match for a given observation, the number of reference pairs which meet the matching tolerance are totaled, but with the restriction that only credit for one i-j pair will be counted for the ith star observation even if several potential candidates for the jth star meet the matching criteria. Clearly, counting several matches from a given i-j pair would be an error and, in this way, the maximum number of matches for a candidate for the ith star will be limited to N-1, where N is the total number of star observations.

After passing through all the pair combinations for the N observations, all candidates with fewer than a preset number of matches are removed from consideration and another pass through the remaining candidates is performed. The minimum number of matches for reliable star identification depends on the number of reference stars observed and the noise in the observations. This issue will be addressed in Section III.

The result is an iterative method, which was chosen for several reasons. The algorithm is simplified in comparison to methods which keep track of more information and can operate in one iteration (References 2 and 3). Less code is required for the iterative algorithm and, given reasonable initial attitude knowledge (within several degrees), should not require excessive processing. For some current missions (e. g. SWAS), memory capability onboard is more of a driver than availability of processing power, leading to the desirability of simplicity.

The iteration ceases when no more candidates are being removed, at which point the candidate with the greatest number of matches is chosen as the identified reference star. If there is a tie for a given star observation, several courses of action can be taken. For this paper, the star was determined to not be identified. Ties are generally due to close neighboring stars, and can be eliminated by implementing "nearest neighbor" restrictions on the reference star catalog.

If the algorithm is being used for single frame identification (all the star observations are taken at the same time), the number of matches should be equal to the number of identified stars minus 1. In the all-sky simulation discussed in Section IV, this is assumed to be the case. A match is considered to have been accomplished only if the final set of identified stars meet this criteria. Onboard star identification processes are usually single frame.

Note that a "mirror image test" can be used to eliminate possible mismatches where the reference stars generate a reflection or mirror-image of the observed stars. This reduces the probability of mis-identifying stars by half. If the expected noise of the star sensor observations is low (on the order of 10 arcsecs) and a reasonable number of reference stars is observed in the field of view of the sensor, this probability is so low that the mirror test is not necessary. For the all-sky results included in this paper, extreme cases were investigated where the probability of mis-identification was significant and the mirror image test was implemented.

III. Probability of Mis-identification

The 2 and 3 star pattern cases will be considered. The positions of the reference stars are not randomly distributed, but in order to develop an estimate of pattern match mis-identifications, it will be assumed that the reference stars are evenly and randomly distributed in the sky.

Let the angular separation between two observed stars be ***R*** (radians) and let the maximum angular error given by the sensor noise be ***E*** (per axis of the star sensor, radians). The needed tolerance on pairwise matching to include all errors due to sensor noise is given by $T = 2\sqrt{2} \bullet E$. With the assumption that the reference stars are randomly and evenly distributed over the celestial sphere, the expected number of random matches to two observations separated by an angle ***R*** is given by

$$\frac{2 \bullet \pi \bullet \sin(R) \bullet [2 \bullet T] \bullet NSTARS}{4 \bullet \pi} \bullet NSTARS \qquad (1)$$

where NSTARS is the number of reference stars visible to the sensor. The SKYMAP stars brighter than the predicted instrumental magnitude of 5.5 (for the Ball CT-601 CCD) were used as a test case, giving a total of 7306 stars in the test catalog. The estimated number of matches for a pairwise matching tolerance of 4 arcsecs versus the actual number seen in the catalog are tabulated below:

R (degrees)	Predicted # Matches	Observed # Matches
1	18.1	22
4	72.3	102
8	144.5	144
11.31	204.4	216

Table 1: Predicted Vs. Observed Matches for Star Pairs

Reasonable agreement between the theoretical and actual results is seen given the simplifying assumptions. The goal is to reach an order of magnitude estimate of the reliability of a pattern match algorithm. Note that the above table gives the number of stars matching the given separation ***R*** over the entire sky. If there is some a priori attitude information, the number of matches is reduced by a factor equal to the actual fraction of the sky which is searched for potential reference star matches.

Emboldened by the success of this simplistic approach, now consider a 3 star pattern. Let the 2 stars with the larger separation provide the base for the 3 star pattern (which will be a triangle unless the stars are co-linear). Assume that we have two stars which meet the pairwise matching tolerance for the base stars (with an angular separation of approximately **R**). Then the conditional probability of a mis-identification (given that the base stars have already been mis-identified) is the probability of a reference star existing near the expected location of the 3rd observed star given the error tolerance **T** on the pair matching algorithm. This area is depicted below (using plane geometry as an approximation to the spherical case). The shaded area in Figures 1 and 2 is intended to represent the intersection of two error bands, where the center of each error band is one of the two base stars.

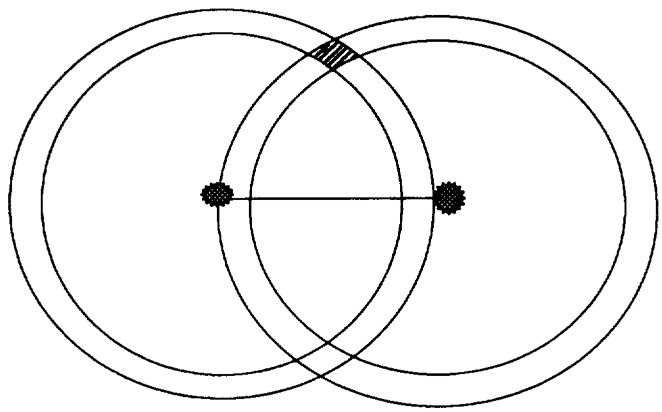

Figure 1. Third Star Area to Pass Pairwise Tolerance Test: Good Geometry Case

The probability of mis-identification of the 3rd star depends on the geometry of the 3 observed stars. If the 3 stars are non-linear, the expected number of reference stars which will meet the pairwise match constraints approaches (for the best geometry cases)

$$\frac{[2 \bullet T]^2}{4 \bullet \pi} \bullet NSTARS \qquad (2)$$

However, if the stars are co-linear, the area where stars will pass the pairwise matching test increases dramatically. This is illustrated in the figure below:

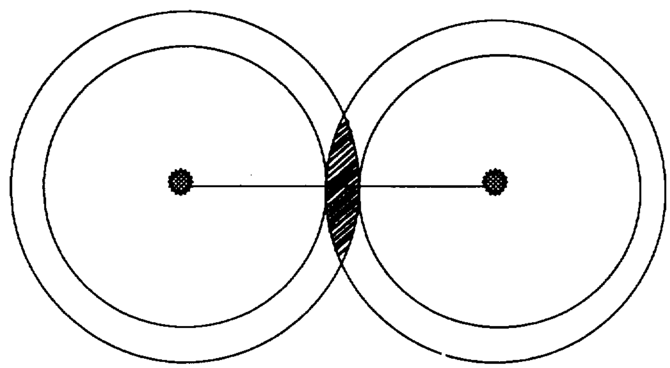

Figure 2. Third Star Area to Pass Pairwise Tolerance Test: Bad Geometry Case

This results in a significant increase in the number of mis-identified stars. For the worst case geometry (the third star co-linear and equidistant from the 2 base stars), the expected number of reference stars which will meet the pairwise match constraints is approximated by the following expression,

$$2 \bullet T \bullet \frac{\sqrt{R \bullet T}}{4 \bullet \pi} \bullet NSTARS \tag{3}$$

where, as before, ***R*** is the separation of the 2 base stars. to ensure that the probability of 3 star pattern mis-identification is kept small, the worst case geometry must be considered when computing the expected number of mis-identified stars.

It is important to note that each additional star which is matched to a reference star via the hybrid match algorithm will decrease the probability of mis-identification by a factor dependent on the pairwise matching tolerance **T** (to a 3/2 power, for the worst case geometry). The less noise in the star observations, the smaller the pairwise matching tolerance **T**. For the 5.5 magnitude test catalog used for Table 1, let the pairwise matching tolerance be 4 arcsec and assume 4 degrees for the 2 base star separation. The expected number of mis-identified stars, over the entire sky, is .003 using equations (1) and (3). This is a worst case number, showing that the identification of 3 reference stars from our test star catalog for a low noise sensor will be over 99.7% successful. Our result assumed a poor geometry situation and no a priori attitude information - thus grossly overestimating the probability of mis-identification. However, if a 4th star were matched, using equation (3) for a conservative estimate of the expected number of stars matching the 4th observation gives about 10^{-7} - about a 99.99999% success rate. As will be seen in the all-sky results, high levels of noise in the sensor observations can be countered if more stars are available for identification.

IV. All-sky Results

In order to test the hybrid match algorithm under extreme conditions, no a priori attitude information is assumed. In this case the hybrid algorithm is no different in concept from a standard pattern match technique. All stars in the reference catalog are taken as candidate matches for each observation.

The reference star catalog for this simulation is determined by choosing all stars with instrumental magnitudes of 5.0 or less from Version 3 of the SKYMAP catalog - a total of 4322 stars. Note that the tracker sensitivity can be selected, making this choice of catalog reasonable. The impact of observing non-catalog stars is discussed later. No other magnitude criteria is used to help identify stars, thus providing a greater challenge to the hybrid identification algorithm. Data from XTE has shown observed magnitude differences greater than 1.0 relative to the predicted magnitudes, so relying on magnitude criteria for all-sky matching can be ill-advised. Test cases are generated by evenly distributing tracker pointing attitudes about the celestial sphere and using those cases which contained the required number of reference stars.

As discussed in the previous section, given a star sensor's characteristics (observation noise and field of view size), rough estimates of the pattern match reliability in star identification can be made. The tolerance T for pairwise matching must be at least $2\sqrt{2}$ times the sensor noise (per axis) in order to accept all valid pairs. For the simulation, reference vectors from the SKYMAP catalog have random noise added to each component of the pointing direction, with the noise limit given by the "noise" value in column 1 of Table 2.

Using equations (1) and (3) with R = 4 degrees, leads to the following estimates for the expected number of mis-identified star patterns for the shown values of **T**.

Noise/T (degrees)	3-Star Patterns	4-Star Patterns	5-Star Patterns
0.00/0.001	0.0003	$4*10^{-9}$	$5*10^{-14}$
0.005/0.015	0.26	.0002	$2*10^{-7}$
0.05/0.15	83	2	.05

Table 2: Expected Number of Mis-identified Patterns

Using the pattern match algorithm with no a priori attitude information, the following results were obtained. There were 114 3-star cases, 104 4-star cases, and 86 5-star cases. The frequency of mis-identified patterns in the simulation is reported as a fraction in the table below. Cases with expected number of mis-identifications greater than 1 are not simulated as each case would likely be mis-identified. If the expected number of mis-identified patterns is small, the probability of mis-identification is approximately equal to the expected number of mis-identified patterns. Thus, the expected number of mis-identified patterns should be a rough estimate of the fraction of mis-identified patterns seen (up to values on the order of a few tenths).

Noise/T (degrees)	3-Star Patterns	4-Star Patterns	5-Star Patterns
0.00/0.001	0.00	0.00	0.00
0.005/0.015	0.09	0.00	0.00
0.05/0.15	-	-	0.02

Table 3: Fraction of Mis-identified Patterns

In the statistics compiles above, cases where star patterns were correctly identified but a reference star was ambiguous due to the existence of multiple reference stars within the tolerance ***T*** of the correct reference star were deemed successful - enough stars were identified to allow computation of an accurate attitude. The algorithm is not required to distinguish between multiple reference stars closer than the assumed sensor noise. Reasonable correspondence of the simulated results with the estimates of reliability is seen. The simulated results show better reliability (lower frequencies of mis-identification) than the estimates and this is expected as the estimated reliability assumed a worst case geometry for the star pattern.

The algorithm should behave well even if the sensor provides spurious observations or tracks non-catalog stars. As a test of this, the medium noise level (0.05 degrees) 5-star case was rerun using a catalog which was missing one of the reference stars for each 5-star pattern. In all cases, the remaining stars were identified correctly, duplicating the expected performance of the 4-star, medium noise case. As for all pattern match algorithms, the algorithm successfully eliminates observations which lack catalog stars and will still identify the reference stars which are available.

V. Implementation for Ground Systems

The implementation of star identification processing in a ground attitude determination system can lead to some additional problems. In ground systems, data over a long time might be accumulated using gyro data. The gyro data is used to form "clumps" of observations (which are assumed to represent observations from a single star) and to propagate these groups to a common time before transforming all the observation groups to the reference inertial frame. Then, the quality of the gyro data becomes the biggest factor in determining the parameters of the star identification procedure. The hybrid algorithm has been implemented in a test version of a ground system and spacecraft data from XTE has been processed. During a large angle slew, the number of distinct stars seen by XTE's 2 star trackers can number in the hundreds. To avoid the computing loads of testing all the pair combinations which increases geometrically with the number of stars observed, it was useful to feed the star observations in to the pattern match portion of the hybrid algorithm in smaller chunks (about 10 stars at a time). This has provided quick and accurate response. As a practical consideration, if the gyro propagation introduces a significant degree of error, it is important to increase the minimum number of star matches needed for star identification as the pairwise noise tolerance factor is increased.

For XTE, the matching tolerance **T** can be set to about 10 arcsecs while the spacecraft is inertial (and a minimum of 3 stars should be tracked for high reliability). When spacecraft maneuvers occur, the clumping errors force the tolerance **T** to be increased. As an example, using uncalibrated gyros on XTE to propagate observations (with approximately an 1 degree per hour uncompensated bias), the matching tolerance **T** needed to be increased to 125 arcsec to allow identification of all reference stars. The minimum allowable number of matches was concurrently increased to 5, thus boosting the reliability of the star identification algorithm back to a high level. A feature of the hybrid algorithm is to provide adaptability to both high and low noise observations.

VI. Conclusions

The hybrid algorithm is robust with regard to inaccuracy in the a priori attitude and provides adaptability to extremes in gyro propagation errors and tracker noise. These features make it attractive for implementation in ground systems.

For onboard systems, current star sensors have the capability to track multiple stars simultaneously. Three or more stars are available over most of the sky (97% of the random attitudes used in the all-sky simulation had at least 3 stars brighter than instrumental magnitude 5.0 within 4 degrees of the sensor boresight). Missions such as XTE and SWAS are still using the direct match method. This method was implemented for spacecraft using trackers which could only track one star at a time and does not fully take advantage of the multi-star tracking.

The direct match technique leads to tight restrictions on spacecraft attitude determination accuracy over maneuvers, where the spacecraft typically is using gyro rate information only. An example of this is XTE, which must be within 200 arcsec of the target attitude after a maneuver in order for the onboard star identification to perform. Also, the spacecraft operators must ensure that the observed stars in the field of view after the maneuver have no other stars close enough to cause confusion - a "nearest neighbor" restriction. Depending on the expected accuracy of the spacecraft gyros, the nearest neighbor restriction can impose complex requirements on the spacecraft operators (SWAS is a good example of this, Reference 4). If a more sophisticated star identification algorithm were to be used onboard the spacecraft, these restrictions would be greatly eased. The hybrid algorithm is put forward as an example of an "add-on" to current onboard attitude determination software which would provide the robustness of pattern matching with only a modest increase in resource usage.

References

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