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Variations on the Davenport Gyroscope Calibration Algorithm

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ABSTRACT

This paper presents a number of variations on the Davenport algorithm for in-flight gyroscope recalibration, or first-order initial calibration, specifically tailored for use with a minimum amount of satellite telemetry data. Central to one of the techniques described is the use of onboard integration of gyroscope data together with a detailed model of scheduled satellite slew profiles. Methods are presented for determining adjustments to either parameters for the standard linear model (i.e., a drift rate bias vector and/or a scale factor / alignment transformation matrix) or individual gyroscope scale parameters, both linear and nonlinear, in cases where the alignments are well known. The results of applying the methods in an analysis of the temporal evolution and nonlinear response of the gyroscopes installed on the Hubble Space Telescope following its first servicing mission are discussed. The two effects, when working coherently, have been found to result in slew errors of almost 1 arcsecond per degree. Procedures for selecting optimal operational gyroscope parameters subject to the constraint of using a linear model are discussed.

Introduction

Reference 1 presents a derivation of the Davenport gyroscope calibration algorithm, which has been used for the in-flight calibration of gyroscopes for a number of spacecraft missions, including those of the High Energy Astrophysics Observatories and the Hubble Space Telescope (HST). As usually implemented, and, in particular, as implemented for the HST mission (Reference 2), the algorithm assumes that the user has available for use in the calibration process a continuous and complete set of gyroscope data extending from an initial to a final spacecraft attitude (as determined by independent reference sensors) for an adequately large number of maneuvers. Empirically, we find that this constraint causes gyroscope scale factor and alignment calibration to be one of the more labor- and data-intensive activities needed in support of mission operations. Fortunately, we also have found that the scale factor and alignment parameters for the gyroscopes used for the HST mission are fairly stable; calibration is usually required only following initial deployment of gyroscopes (i.e., following HST's initial deployment in April 1990, activation of reserve gyroscopes in response to gyroscope failures, and installation of new gyroscopes during the first HST servicing mission in December 1993).

Although the HST gyroscopes are "fairly stable," a performance analysis conducted in September 1995 (Reference 3) has indicated that in the 18 months following the first HST servicing mission, the gyroscope response has changed systematically, the errors being most manifest in negative yaw maneuvers wherein systematic errors of roughly 0.8 arcsecond per degree occur.

Given this recent experience with the HST gyroscopes, we have found it desirable to develop an algorithm that permits recalibration of the gyroscopes, at least to first order in the change parameters, using a data set that is both much reduced in volume and readily available during normal mission operations. We also have found it useful to extend the algorithm to allow study of both isolated and nonlinear scale corrections. The algorithm that we present here requires as input from telemetry only the attitude error measurements determined by the onboard computer (OBC) pointing control subsystem following large vehicle maneuvers. All other required input can be obtained from the schedule of commanded maneuvers and the spacecraft parameters characterizing those maneuvers.

The body of this paper is divided into six sections, excluding this introduction. These include (1) background on the basics of the Davenport algorithm, (2) a reformulation taking advantage of OBC integration of gyroscope data and modeling of planned maneuver profiles, (3) some comments on the calibration of gyro bias, (4) an extension to both isolated and nonlinear scale factor corrections, (5) a discussion of selection of measurement weights to be used in the algorithms, and (6) an application of the algorithm to data accumulated for the gyroscopes used for the HST mission.

The Davenport gyroscope calibration algorithm, as well as the variations of it discussed in this paper, are envisioned as applied in a batch mode least-squares algorithm. Batch mode processing is strictly appropriate only if the time scale for collection of the calibration data is short compared with the time scale for any variation that may apply to the state vector parameters. Empirically, in the case of the gyroscopes used on HST, we have found the scale factor and alignment parameters sufficiently stable that a batch mode approach for their calibration is operationally viable. In cases where this fails to be true, reformulating the calibration equations presented here in terms of a Kalman filter (e.g., Reference 4) should be considered.

Section 1 - Background on the Basics of the Davenport Algorithm

Reference 1 presents the gyroscope calibration algorithm that is used in the HST mission for the calibration of scale factors, alignments, and biases of the gyroscopes when one or more gyroscopes are first activated for operational use. The basic equations are as follows. Consider a satellite gyro system consisting of N_G single-axis gyroscopes. In response to some angular motion of the satellite, the output response column matrix of gyro counts, $\mathbf{C}$, consists of the N_G individual gyro readings. The response vector is translated into a measured angular velocity, $\mathbf{\Omega}_M$, in the spacecraft frame via

$$\mathbf{\Omega}_M = \boldsymbol{G}_0 \mathbf{C} - \mathbf{D}_0 \tag{1}$$

where $\boldsymbol{G}_0$ is the $3\mathrm{x}N_G$ scale factor / alignment matrix, and $\mathbf{D}_0$ is the gyro system drift rate bias expressed in the spacecraft frame. The goal of the algorithm is to determine correction matrices $\boldsymbol{m}$ and $\mathbf{d}$ that may be applied to $\boldsymbol{G}_0$ and $\mathbf{D}_0$ so that a modified equation (1) will yield the true angular rate, $\mathbf{\Omega}$, as indicated in equations (2a) - (2c).

$$\boldsymbol{G} = (\boldsymbol{I}_3 + \boldsymbol{m})\, \boldsymbol{G}_0 \tag{2a}$$

$$\mathbf{D} = (\boldsymbol{I}_3 + \boldsymbol{m})\, \mathbf{D}_0 + \mathbf{d} \tag{2b}$$

$$\mathbf{\Omega} = \boldsymbol{G}\, \mathbf{C} - \mathbf{D} = (\boldsymbol{I}_3 + \boldsymbol{m})\, \mathbf{\Omega}_M - \mathbf{d} \tag{2c}$$

where $\boldsymbol{I}_3$ is the 3x3 identity matrix. Gyroscope miscalibration information is sampled through any given maneuver via the error quaternion

$$\delta\mathbf{Q} \equiv \mathbf{Q}_R \mathbf{Q}_G^{-1} \tag{3}$$

where $\mathbf{Q}_R$ represents the true vehicle rotation as determined from reference star measurements, and $\mathbf{Q}_G$ represents the vehicle rotation inferred from the gyroscope measurements. As discussed in Reference 1, $\delta\mathbf{Q}$ represents a rotation from the gyro-inferred to the true postmaneuver attitude, expressed in the premaneuver reference frame. The information content of $\delta\mathbf{Q}$ is related to $\boldsymbol{m}$ and $\mathbf{d}$ via the sensitivity equation

$$\mathbf{Z} = 1/2 \int \boldsymbol{T} (\mathbf{\Omega} - \mathbf{\Omega}_M)\, dt \tag{4a}$$

$$= 1/2 \int \boldsymbol{T} (\boldsymbol{m}\, \mathbf{\Omega}_M - \mathbf{d})\, dt \tag{4b}$$

where $\mathbf{Z}$ is the vector component of $\delta\mathbf{Q}$, $\boldsymbol{T}$ is the matrix that transforms vectors to premaneuver spacecraft coordinates, and the time integral is over the whole maneuver. Because equation (4b) is linear in $\boldsymbol{m}$ and $\mathbf{d}$, it can be used as the basis for a linear least-squares algorithm to provide estimates for $\boldsymbol{m}$ and $\mathbf{d}$. If a solution for all 12 correction terms is needed, at least 4 independent "maneuvers" are required to perform the calibration. The maneuvers must provide a reasonable sample of pitch, roll, and yaw variation, as well as an independent sample for bias determination; the latter is permitted to be a period of essentially constant attitude.

Although the information content is unchanged, it is often more convenient to reexpress $\mathbf{Z}$ in terms of an error vector, ζ, representing the rotation from the true postmaneuver attitude to the intended (and gyro-inferred, assuming closed-loop control) postmaneuver attitude, i.e., the rotation that the spacecraft must perform after it determines its postmaneuver error. The vector ζ is related to $\mathbf{Z}$ and $\{\boldsymbol{m}, \mathbf{d}\}$ via

$$\zeta = - \boldsymbol{T}_\tau^{-1} \mathbf{Z}$$

$$= - 1/2\, \boldsymbol{T}_\tau^{-1} \int \boldsymbol{T} (\boldsymbol{m}\, \mathbf{\Omega}_M - \mathbf{d})\, dt \tag{5}$$

where τ represents the maneuver duration time, and its use as a subscript on T_τ means that T is to be evaluated at the maneuver end-time. The matrix T_τ^{-1} (which equals T_τ^T) is thus the premaneuver to postmaneuver reference frame transformation matrix.

Section 2 - Use of OBC Gyro Data Integration and Model Maneuver Profiles

As discussed in Reference 1, equation (5) is accurate only to first order in m and $\mathbf{d}$, implying that the associated least-squares algorithm is intrinsically iterative. The matrix terms ζ, T, and Ω_M must be reevaluated on each iteration. Multiple iterations can only be applied if a complete set of gyroscope data from throughout each of the calibration maneuvers is available. In this section we discuss a procedure that excludes the possibility of multiple iterations, the gain being a drastic reduction in the total volume of data required to perform the calibration. This can be significant if either (1) the sheer volume of data for frequent, normal calibrations becomes unwieldy or (2) the standard telemetry format used does not contain an adequately dense sampling of gyro data for accurate integration.

If calibration needs are adequately met via a first order correction, it is possible to implement an algorithm with drastically lower data requirements. Integration of the full set of gyroscope data is required at two points in the use of equation (5): first in the determination of Q_G for the construction of ζ, and then in the time integral over $(Tm\Omega_M)$. Ground processing of gyro data to determine Q_G can be eliminated if the spacecraft OBC maintains and transmits an estimate of the spacecraft attitude based solely on gyroscope data, at least through the time period between the accumulation of star sensor data for pre- and postmaneuver definitive attitude estimation. Sampling the OBC's pre- and postmaneuver attitude estimates then allows construction of Q_G as the connecting eigenvector rotation between the two. Ground processing of the gyro data for use in the integral over $(Tm\Omega_M)$ can be eliminated if a sufficiently precise model of the maneuver profile is available. This follows because, to first order in the correction terms, equation (5) is unchanged if $m\Omega_M$ is replaced with $m\Omega_P$, Ω_P being the planned angular velocity as a function of time based on spacecraft design parameters. We make the latter substitution in what follows.

The simplifications noted in the preceding paragraph allow the elimination of all ground processing of the raw gyro data. The elimination of ground processing of the reference star data may also be possible, although this results in a smaller gain. For many satellites, the OBC generates an attitude error estimate based upon postmaneuver reference star measurements and uses this estimate to generate an error nulling maneuver. If the vehicle attitude is maintained accurately by the onboard pointing control system during the periods between maneuvers, the postmaneuver error nulling maneuver will correspond to the error vector ζ required for our analysis. If this error vector is included in telemetry, no other spacecraft data are required.

We assume finally that each maneuver is a pure eigenaxis maneuver. This allows the analysis to be done in a coordinate system, designated here with a prime ('), in which the x'-axis is aligned along the maneuver axis. Expressed in the primed frame, equation (5) becomes

$$R(2\zeta) = -T'^{T}_{\tau} \int T'(m'\Omega'_P - \mathbf{d}')\,dt \qquad (6a)$$

$$T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos[\theta(t)] & -\sin[\theta(t)] \\ 0 & \sin[\theta(t)] & \cos[\theta(t)] \end{bmatrix}$$

$$= R\,T\,R^{-1} = R\,T\,R^{T} \qquad (6b)$$

$$m' = R\,m\,R^{-1} = R\,m\,R^{T} \qquad (6c)$$

$$\mathbf{d}' = R\,\mathbf{d} \qquad (6d)$$

$$\Omega'_P = [\,\omega(t),\ 0,\ 0\,]^T = R\,\Omega_P \qquad (6e)$$

where R is the transformation matrix that converts premaneuver spacecraft coordinates to the premaneuver primed frame, $\theta(t)$ is the maneuver angle as a function of time, and $\omega = d\theta/dt$. The form of $\theta(t)$ will depend upon the total maneuver angle, Θ, and design parameters governing the execution of maneuvers. To first order, R may be based on the planned maneuver quaternion, Q_P. The eigenvector and rotation angle set, $\{\mathbf{r}, \varphi\}$, defining the quaternion representation of R is constructed from the spacecraft frame Q_P eigenvector, η, and the spacecraft frame standard unit vectors, $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$, using

$$\mathbf{r} = (\mathbf{x} \times \eta) / |\mathbf{x} \times \eta|$$

$$= (-\mathbf{y}\eta_3 + \mathbf{z}\eta_2) / (\eta_2^2 + \eta_3^2)^{1/2} \qquad (7a)$$

$$\varphi = \cos^{-1}(\mathbf{x} \cdot \eta) = \cos^{-1}(\eta_1) \qquad (7b)$$

The simple forms of equations (6b) and (6e) allow equation (6a) to be reexpressed as

$$\boldsymbol{R}\,(2\,\zeta) = -\,\boldsymbol{T'}_\tau^{T}\;[\,\boldsymbol{K}_1\,[\boldsymbol{m'}]_1\; - \;\boldsymbol{K}_0\,\mathbf{d'}\,] \qquad (8a)$$

$$\boldsymbol{K}_k \;\equiv \begin{bmatrix} K_k & 0 & 0 \\ 0 & K_{ck} & -K_{sk} \\ 0 & K_{sk} & K_{ck} \end{bmatrix} \qquad (8b)$$

$$K_k \;\equiv \int \omega^k\,dt \qquad (8c)$$

$$K_{ck} \;\equiv \int \cos(\theta)\,\omega^k\,dt \qquad (8d)$$

$$K_{sk} \;\equiv \int \sin(\theta)\,\omega^k\,dt \qquad (8e)$$

where $[\boldsymbol{m'}]_1$ indicates the column matrix formed from the first column of $\boldsymbol{m'}$. Note that the elements of $\boldsymbol{K}_1$ are analytic, i.e., $K_1 = \Theta$, $K_{c1} = \sin(\Theta)$, and $K_{s1} = [1-\cos(\Theta)]$, whereas K_0 is equal to the maneuver duration, τ. The functional form of $\theta(t)$ enters only via K_{c0} and K_{s0}.

The multiplication of $\boldsymbol{T'}_\tau^{T}$ into $\boldsymbol{K}_1$ and $\boldsymbol{K}_0$ in equation (8a), together with an application of the sine and cosine laws for two angle sums, produces

$$\boldsymbol{R}\,(2\,\zeta) = -\,[\,\boldsymbol{K^*}_1\,[\boldsymbol{m'}]_1\; - \;\boldsymbol{K^*}_0\,\mathbf{d'}\,] \qquad (9a)$$

$$\boldsymbol{K^*}_k \;\equiv \begin{bmatrix} K_k & 0 & 0 \\ 0 & K^*_{ck} & K^*_{sk} \\ 0 & -K^*_{sk} & K^*_{ck} \end{bmatrix} \qquad (9b)$$

$$K^*_{ck} \;\equiv \int \cos(\Theta-\theta)\,\omega^k\,dt \qquad (9c)$$

$$K^*_{sk} \;\equiv \int \sin(\Theta-\theta)\,\omega^k\,dt \qquad (9d)$$

Because $\boldsymbol{K}_1$ depends only on Θ and not the form of $\theta(t)$, it can be shown that $\boldsymbol{K^*}_1 = \boldsymbol{K}_1^{T}$. This relationship holds for $\boldsymbol{K^*}_0$ as well (actually, for all k) if $\omega(t)$ is an even function of time about the maneuver midpoint. This constraint, which is fairly standard for spacecraft maneuver profiles, also yields the following convenient equations for K_{c0} and K_{s0} (expressed for general k):

$$K_{ck} = \cos(\Theta/2)\,F_k(\Theta) \qquad (10a)$$

$$K_{sk} = \sin(\Theta/2)\,F_k(\Theta) \qquad (10b)$$

$$F_k(\Theta) \;\equiv \int \cos\,[\theta(t) - \Theta/2]\;\omega^k\,dt \qquad (10c)$$

We now need to transform equation (9a) back into the spacecraft frame so as to have ζ related to $\boldsymbol{m}$ and $\mathbf{d}$ rather than to $\boldsymbol{m'}$ and $\mathbf{d'}$. Defining $\mathbf{m}$ as the 9-by-1 column matrix $[[\boldsymbol{m}]_1^{T}, [\boldsymbol{m}]_2^{T}, [\boldsymbol{m}]_3^{T}]^{T}$ and using equations (6b) and (6c), we can rewrite equation (9a) as

$$2\,\zeta = -\,\boldsymbol{R}^{T}\boldsymbol{K^*}_1\,\boldsymbol{B}\,\mathbf{m} \; + \boldsymbol{R}^{T}\boldsymbol{K^*}_0\,\boldsymbol{R}\,\mathbf{d} \qquad (11a)$$

$$B_{i\;j+3(n-1)} \;\equiv R_{ij}\,R_{1n} \qquad (11b)$$

where equation (11b) defines the elements of the 3-by-9 matrix $\boldsymbol{B}$. Equation (11a) is our new least-squares algorithm sensitivity equation. Its use removes the need for an integration of the gyro telemetry data. The only required time integrations are for K^*_{c0} and K^*_{s0}, or, more simply, $F_0(\Theta)$ if the symmetry constraint on $\omega(t)$ is applied. Appexdix A presents a specific, fairly common maneuver profile usable in the latter evaluation.

Section 3 - Bias-Only Calibration Assuming Fixed Scale and Alignment

We consider now the application of the algorithm of Section 2 to a bias-only calibration. This begins with the constraining assumption $\boldsymbol{m} = \boldsymbol{0}$. This constraint is reasonable for many operational scenarios; empirically, it has been found that spacecraft gyro biases can change significantly within as little as a single day, whereas time scales for scale factor and alignment are considerably longer. For this situation, equation (11a) reduces to

$$2\,\zeta = \boldsymbol{R}^{T}\boldsymbol{K^*}_0\,\boldsymbol{R}\,\mathbf{d} \qquad (12)$$

Two data gathering scenarios are of possible interest for this calibration. For the first scenario, the spacecraft is held at constant, or nearly constant, attitude over the time period of interest. "Nearly constant" in this context means that the magnitude of any net maneuver angle must be smaller than the product $\delta d \Delta t$, where δd is the maximum permitted error in the estimate for $\mathbf{d}$, and Δt is the time period between two reference attitude measurements. For this case, $[\boldsymbol{R}^{T}\boldsymbol{K^*}_0\,\boldsymbol{R}]$ reduces to $\boldsymbol{I}_3\Delta t$, and equation (12) becomes

$$\mathbf{d} = 2\,\zeta\,/\,\Delta t \qquad (13)$$

We have used Δt rather than τ here because there is no scheduled or executed maneuver for which we can evaluate $\tau(\Theta)$. The vector ζ may be constructed from separate initial and final reference star measurements or from an OBC-determined attitude error at the end of the time period if the spacecraft applied an attitude correction at the start of the period. In the latter case, care must be taken to ensure that the onboard attitude propagation across the time period involved the use of gyroscope data only, i.e., no control-law feedback based on reference star data.

The second data gathering scenario uses the procedures outlined in Section 2 applied to equation (12). The potential operational advantage of using this approach arises if a set of dual mode gyroscopes, i.e., sensors with high-rate and low-rate modes, is being used -- the high-rate mode being used during large maneuvers to allow greater dynamic range, and the low-rate mode during periods of near-constant attitude to allow greater precision. For such gyroscopes, equation (13) can be used to calibrate the high-rate mode bias only if the gyroscopes are commanded to remain in high-rate mode during the calibration period, implying that dedicated spacecraft time would be required for the calibration. In contrast, the use of equation (12) would allow relatively frequent high-rate mode bias calibrations based on serendipitous maneuvers.

A caveat pertains here -- one of relevance to the next section. An estimate of the bias based on equation (12) applied to a single maneuver may fail to be good if the estimate for the scale factor / alignment matrix G is insufficiently accurate, because of either poor initial calibration or an actual change in the gyroscope parameters since the time of calibration. The effect of errors in estimates for linear scale factors would tend to cancel each other in the estimate for **d** if the least-squares fit is performed using an ensemble of randomly directed maneuvers or paired sets of oppositely directed maneuvers. Taking advantage of this fact to reduce the influence of possible scale factor errors may be desirable.

Section 4 - Isolated and Nonlinear Scale Factor Calibration

The original Davenport algorithm combines the observable aspects of alignment and scale factor changes into the single change matrix $\boldsymbol{m}$. It also assumes that gyroscope response is purely linear. We have found it useful to be able to study the indiviual gyroscope response curves, with respect to both nonlinear corrections as well as temporal variations of the dominant (i.e., linear) terms. In this section we discuss an extension of the algorithm presented in Section 2 designed for this purpose.

To keep the initial discussion simple, we will assume that the state vector for our problem is restricted to scale factor adjustments. Specifically, we assume that the gyro alignments are well known and fixed, and that the adjustment to the operational drift rate bias is restricted to that associated with scale factor corrections (i.e., the "$\boldsymbol{m}\mathbf{D}_0$" term in equation (2b)), with no intrinsic bias changes permitted (i.e., no **d** term in the state vector). We will relax both of these simplifications eventually. We will, however, not attempt to model ongoing temporal changes in the drift rate bias that occur during the time period over which the calibration data are accumulated. Given that bias changes occur relatively rapidly and are likely to be significant over the data accumulation time period, this last simplification may at first glance seem inappropriate. If, however, a good estimate of the changing bias vector associated with the operational alignment and scale calibration is maintained throughout the period of data accumulation (using the methods of Section 3), the effect of the bias will have been removed on an ongoing basis. From this perspective, we see that we are not actually neglecting the changing bias; rather, the bias effects have been precorrected as part of ongoing operations.

To allow for nonlinear scale effects, we assume a model for gyroscope responsivity of the form

$$C_n = S_n \left[\Omega_{m,n} + \sum s_{nk}\, g_k(\Omega_{m,n}) \right] \tag{14}$$

where subscript n indicates the nth gyroscope, C_n is the resultant gyro reading, S_n is the nominal (or current best estimated) gyro scale factor, $\Omega_{m,n}$ is the spacecraft angular rate projected onto the gyro input axis, and the summation over k represents a set of small corrections to the predominantly linear relationship between $\Omega_{m,n}$ and C_n. The parameters s_{nk} are correction coefficients applied to the functions $g_k(\Omega_{m,n})$. The latter can be any convenient set of functions, subject only to the constraint that the same set of functions be used for all of the gyroscopes. To minimize the eventual size of the least-squares state vector, the functions should be selected so that a good fit can be found with as few correction functions as possible. For our HST analysis, we have found it convenient to use two: $g_1(\Omega) = \Omega$ and $g_2(\Omega) = g_{11}(\Omega) \equiv |\Omega|$. In this model, s_{n1} represents an average linear correction, and s_{n2} represents the difference between scale factors for positive and negative maneuvers.

We assume next that an acceptably accurate inverse to equation (14) can be written in the form

$$\Omega_{in,n} = C_n/S_n + \Sigma\, \sigma_{nk}\, g_k(C_n/S_n) \qquad (15)$$

In principle, each σ_{nk} is a function of the full set $\{s_{n1}, s_{n2}, \ldots\}$. However, if the sum $\Sigma s_{nk} g_k(\Omega_{in,n})$ and all of its individual terms are small relative to $\Omega_{in,n}$, and if the correction functions g_k vary continuously, then $\sigma_{nk} = -s_{nk}$ to first order in the correction terms for all n and k values. We will be using this approximation in what follows.

For notational compactness, equation (14) can be rewritten as

$$\mathbf{C} = \boldsymbol{S}\,(\boldsymbol{A}\,\boldsymbol{\Omega} + \Sigma\, \boldsymbol{s}_k\, [^{g_k}\cdot(\boldsymbol{A}\boldsymbol{\Omega})]\,) \qquad (16)$$

where $\mathbf{C}$ is a N_G-by-1 column matrix (the C_n's), $\boldsymbol{S}$ and $\boldsymbol{s}_k$ are N_G-by-N_G diagonal matrices (the S_n's and s_{nk}'s), $\boldsymbol{A}$ is the N_G-by-3 matrix of gyro input axis unit direction vectors, and the symbol $[^{g_k}\cdot]$ is defined such that

$$[^{g_k}\cdot \mathbf{V}] \equiv [g_k(V_1), g_k(V_2), \ldots, g_k(V_N)]^T \qquad (17)$$

for any N-by-1 column matrix $\mathbf{V}$. We also need a matrix version of equation (15) that gives $\boldsymbol{\Omega}_M$ as a function of $\mathbf{C}$. If N_G exceeds 3, our equation must include a weighting scheme for how the gyro data are to be combined in forming $\boldsymbol{\Omega}_M$. We use the following equation:

$$\boldsymbol{\Omega}_M = [\boldsymbol{A}^T\boldsymbol{A}]^{-1}\boldsymbol{A}^T\,\boldsymbol{\Omega}_{in}$$

$$= \boldsymbol{\aleph}\,(\,\boldsymbol{S}^{-1}\mathbf{C} + \Sigma\, \boldsymbol{\sigma}_k\, [^{g_k}\cdot(\boldsymbol{S}^{-1}\mathbf{C})]\,) \qquad (18)$$

where $\boldsymbol{\Omega}_{in}$ is the N_G-by-1 matrix formed from the various $\Omega_{in,n}$ estimates, $\boldsymbol{\aleph} \equiv [\boldsymbol{A}^T\boldsymbol{A}]^{-1}\boldsymbol{A}^T$, and $\boldsymbol{\sigma}_k$ is an N_G-by-N_G diagonal matrix (the σ_{kn}'s). By using equation (18) as the mechanism for constructing $\boldsymbol{\Omega}_M$ from $\mathbf{C}$, we have selected a convention whereby equal weight is given to each of the components of $\boldsymbol{\Omega}_{in}$. This is a change from the more typical convention in constructing the matrix $\boldsymbol{G}_0$ for equation (1) whereby equal weight is given to each component of $\mathbf{C}$.

At this point we should clarify notation a bit in preparation for constructing the least-squares algorithm for a recalibration of the s_{nk} coefficients. Equation (16) should be viewed as applying the true s_{nk} values; it represents the actual response of the sensors. In contrast, equation (18) represents the users interpretation of the counts; thus the σ_{nk} are functions of $\{s_{n1,0}, s_{n2,0}, \ldots\}$, where the subscript 0 indicates current estimate. The "small correction terms" approximation thus leads to $\sigma_{nk} = -s_{nk,0}$. The least-squares state vector will be the set $\{\, \delta s_{nk}\, \}$ for all n and k, where $\delta s_{nk} \equiv s_{nk} - s_{nk,0}$.

To proceed with a formulation of an extended least-squares algorithm based on equation (4a), we require an expression for $(\,\boldsymbol{\Omega} - \boldsymbol{\Omega}_M\,)$ linear in the correction terms $\delta \boldsymbol{s}_k$. Combining equations (16) and (18) yields

$$\boldsymbol{\Omega}_M = \boldsymbol{\aleph}\,\{\,\boldsymbol{A}\boldsymbol{\Omega} + \Sigma\, \boldsymbol{s}_k\, [^{g_k}\cdot \boldsymbol{A}\boldsymbol{\Omega}] + \Sigma\, \boldsymbol{\sigma}_k\, [^{g_k}\cdot (\boldsymbol{A}\boldsymbol{\Omega} + \Sigma\, \boldsymbol{s}_\kappa\, [^{g_\kappa}\cdot \boldsymbol{A}\boldsymbol{\Omega}])]\,\} \qquad (19)$$

The assumptions that the g_k functions vary continuously and that all of the $\boldsymbol{s}_k$ and $\boldsymbol{\sigma}_k$ elements are small imply that terms of the form $\boldsymbol{\sigma}_k\, [^{g_k}\cdot (\boldsymbol{A}\boldsymbol{\Omega} + \Sigma\, \boldsymbol{s}_\kappa\, [^{g_\kappa}\cdot \boldsymbol{A}\boldsymbol{\Omega}])]$ are equal to $\boldsymbol{\sigma}_k\, [^{g_k}\cdot \boldsymbol{A}\boldsymbol{\Omega}]$ to first order. Using this simplification and setting $\boldsymbol{\sigma}_k = -\,\boldsymbol{s}_{k,0}$ yields

$$(\,\boldsymbol{\Omega} - \boldsymbol{\Omega}_M\,) = -\boldsymbol{\aleph}\, \Sigma\, \{\, \delta \boldsymbol{s}_k\, [^{g_k}\cdot \boldsymbol{A}\boldsymbol{\Omega}]\,\} \qquad (20)$$

To be able to follow our analytic maneuver model approach as developed in Section 2, we insert equation (20) into equation (4a) and apply appropriate transformations to the "primed" reference frame. The resulting sensitivity equation is

$$2\,\zeta = \boldsymbol{R}^T \boldsymbol{T}'^{\,T}_{\tau} \int \boldsymbol{T}'\,(\boldsymbol{R}\boldsymbol{\aleph}\,\Sigma\{\delta \boldsymbol{s}_k\, [^{g_k}\cdot [\boldsymbol{A}\boldsymbol{R}^T]\boldsymbol{\Omega}']\})\mathrm{dt}$$

$$= \boldsymbol{R}^T \Sigma\{\boldsymbol{T}'^{\,T}_{\tau}\!\int \boldsymbol{T}'\,(\boldsymbol{R}\boldsymbol{\aleph}\, [^{g_k}\cdot [\omega \boldsymbol{A}\boldsymbol{R}^T]_1]_D)\mathrm{dt}\}[\delta \boldsymbol{s}_k]_C \qquad (21)$$

where $[^{g_k}\cdot [\omega \boldsymbol{A}\boldsymbol{R}^T]_1]_D$ indicates a diagonal matrix formed from the elements of $[^{g_k}\cdot [\omega \boldsymbol{A}\boldsymbol{R}^T]_1]$, and $[\delta \boldsymbol{s}_k]_C$ indicates the column matrix formed from the diagonal elements of $\delta \boldsymbol{s}_k$ (recall that $\delta \boldsymbol{s}_k$ is a diagonal matrix). If we impose the additional constraint on each g_k that it satisfy the commutivity relation $g_k(ab) = g_k(a)g_k(b)$, equation (21) can be written in the convenient form

$$2\,\zeta = \boldsymbol{R}^T \Sigma\, \boldsymbol{K}^*_{gk}\,(\boldsymbol{R}\,\boldsymbol{\aleph}\, [^{g_k}\cdot [\boldsymbol{A}\boldsymbol{R}^T]_1]_D)\, [\delta \boldsymbol{s}_k]_C \qquad (22)$$

where the $\boldsymbol{K}^*_{gk}$ matrices are defined analogously to the $\boldsymbol{K}^*_k$ matrices discussed in Section 2, with $g_k(\omega)$ replacing ω^k in defining the required components. For the case of $\{g_1(\Omega) \equiv \Omega;\ g_2(\Omega) \equiv g_{|1|}(\Omega) \equiv |\Omega|\}$, equation (22) becomes

$$2\,\zeta = \boldsymbol{R}^{T}\boldsymbol{K}^{*}_{1}\,\boldsymbol{R}\,\aleph\,(\,[\,[\boldsymbol{AR}^{T}]_{1}]_{D}\;[\delta \boldsymbol{s}_{1}]_{C} + [^{||\cdot}[\boldsymbol{AR}^{T}]_{1}]_{D}\;[\delta \boldsymbol{s}_{1|}]_{C}\,) \qquad (23)$$

where the symbol $[^{||\cdot}\,]$ in the last term implies that the absolute value operation is applied to all of the elements of $[\boldsymbol{AR}^{T}]_{1}$. The $\boldsymbol{K}^{*}_{1}$ matrix applies to the last term with no adjustments because ω is by definition positive in the primed reference frame.

For each maneuver used in the calibration process, equation (22) provides three linear equations in the $N_G \cdot k_{max}$ unknowns $\{\delta s_{nk}\}$. To get proper visibility for accurately measuring all of the $\{\delta s_{nk}\}$ elements, a range of both positive and negative maneuvers in all of the pitch, roll, and yaw directions must be sampled. With an appropriately large number of maneuvers sampled, equation (22) can be used as the basis for a standard least-squares algorithm to determine estimates for the correction terms.

As with the original Davenport approach to the calibration problem, adjustments to the scale factor calibration imply an associated adjustment to the current estimate for the drift rate bias vector. In equation (2b) this adjustment is represented by the quantity $\boldsymbol{m}\mathbf{D}_{0}$. The analogous correction for the derivation in this section, which we will here call $\mathbf{d}_{s}$, is given by

$$\mathbf{d}_{s} = -\aleph\,\Sigma\,\{\,\delta s_{k}\,[^{gk}\cdot \boldsymbol{A}\mathbf{D}_{0}]\,\} \qquad (24)$$

which follows from equation (20) by replacing $(\mathbf{\Omega} - \mathbf{\Omega}_{M})$ with $\mathbf{d}_{s}$ on the left-hand-side and $\mathbf{\Omega}$ with $\mathbf{D}_{0}$ on the right-hand-side. The $\mathbf{D}_{0}$ value to be inserted into the equations is the most recent value determined for operational use.

Equation (22) can be generalized to allow for alignment and/or bias adjustments within the calibration state vector. This is done by simply combining equations (11a) and (22), with the restriction that the summation over k exclude the linear scale factor corrections, i.e.,

$$2\,\zeta = -\boldsymbol{R}^{T}\boldsymbol{K}^{*}_{1}\,\boldsymbol{B}\,\mathbf{m} + \boldsymbol{R}^{T}\boldsymbol{K}^{*}_{0}\,\boldsymbol{R}\,\mathbf{d} + \boldsymbol{R}^{T}\Sigma\,\boldsymbol{K}^{*}_{gk}\,(\boldsymbol{R}\,\aleph_{0}\,[^{gk}\cdot[\boldsymbol{A}_{0}\boldsymbol{R}^{T}]_{1}]_{D}\,)\,[\delta s_{k}]_{C} \qquad (25)$$

with the set $\{\delta s_{k}\}$ restricted to nonlinear terms. The 0 subscript on $\aleph_{0}$ and $\boldsymbol{A}_{0}$ indicates that the current estimate for the gyro alignments is used in constructing the nonlinear correction coefficients. After the calibration set $\{\mathbf{m}, \mathbf{d}, \{\delta s_{k}\}\}$ has been determined, equations (26a) - (26d) can be used to calculate $\mathbf{\Omega}$.

$$\mathbf{\Omega} = (\boldsymbol{G}\,\mathbf{C} - \mathbf{D}) - \aleph_{0}\,\Sigma\,\{(s_{k,0} + \delta s_{k})[^{gk}\cdot \boldsymbol{A}_{0}\,(\boldsymbol{G}\,\mathbf{C} - \mathbf{D})]\,\} \qquad (26a)$$

$$\boldsymbol{G} = (\boldsymbol{I}_{3} + \boldsymbol{m})\,\boldsymbol{G}_{0} \qquad (26b)$$

$$\mathbf{D} = (\boldsymbol{I}_{3} + \boldsymbol{m})\,\mathbf{D}_{0} + \mathbf{d} - \aleph_{0}\,\Sigma\,\{\,\delta s_{k}\,[^{gk}\cdot \boldsymbol{A}_{0}\mathbf{D}_{0}]\,\} \qquad (26c)$$

$$\boldsymbol{G}_{0} = \aleph_{0}\,\boldsymbol{S}^{-1} \qquad (26d)$$

Although straightforward to use as the basis for a least-squares algorithm (i.e., to solve for the state vector $\{\mathbf{m}, \mathbf{d}, \{\delta s_{k}\}\}$ given an error set $\{\zeta\}$), equation (25) is somewhat unaesthetic in that it mixes a set of parameters pertaining to the combined gyro system (i.e., $\{\mathbf{m}, \mathbf{d}\}$) with another set pertaining to the individual gyroscopes (i.e., $\{\delta s_{k}\}$). For elegance in presentation and to support engineering analysis of individual gyro behavior, having a state vector consisting solely of specific parameters of the individual gyros would be desirable. Equation (25) could be so recast if we were dealing only with sets of three gyros. However, for gyro sets containing more than three gyros, the parameter set $\{\mathbf{m}, \mathbf{d}\}$ captures all of the functionally observable information available in the maneuver measurements. (Of course, if the full set of gyro data is available, the data can be processed for each combination of three gyros and the individual gyro parameters extracted, but this defeats the processing simplifications discussed herein.)

This point concerning observability raises a question: for how many gyroscopes can unique scale factor information be obtained when equation (22) is applied together with the constraint of fixed gyro alignments? This question may be readily answered for the case where the state vector is restricted to linear scale corrections, i.e., δs_{1}. In this case, the δs_{1} matrix transforms to an equivalent $\boldsymbol{m}$ matrix via

$$\boldsymbol{m} = -\aleph\;\delta s_{1}\,\boldsymbol{A} = -\,[\boldsymbol{A}^{T}\boldsymbol{A}]^{-1}[\boldsymbol{A}^{T}\,\delta s_{1}\,\boldsymbol{A}] \qquad (27)$$

Both $[\boldsymbol{A}^{T}\boldsymbol{A}]$ and $[\boldsymbol{A}^{T}\,\delta s_{1}\,\boldsymbol{A}]$ can be shown to be 3-by-3 symmetric matrices, implying that the product $[\boldsymbol{A}^{T}\boldsymbol{A}]^{-1}[\boldsymbol{A}^{T}\,\delta s_{1}\,\boldsymbol{A}]$ is as well. The change matrix $\boldsymbol{m}$ therefore has only six independent elements, from which we conclude that the techniques of this section can provide independent scale parameter corrections for at most six gyroscopes. ("At most" applies because any coaligned gyroscopes will have degenerate corrections irrespective of the total number).

Section 5 - Least-Squares Solution and Weight Matrix Specification

For completeness, we present in this section a few points pertaining to the selection of weights to be applied to the input measurements. As discussed in many references on least-squares algorithms (e.g., Reference 4), the solution for the batch linear least-squares problem associated with a matrix equation $\boldsymbol{H}\,\mathbf{X} = \mathbf{Y}$ can generally be written as

$$\mathbf{X} = (\boldsymbol{H}^T \boldsymbol{W} \boldsymbol{H} + \boldsymbol{W}_A)^{-1} (\boldsymbol{H}^T \boldsymbol{W}\,\mathbf{Y} + \boldsymbol{W}_A \mathbf{X}_A) \quad (28)$$

For our problem, $\mathbf{X}$ (the state vector) will be some combination of $\mathbf{m}$, $\mathbf{d}$, and/or $\{\delta \mathbf{s}_k\}$, $\mathbf{Y}$ is $[\zeta_1^T, \ldots, \zeta_N^T]^T$ for N maneuvers, $\boldsymbol{H}$ is a matrix of state vector multiplying elements constructed from appropriate pieces of equation (25), $\boldsymbol{W}$ is a 3N-by-3N weight matrix for the error measurements (the elements of $\mathbf{Y}$), and $\boldsymbol{W}_A$ is a weight matrix associated with the a priori state vector estimate, $\mathbf{X}_A$. For our problem, because the state vector consists of differential changes from the previous best estimate, we set $\mathbf{X}_A = \mathbf{0}$. Our only remaining concern, therefore, is to establish reasonable estimates for $\boldsymbol{W}$ and $\boldsymbol{W}_A$.

Often it is both convenient and reasonable to simply set $\boldsymbol{W}$ to $\boldsymbol{I}_{3N}$ and $\boldsymbol{W}_A$ to $\boldsymbol{0}$. (We used this approach in our analysis of the HST maneuver data and have found it operationally acceptable.) Implicit in the approach are the following five assumptions: (1) the state vector correction terms are fairly stable over the time period of data collection, (2) the degree of correlation between measurement error components is fairly small, (3) the expected error component magnitudes are all approximately the same, (4) the data set spans the domain of state vector sensitivity sufficiently well that observability is not a problem, and (5) a sufficiently extensive data set has been accumulated that neglect of a priori information does not undermine operations. The first three points relate to setting $\boldsymbol{W}$ to $\boldsymbol{I}_{3N}$, whereas the last two relate to setting $\boldsymbol{W}_A$ to $\boldsymbol{0}$.

If any of the conditions indicated in the previous paragraph are significantly violated, a more sophisticated weighting scheme is required. We present here a method for specifying $\boldsymbol{W}$ that retains assumption 1, eliminates assumption 3, and replaces assumption 2 with a less restrictive one (called 2a) that the measurement errors associated with each maneuver are uncorrelated with those of all others. We will not consider the possible advantages of a nonzero $\boldsymbol{W}_A$. Assumptions 1 and 2a allow $\boldsymbol{W}$ to be expressed as a block diagonal matrix, with each block being a 3-by-3 matrix, $\boldsymbol{w}$, associated with a specific maneuver. Given the block diagonal form, each $\boldsymbol{w}$ can be written as $(\boldsymbol{p}_R + \boldsymbol{p}_G)^{-1}$, where $\boldsymbol{p}_R$ is the covariance associated with reference attitude errors, and $\boldsymbol{p}_G$ is the covariance associated with random gyro errors. The attitude covariance matrix is given by

$$\boldsymbol{p}_R = \boldsymbol{T}_\tau^T\, \boldsymbol{p}_i\, \boldsymbol{T}_\tau + \boldsymbol{p}_f \quad (29)$$

where $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ are the initial and final attitude covariance matrices in the instantaneous spacecraft frame, and $\boldsymbol{T}_\tau$ is as used in equation (5). Reference 5 specifies an equation for attitude covariance matrices such as $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$. This equation, which depends upon the reference star distribution and the measurement and catalog errors for each star, is

$$\boldsymbol{p}_{i/f} = \sigma_t^2\, [\boldsymbol{I}_3 - \Sigma(\sigma_t^2/\sigma_j^2) \mathbf{V}_j \mathbf{V}_j^T]^{-1} \quad (30)$$

where $\sigma_t^2 \equiv [\,\Sigma \sigma_j^{-2}\,]^{-1}$, σ_j is the root-mean-square combined measurement and catalog error for the jth star, $\mathbf{V}_j$ is the jth star vector expressed in the spacecraft frame, and the sums are over all observations. This expression can be simplified for processing purposes in the case of observations from a number of well-separated star sensors with fairly narrow fields-of-view (narrow relative to the field-of-view separations). In this case, each $\mathbf{V}_j$ can be replaced with the boresight direction vector for the jth sensor expressed in spacecraft coordinates, with σ_j then indicating typical error size for that sensor. This substitution eliminates ground processing of the reference star data.

A reasonable, albeit heuristic, model for the covariance associated with gyro errors is

$$\boldsymbol{p}_G \approx \boldsymbol{I}_3\, [\, \sigma_{gd}^2\, \tau^2 + \sigma_{gsa}^2\, \Theta^2\,] \quad (31)$$

where σ_{gd} is the typical single-axis standard deviation of the gyro drift rate bias, and σ_{gsa} is the typical scale factor / alignment maneuver error. Equation (31) does not attempt to model the physical mechanism that produces gyro noise, but rather requires the user to provide parameters σ_{gd} and σ_{gsa} based on typical spacecraft performance. Empirically, for the HST gyroscopes working as a set, we find $\sigma_{gd} \sim 0.01$ arcsecond per second and $\sigma_{gsa} \sim 0.2$ arcsecond per degree.

Section 6 - HST Gyroscope Behavior

The HST gyroscope system comprises three rate gyro assemblies (RGAs) manufactured by AlliedSignal Government Electronic Systems. Each RGA consists of two single-degree-of-freedom, dual-mode, rate integrating, mechanical gyroscopes. The high-rate mode has a range of ±1800 degrees per hour and a resolution of 7.5 milliarcseconds per 40-hertz sample; the low-rate mode has a range of ±20 degrees per hour and a resolution of 0.125 milliarcsecond per 40-hertz sample. The gyro alignments are such that any three can be used to completely sample rotations of the spacecraft. The onboard system is configured to nominally use four gyroscopes simultaneously, keeping the remaining two as backups.

RGA units 2 and 3 (those housing gyros 3, 4, 5, and 6) were replaced in December 1993 during the first HST servicing mission. All six gyroscopes were activated for the servicing mission and early on-orbit verification and calibration phase. The iterative calibration procedure described in References 1 and 6 was followed until convergence was achieved. Thereafter, the two gyros in RGA unit 1 were deactivated, leaving HST operating with four new, freshly calibrated gyroscopes. The active gyros are mounted with input-axis unit vectors of approximately (±0.586, ±0.617, -0.525), with the sign sense for the first two components being (--, ++, -+, +-), for gyros 3, 4, 5, and 6, respectively. The symmetry of these vectors about the yaw axis is significant for understanding the specific manifestation of an observed growing scale error.

As is typical with spacecraft gyroscopes, the biases vary fairly rapidly. For the HST gyroscopes, the change in the drift rate bias for both high- and low-rate modes has been found to be about 7 arcseconds per hour per day. The temporal variation of the high-rate mode drift bias vector (i.e., as measured in vehicle space) has been found to track the low-rate mode vector variations quite closely. This allowed implementation of an operational procedure whereby only the low-rate mode bias is measured frequently, based on data accumulated during science pointing with the spacecraft pointing control system locked on fine guidance sensor guide stars. The high-rate mode bias is then determined from the low-rate mode bias via an additive offset, which is monitored for constancy once every 4 to 6 weeks. The algorithm used for monitoring the offset had been, until recently, essentially that discussed in Section 3 in association with equation (13). The spacecraft pointing control system is commanded to place the gyroscopes in high-rate mode while maintaining a constant attitude for approximately one orbit (about 95 minutes). Fixed-head star tracker star measurements are obtained at the beginning and end of this constant attitude period and used to determine the true attitude change.

In HST operations, most large maneuvers are predominantly about the yaw axis. The predominant symptom of the scale factor problem discovered in August 1995 was a substantially larger postslew pointing error for negative yaw maneuvers than for positive yaw maneuvers. Upon examining the quantity $E \equiv (2\zeta \cdot \eta / \Theta)$ for maneuvers between the time of the first servicing mission and August 1995 with $|\eta_3| > 0.9$ and $\Theta > 90$ degrees, we found that although the average value of E for positive yaw maneuvers stayed near zero, its value for negative yaw maneuvers was fairly well fit by the curve

$$\overline{E} \approx 0.2 + 0.6\,(1 - e^{-t/T}) \quad \text{arcseconds per degree} \tag{32a}$$

$$T \approx 6 \text{ months} \tag{32b}$$

The sense of the error for negative yaw maneuvers was such that the spacecraft fell short of its intended destination. The random scatter for E is about 0.3 arcsecond per degree (3σ).

The analysis techniques described in this paper were developed to study the temporal change that was seen to have occurred in the HST RGAs. As part of our study, we have come to realize that the effects of gyroscope nonlinearities are as important as the temporal changes that precipitated the study. We applied our analysis to a combined set of 83 maneuvers collected in August 1994 and August 1995. (Our data indicate that the scale factors had stopped changing by August 1994.) For some of our analysis runs, we also included a 1-hour period of constant attitude. We find that studying the fit residuals associated with the constant attitude period is important for constructing a high-fidelity model of gyroscope response. The results of our analysis are specified below.

(1) To study the change in average linear scale relative to the original post-servicing-mission calibration, we performed a fit using the high mode bias offset vector and gyro frame linear scale factors as our state vector. The best fit values for this case are given in equations (33a) and (33b).

$$\mathbf{d}_{OFFSET} = [-1.8\times10^{-2}, 3.4\times10^{-2}, -7.7\times10^{-2}]^T \pm 1\times10^{-2} \text{ arcsecond per second} \quad (33a)$$

$$[\delta s_{\mathrm{I}}]_C = [5.7\times10^{-5}, 4.2\times10^{-5}, 8.4\times10^{-5}, 1.74\times10^{-4}]^T \pm 1\times10^{-5} \quad (33b)$$

As will be discussed shortly, the bias offset adjustment is that required to compensate for gyroscope nonlinearities, the "true" bias at constant attitude already having been eliminated by the standard operational procedures. The $[\delta s_{\mathrm{I}}]_C$ elements represent the average change in the high-rate mode scale factors. The sign sense indicates that the gyros have become more sensitive (more counts per degree of actual slew). The largest single change, that for gyro 6, corresponds to an error of 56 arcseconds for a 90-degree slew about the input axis.

(2) Because of the difference in response for positive and negative slews, together with the fact that the bias determination procedure had been tuned to work accurately at zero angular rate, it seemed likely that some scale nonlinearity was involved. Taking $\mathbf{d} = \mathbf{0}$ as a constraint effectively imposed by the operational procedures, we investigated potential nonlinearities by solving for a state vector consisting of $[\delta s_{\mathrm{I}}]_C$ and $[\delta s_{\mathrm{II}}]_C$. The best-fit results in this case are

$$[\delta s_{\mathrm{I}}]_C = [6.0\times10^{-5}, 2.9\times10^{-5}, 1.27\times10^{-4}, 1.48\times10^{-4}]^T \pm 1\times10^{-5} \quad (34a)$$

$$[\delta s_{\mathrm{II}}]_C = [0.8\times10^{-5}, 6.1\times10^{-5}, 1.95\times10^{-4}, 7.8\times10^{-5}]^T \pm 1\times10^{-5} \quad (34b)$$

Comparing the nonlinear correction values with the average change values indicated for the first case, we see that the error associated with not taking the nonlinear effect into account can be as large as the temporal change. We also determined fit parameters for two other cases, one including $\mathbf{d}$ in the state vector and another using $g_2(\Omega) = \Omega^2$ rather than $|\Omega|$. The former showed a slight reduction in the fit residuals, whereas the latter showed a slight increase in the fit residuals; the changes in residuals in both cases were insignificant.

Given our findings regarding scale factor nonlinearities, the spacecraft pointing control logic should ideally include compensation for this effect when estimating spacecraft angular rates. Although the HST pointing control system does not model scale factor nonlinearities, we can compensate to a significant degree for the nonlinearities by allowing the low-to-high bias offset to absorb the average effect of the gyroscope nonlinearities as weighted by the actual distribution of maneuvers scheduled for the HST mission. This is effectively what happens with the fit procedure associated with equation (33a). The large negative third component for the bias in equation (33a) is associated with the positive sign of the components of $[\delta s_{\mathrm{II}}]_C$ in equation (34b), together with the fact that gyros 3 - 6 are, on average, pointing along the negative yaw axis. This weighting for mission maneuver distribution will also affect the estimated average scale factors, as can be seen by comparing equations (33b) and (34a). Empirically, it appears that adequate HST mission performance is achieved with this approach during normal operations. We note, however, that this approach does not give optimized performance for high-rate mode, inertial hold conditions, the implied spurious drift being about 300 arcseconds per hour.

Using the bias vector to absorb the average effect of gyroscope nonlinearities weighted according to the profile of mission maneuvers could be problematic for spacecraft that use single-mode gyroscopes. For such spacecraft, science operations would likely require the bias vector to be selected so that pointing performance is optimized with respect to constant attitude periods. Adjusting the bias to improve maneuver performance is therefore not an option. Mission engineers designing the pointing control and sensor calibration algorithms for such missions should consider including compensation for gyroscope nonlinearities, particularly if slewing accuracies better than 1 arcsecond per degree are required.

(3) As part of our analysis of the HST gyroscope changes, we also considered the possibility that the changes were associated with the gyroscope alignment matrix. We therefore performed a fit for a scale factor / alignment correction matrix ($\boldsymbol{m}$) together with a bias adjustment ($\mathbf{d}$) based on equation (11a). We found that including the alignment adjustments did not significantly improve the residuals relative to those associated with the fit restricted to state vector $\{\mathbf{d}, [\delta s_{\mathrm{I}}]_C\}$. We specifically found that the alignment terms did not allow us to simultaneously obtain improved residuals for the maneuver data while maintaining small residuals for the constant attitude data. Our results are consistent with there being no significant change in the gyroscope alignments during the 18 months following the first HST servicing mission.

Conclusions

This paper has presented a number of variations on the Davenport algorithm for gyroscope calibration specifically designed to (1) allow analysis with a drastically restricted quantity of telemetry data and (2) extend the state vector domain to allow study of both isolated and nonlinear scale factor corrections. We have applied the techniques to data obtained during normal operations of HST as part of a study of temporal variations of the HST gyroscope scale factors. We have found that the HST replacement gyroscopes experienced significant change over the first 6 to 8 months following the first HST servicing mission, the largest individual change corresponding to an error in estimated projected rate about the input axis of about 56 arcseconds per 90 degrees. We have found scale factor nonlinearities that, when characterized as differences between scale factors associated with positive and negative rotations, are as large as 2 parts in 10000, i.e., about 65 arcseconds per 90 degrees. For spacecraft, such as HST, that use dual-mode gyroscopes, the effects of the nonlinearities can be accommodated to a significant degree via adjustments to the high-rate mode drift rate bias vector. This approach may be inadequate for missions using single-mode gyroscopes. Finally, we find, to within the accuracy of our data set, that no significant changes have occurred to the gyroscope alignments during the first 18 months following the servicing mission.

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Appendix - Model Maneuver Profile

In this appendix we present the details of one fairly common maneuver model. In addition to the total maneuver angle, the model uses three input parameters characterizing the spacecraft's maneuver execution algorithm. These parameters can be selected as the maximum jerk magnitude (J_m), the jerk pulse duration (δ), and the maximum angular velocity magnitude (ω_m). The maneuver profile is symmetric about the midtime ($\tau/2$); it is therefore sufficient to construct the maneuver profile through that time. Throughout the maneuver, the angle (θ), rate (ω), and acceleration (a) are continuous, and the jerk (the third time derivative of θ) takes on one of three values: J, 0, or -J. The maneuver through its midpoint is composed of two, three, or four segments, depending upon the value of Θ. The construction for each solution type is presented below.

Operationally, three auxiliary parameters are first calculated from the three input parameters:

$$\varepsilon_{max} = \omega_m / J_m \delta - \delta \tag{A.1a}$$

$$\Theta_a = 2 J_m \delta^3 \tag{A.1b}$$

$$\Theta_b = \Theta_a \{[(\varepsilon_{max}^2 + 3\delta\varepsilon_{max}) / 2\delta^2] + 1\} \tag{A.1c}$$

These three equations will be derived below. The determination of whether a two-, three-, or four-segment half-maneuver pertains depends upon where Θ falls relative to Θ_a and Θ_b; a two-segment solution pertains for Θ in the range $[0,\Theta_a]$, a three-segment solution for the range $[\Theta_a,\Theta_b]$, and a four-segment solution for $[\Theta_b,\pi]$.

Two-segment solution

The two-segment solution assumes that the jerk is equal to some positive value J for a time period δ and equal to -J for a subsequent equal period. The functions a(t), ω(t), and θ(t) are each required to be continuous through the point of discontinuous jerk. The angular velocity reaches its maximum value at exactly the midpoint of the maneuver, i.e., at $\tau/2 = 2\delta$. The solution for the two segments is specified below.

Segment 1: $0 < t < \delta$

$$J(t) = J \quad \text{(J yet unknown)} \tag{A.2a}$$

$$a(t) = J\,t \tag{A.2b}$$

$$\omega(t) = 1/2\, J\, t^2 \tag{A.2c}$$

$$\theta(t) = 1/6\, J\, t^3 \tag{A.2d}$$

Segment 2: $\delta < t < 2\delta$

$$J(t) = -J \tag{A.2e}$$

$$a(t) = J\delta - J(t-\delta) \tag{A.2f}$$

$$\omega(t) = 1/2\, J\delta^2 + J\delta(t-\delta) - 1/2\, J(t-\delta)^2 \tag{A.2g}$$

$$\theta(t) = 1/6\, J\delta^3 + 1/2\, J\delta^2(t-\delta) + 1/2\, J\delta(t-\delta)^2 - 1/6\, J(t-\delta)^3 \tag{A.2h}$$

The unknown J is determined by the requirement that $\theta(\tau/2) = \Theta/2$. Substituting $t = 2\delta$ in equation (A.2h) yields

$$J = \Theta / 2\delta^3 \tag{A.2i}$$

The two-segment solution applies until equation (A.2i) produces a value of J greater than J_m. This gives the limiting angle Θ_a, indicated in equation (A.1b).

Three-segment solution

For maneuvers with angle Θ exceeding Θ_a, the two periods of constant jerk are separated by a period of zero jerk, of duration ε (to be determined). For convenience, let us define a time point $\Delta = \delta+\varepsilon$. The solution for the three segments is specified below.

Segment 1: $0 < t < \delta$

$$J(t) = J_m \quad \text{(A.3a)}$$
$$a(t) = J_m t \quad \text{(A.3b)}$$
$$\omega(t) = 1/2\, J_m t^2 \quad \text{(A.3c)}$$
$$\theta(t) = 1/6\, J_m t^3 \quad \text{(A.3d)}$$

Segment 2: $\delta < t < \Delta$

$$J(t) = 0 \quad \text{(A.3e)}$$
$$a(t) = J_m \delta \quad \text{(A.3f)}$$
$$\omega(t) = 1/2\, J_m \delta^2 + J_m \delta\,(t - \delta) \quad \text{(A.3g)}$$
$$\theta(t) = 1/6\, J_m \delta^3 + 1/2\, J_m \delta^2 (t - \delta) + 1/2\, J_m \delta\,(t - \delta)^2 \quad \text{(A.3h)}$$

Segment 3: $\Delta < t < \Delta+\delta$

$$J(t) = -J_m \quad \text{(A.3i)}$$
$$a(t) = J_m \delta - J_m(t - \Delta) \quad \text{(A.3j)}$$
$$\omega(t) = 1/2\, J_m \delta^2 + J_m \delta\, \varepsilon + J_m \delta\,(t - \Delta) - 1/2\, J_m(t - \Delta)^2 \quad \text{(A.3k)}$$
$$\theta(t) = 1/6\, J_m \delta^3 + 1/2\, J_m \delta^2 \varepsilon + 1/2\, J_m \delta\, \varepsilon^2 + 1/2\, J_m \delta^2 (t - \Delta) + J_m \delta\, \varepsilon\,(t - \Delta) + 1/2\, J_m \delta\,(t - \Delta)^2 - 1/6\, J_m(t - \Delta)^3 \quad \text{(A.3l)}$$

The unknown ε is determined by the requirement that $\theta(\tau/2) = \Theta/2$. Substituting $t = \Delta+\delta$ in equation (A.3l) yields the quadratic equation

$$\varepsilon^2 + 3\,\delta\,\varepsilon - 2\delta^2 (\Theta/\Theta_a - 1) = 0 \quad \text{(A.3m)}$$

the solution for which is

$$\varepsilon = 3/2\, \delta\, \{[\,1 + 8/9\, (\Theta/\Theta_a - 1)]^{1/2} - 1\}$$

$$= 1/2\, \delta\, \{[\,1 + 8\, \Theta/\Theta_a]^{1/2} - 3\} \quad \text{(A.3n)}$$

The three-segment solution applies until equation (A.3k), combined with equation (A.3n), produces a value of ω greater than ω_m. The maximum permitted value of ε can be found by setting $\omega(t)$ in equation (A.3k) to ω_m at $t = 2\delta+\varepsilon$. This results in

$$\varepsilon_{max} = \omega_m / J_m \delta - \delta \quad \text{(A.3o)}$$

Note that for the progression of solutions to be consistent, we require $\omega_m \geq J_m\delta^2$. The maximum maneuver angle permitted for the three-segment model can be found by substituting ε_{max} for ε in equation (A.3m); the result is equation (A.1c).

Four-segment solution

For maneuvers with angle Θ exceeding Θ_b, the third segment is followed by a period of constant angular rate at the maximum permitted value. This fourth segment lasts until the maneuver reaches the halfway point, i.e., until $\theta(t) = \Theta/2$. The result is that the maneuver profile for the first three segments is the same as that appropriate for a three-segment solution with $\varepsilon = \varepsilon_{max}$, and during the fourth segment it is given by

$$J(t) = 0 \quad \text{(A.4a)}$$
$$a(t) = 0 \quad \text{(A.4b)}$$
$$\omega(t) = \omega_m \quad \text{(A.4c)}$$
$$\theta(t) = \Theta_b/2 + \omega_m\,[t - (2\delta+\varepsilon_{max})] \quad \text{(A.4d)}$$

The total maneuver duration in this case is determined by the requirement that $\theta(\tau/2) = \Theta/2$. Thus, τ is given in this case by

$$\tau/2 = (\Theta - \Theta_b)/2\omega_m + (2\delta+\varepsilon_{max}) \quad \text{(A.4e)}$$

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