Attitude Estimation Using Modified Rodrigues Parameters

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Abstract

In this paper, a Kalman filter formulation for attitude estimation is derived using the Modified Rodrigues Parameters. The extended Kalman filter uses a gyro-based model for attitude propagation. Two solutions are developed for the sensitivity matrix in the Kalman filter. One is based upon an additive error approach, and the other is based upon a multiplicative error approach. It is shown that the two solutions are in fact equivalent. The Kalman filter is then used to estimate the attitude of a simulated spacecraft. Results indicate that the new algorithm produces accurate attitude estimates by determining actual gyro biases.

Introduction

A widely used parameterization for attitude estimation is the quaternion representation. Advantages of using quaternions include: 1) the kinematic equations are linear with respect to angular velocities, 2) singularities are not present for any eigenaxis rotation, and 3) the attitude matrix is algebraic in the quaternion components. However, since the quaternion parameterization involves the use of four components to represent the attitude motion, the quaternion components are non-minimal (dependent). This leads to a constraint that the quaternion must have unit norm.

The quaternion normalization constraint produces a singularity in the Kalman filter covariance matrix. Three solutions (two of which yield identical results) to this problem are summarized by Lefferts et al. [1]. The first approach uses the transition matrix of the state-error vector to obtain a reduced order representation of the error covariance. The second approach deletes one of the quaternion components in order to obtain a truncated error covariance expression. The third approach uses an incremental quaternion error which results in a representation that is identical to the first approach. This approach is most commonly used to maintain normalization for the estimated quaternion.

Three-dimensional parameterizations of attitude are still useful for many control applications (e.g., see [2-3]). Since spacecraft control algorithms require estimates of attitude and/or rate, it is therefore advantageous to develop a Kalman filter which utilizes a three-dimensional parameterization. A number of three-dimensional parameterizations is shown in an excellent survey by Shuster [4]; including, the Euler angle representation, the Rodrigues parameters, and the modified Rodrigues parameters. It is widely known that all three-dimensional parameterizations have singularities (e.g., the Rodrigues parameters are singular for 180 degree rotations [4]). The choice of parameters depends on a number of factors; for example, the type of rotation maneuver for the spacecraft, computational requirements, physical representation insight, etc.

Most control applications require a parameterization that has the singularity as far from the origin as possible. Specifically, the modified Rodrigues parameters (MRP) [5] have recently been used for spacecraft control applications, since they allow for rotations up to 360 degrees. Tsiotras [6] utilized the MRP to derive a new class of globally asymptotically stabilizing feedback control laws. Schaub et. al. [7] utilized the MRP to estimate external torques by tracking a Lyapunov function. Crassidis and Markley [8] utilized the MRP to develop a sliding mode controller for spacecraft maneuvers. However, the aforementioned control schemes assume that the attitude (i.e., the MRP) is already known. For this reason, a Kalman filter using the modified Rodrigues parameters is developed in this paper.

The organization of this paper proceeds as follows. First, a brief review of the quaternion and MRP kinematic equations is shown. Then, a brief review of the Kalman filter is shown using the quaternion representation. Next, a Kalman filter for attitude estimation is derived using the MRP. Also, the sensitivity matrix is derived using both an additive and a multiplicative approach. Finally, the new algorithm is used to estimate the attitude of the Tropical Rainfall Measurement Mission (TRMM) spacecraft.

Attitude Kinematics

In this section, a brief review of the kinematic equations of motion using the modified Rodrigues parameters is shown. This parameterization is derived by employing a stereographic projection of the quaternions. The quaternion representation is given by

$$\underline{q} \equiv \begin{bmatrix} \underline{q}_{13} \\ \cdots \\ q_4 \end{bmatrix} \tag{1}$$

with

$$\underline{q}_{13} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \underline{\hat{n}} \sin\left(\frac{\theta}{2}\right) \tag{2a}$$

$$q_4 = \cos\left(\frac{\theta}{2}\right) \tag{2b}$$

where $\underline{\hat{n}}$ is a unit vector corresponding to the axis of rotation and θ is the angle of rotation. The quaternion kinematic equations of motion are derived by using the spacecraft's angular velocity ($\underline{\omega}$), given by

$$\underline{\dot{q}} = \frac{1}{2}\Omega(\underline{\omega})\underline{q} = \frac{1}{2}\Xi(\underline{q})\underline{\omega} \tag{3}$$

where $\Omega(\underline{\omega})$ and $\Xi(\underline{q})$ are defined as

$$\Omega(\underline{\omega}) \equiv \begin{bmatrix} -[\underline{\omega}\times] & \vdots & \underline{\omega} \\ \cdots\cdots & \vdots & \cdots\cdots \\ -\underline{\omega}^T & \vdots & 0 \end{bmatrix} \tag{4a}$$

$$\Xi(\underline{q}) \equiv \begin{bmatrix} q_4 I_{3\times3} + [\underline{q}_{13}\times] \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ -\underline{q}_{13}^T \end{bmatrix} \tag{4b}$$

where $I_{3\times3}$ is a 3×3 identity matrix. The 3×3 dimensional matrices $[\underline{\omega}\times]$ and $[\underline{q}_{13}\times]$ are referred to as cross product matrices since $\underline{a}\times\underline{b} = [\underline{a}\times]\underline{b}$, with

$$[\underline{a}\times] \equiv \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{5}$$

Since a three degree-of-freedom attitude system is represented by a four-dimensional vector, the quaternions cannot be independent. This condition leads to the following normalization constraint

$$\underline{q}^T\underline{q} = \underline{q}_{13}^T\underline{q}_{13} + q_4^2 = 1 \tag{6}$$

The modified Rodrigues parameters are defined by [5]

$$\underline{p} = \frac{\underline{q}_{13}}{1+q_4} = \underline{\hat{n}}\tan\left(\frac{\theta}{4}\right) \tag{7}$$

where $\underline{p}$ is a 3×1 vector. The kinematic equations of motion are derived by using the spacecraft's angular velocity ($\underline{\omega}$), given by [4]

$$\underline{\dot{p}} = \frac{1}{4}\left\{\left(1-|\underline{p}|^2\right)\underline{\omega} - 2\underline{\omega}\times\underline{p} + 2(\underline{\omega}\bullet\underline{p})\underline{p}\right\} \tag{8}$$

where $|\;|$ is the norm operator, and $\bullet$ is the dot product. This equation may be re-written as

$$\underline{\dot{p}} = \frac{1}{2}\left\{\frac{1}{2}\left(1-\underline{p}^T\underline{p}\right)I_{3\times3} + [\underline{p}\times] + \underline{p}\,\underline{p}^T\right\}\underline{\omega} \tag{9}$$

The measurement model is assumed to be of the form given by [4]

$$\underline{B}_B = A(\underline{p})\underline{B}_I \tag{10}$$

where $\underline{B}_I$ is a 3×1 dimensional vector of some reference object (e.g., a vector to the sun or to a star, or the Earth's magnetic field vector) in a reference coordinate system, $\underline{B}_B$ is a 3×1 dimensional vector defining the components of the corresponding reference vector measured in the spacecraft body frame, and $A(\underline{p})$ is given by

$$A(\underline{p}) = I_{3\times3} - \frac{4\left(1-\underline{p}^T\underline{p}\right)}{\left(1+\underline{p}^T\underline{p}\right)^2}[\underline{p}\times] + \frac{8}{\left(1+\underline{p}^T\underline{p}\right)^2}[\underline{p}\times]^2 \tag{11}$$

which is the 3×3 dimensional (orthogonal) attitude matrix.

Kalman Filter Development

In this section, a Kalman filter is derived for attitude estimation using the modified Rodrigues parameters. First a brief review of basic principles of the Kalman filter using quaternions is shown (see [1] for more details). The state error vector has seven components consisting of a four-component error quaternion ($\delta\underline{q}$) and a three-vector gyro bias error $\Delta\underline{b}$, given by

$$\Delta\underline{x}_f = \begin{bmatrix} \delta\underline{q} \\ \cdots \\ \Delta\underline{b} \end{bmatrix} \tag{12}$$

The error quaternion is defined as

$$\delta\underline{q} = \underline{q} \otimes \hat{\underline{q}}^{-1} \tag{13}$$

where $\underline{q}$ is the true quaternion and $\hat{\underline{q}}$ is the estimated quaternion. Also, the operator $\otimes$ refers to quaternion multiplication (see [4] for details). Since the incremental quaternion corresponds to a small rotation, Equation (13) can be approximated by

$$\delta\underline{q} \approx \begin{bmatrix} \delta\underline{q}_{13} \\ \cdots \\ 1 \end{bmatrix} \tag{14}$$

which reduces the four-component error quaternion into a three-component (half-angle) representation. Equation (14) is then used to derive a quaternion based Kalman filter (see [1] for details).

The state equation for the new algorithm consists of the modified Rodrigues parameters $(\underline{p})$ and a gyro bias corrections $(\underline{b})$, given by

$$\underline{x} = \begin{bmatrix} \underline{p} \\ \cdots \\ \underline{b} \end{bmatrix} \tag{15}$$

The true angular velocity is assumed to be modeled by

$$\underline{\omega} = \tilde{\underline{\omega}} - \underline{b} - \underline{\eta}_1 \tag{16}$$

where $\underline{\omega}$ is the true angular velocity, $\tilde{\underline{\omega}}$ is the gyro-measured angular velocity, and $\underline{b}$ is the gyro drift vector, which is modeled by

$$\dot{\underline{b}} = \underline{\eta}_2 \tag{17}$$

The 3×1 vectors, $\underline{\eta}_1$ and $\underline{\eta}_2$, are assumed to be modeled by a Gaussian white-noise process with

$$\underline{w} \equiv \begin{bmatrix} \underline{\eta}_1 \\ \cdots \\ \underline{\eta}_2 \end{bmatrix} \tag{18a}$$

$$E\{\underline{w}(t)\} = \underline{0} \tag{18b}$$

$$E\{\underline{w}(t)\underline{w}^T(t')\} = Q\delta(t-t') \tag{18c}$$

The true state-model equation can now be written as

$$\dot{\underline{p}} = \underline{f}(\underline{p},\underline{b},\underline{\tilde{\omega}},t) + \underline{g}(\underline{p},\underline{\eta}_1,t) \tag{19a}$$

$$\dot{\underline{b}} = \underline{\eta}_2 \tag{19b}$$

where

$$\underline{f}(\underline{p},\underline{b},\underline{\tilde{\omega}},t) = \frac{1}{2}\left\{\frac{1}{2}\left(1-\underline{p}^T\underline{p}\right)I_{3\times3} + \left[\underline{p}\times\right] + \underline{p}\,\underline{p}^T\right\}(\underline{\tilde{\omega}} - \underline{b}) \tag{20a}$$

$$\underline{g}(\underline{p},\underline{\eta}_1,t) = -\frac{1}{2}\left\{\frac{1}{2}\left(1-\underline{p}^T\underline{p}\right)I_{3\times3} + \left[\underline{p}\times\right] + \underline{p}\,\underline{p}^T\right\}\underline{\eta}_1 \tag{20b}$$

The extended Kalman filter utilizes a first-order Taylor series expansion for the state-error equation, given by

$$\Delta\dot{\underline{x}} = F\,\Delta\underline{x} + G\,\underline{w} \tag{21}$$

where

$$F = \begin{bmatrix} \dfrac{\partial \underline{f}}{\partial \underline{p}} & \vdots & \dfrac{\partial \underline{f}}{\partial \underline{b}} \\ \cdots & \vdots & \cdots \\ 0_{3\times3} & \vdots & 0_{3\times3} \end{bmatrix} \tag{22a}$$

$$G \equiv \begin{bmatrix} G_{11} & \vdots & G_{12} \\ \cdots & \vdots & \cdots \\ G_{21} & \vdots & G_{22} \end{bmatrix} \tag{22b}$$

and

$$G_{11} = \frac{\partial \underline{g}}{\partial \underline{\eta}_1} = -\frac{1}{2}\left\{\frac{1}{2}\left(1-\underline{p}^T\underline{p}\right)I_{3\times3} + \left[\underline{p}\times\right] + \underline{p}\,\underline{p}^T\right\} \tag{23a}$$

$$G_{12} = G_{21} = 0_{3\times3} \tag{23b}$$

$$G_{22} = I_{3\times3} \tag{23c}$$

The estimated state-error equation is given by

$$\Delta\dot{\hat{\underline{x}}} = F\big|_{\underline{x}=\hat{\underline{x}}}\Delta\hat{\underline{x}} \equiv \hat{F}\,\Delta\hat{\underline{x}} \tag{24a}$$

$$\Delta\underline{\hat{x}} = \underline{x} - \underline{\hat{x}} \tag{24b}$$

The partial derivatives in Equation (24a) for the state-error matrix are given by

$$\left.\frac{\partial \underline{f}}{\partial \underline{p}}\right|_{\underline{x}=\underline{\hat{x}}} = \frac{1}{2}\left\{\underline{\hat{p}}\,\underline{\hat{\omega}}^T - \underline{\hat{\omega}}\,\underline{\hat{p}}^T - [\underline{\hat{\omega}}\times] + \left(\underline{\hat{\omega}}^T\underline{\hat{p}}\right)I_{3\times3}\right\} \tag{25a}$$

$$\left.\frac{\partial \underline{f}}{\partial \underline{b}}\right|_{\underline{x}=\underline{\hat{x}}} = -\frac{1}{2}\left\{\frac{1}{2}\left(1-\underline{\hat{p}}^T\underline{\hat{p}}\right)I_{3\times3} + \left[\underline{\hat{p}}\times\right] + \underline{\hat{p}}\,\underline{\hat{p}}^T\right\} \equiv \hat{G}_{11} \tag{25b}$$

where

$$\underline{\hat{\omega}} = \underline{\tilde{\omega}} - \underline{\hat{b}} \tag{26}$$

State-observable discrete measurements are assumed to be modeled by

$$\underline{z}_k = \underline{h}_k\left(\underline{x}_k\right) + \underline{v}_k \tag{27}$$

where

$$\underline{h}_k\left(\underline{x}_k\right) = A\left(\underline{p}_k\right)\underline{B}_{I_k} \tag{28}$$

and $\underline{v}_k$ is assumed to be modeled by a zero-mean Gaussian process with

$$E\left\{\underline{v}_k\right\} = \underline{0} \tag{29a}$$

$$E\left\{\underline{v}_k\,\underline{v}_l^T\right\} = R\delta_{kl} \tag{29b}$$

The sensitivity matrix can be written as

$$H_k = \left[L \;\vdots\; 0_{3\times3}\right] \tag{30}$$

where L can be derived using an additive approach or a multiplicative approach. The additive approach expands $\underline{h}_k(\underline{x}_k)$ in a power series about $\underline{\hat{x}}_k$, given by

$$\underline{h}_k(\underline{x}_k) = \underline{h}_k(\underline{\hat{x}}_k) + \left.\frac{\partial \underline{h}_k}{\partial \underline{p}}\right|_{\underline{x}_k=\underline{\hat{x}}_k}\Delta\underline{x}_k \tag{31}$$

The brute force differentiation in Equation (31) can be shown to be given by

$$\begin{aligned}\left.\frac{\partial \underline{h}_k}{\partial \underline{p}}\right|_{\underline{x}_k=\underline{\hat{x}}_k} = & \left.\frac{4}{\left(1+\underline{\hat{p}}^T\underline{\hat{p}}\right)^2}\left\{[\underline{B}_I\times]\left(\left(1-\underline{\hat{p}}^T\underline{\hat{p}}\right)\; I_{3\times3} - 2\underline{\hat{p}}\,\underline{\hat{p}}^T\right) + 2\underline{\hat{p}}\,\underline{B}_I^T - 4\underline{B}_I\underline{\hat{p}}^T + 2\left(\underline{\hat{p}}^T\underline{B}_I\right)I_{3\times3}\right\}\right|_{t_k} \\ & + \left.\frac{16}{\left(1+\underline{\hat{p}}^T\underline{\hat{p}}\right)^3}\left\{2\left(\underline{\hat{p}}^T\underline{\hat{p}}\right)\underline{B}_I\underline{\hat{p}}^T - \left(1-\underline{\hat{p}}^T\underline{\hat{p}}\right)\;[\underline{B}_I\times]\underline{\hat{p}}\,\underline{\hat{p}}^T - 2\underline{\hat{p}}\,\underline{\hat{p}}^T\underline{B}_I\underline{\hat{p}}^T\right\}\right|_{t_k}\end{aligned} \tag{32}$$

which is somewhat complicated. The multiplicative approach assumes that the true parameters are given by

$$\underline{p} = \delta\underline{p} \otimes \underline{\hat{p}} \tag{33}$$

where $\delta\underline{p}$ is the error MRP. The composition rule for the MRP leads to the following

$$\underline{p} = \frac{\left(1-|\underline{\hat{p}}|^2\right)\delta\underline{p} + \left(1-|\delta\underline{p}|^2\right)\underline{\hat{p}} - 2[\delta\underline{p}\times]\underline{\hat{p}}}{1+|\delta\underline{p}|^2|\underline{\hat{p}}|^2 - 2\delta\underline{p}\bullet\underline{\hat{p}}} \tag{34}$$

For small $\delta\underline{p}$, Equation (34) can be approximated using

$$\begin{aligned}\underline{p} &\approx \left(1+2\delta\underline{p}\bullet\underline{\hat{p}}\right)\left[\left(1-|\underline{\hat{p}}|^2\right)\delta\underline{p} + \underline{\hat{p}} - 2[\delta\underline{p}\times]\underline{\hat{p}}\right] \\ &\approx \underline{\hat{p}} + \left[\left(1-|\underline{\hat{p}}|^2\right)I_{3\times3} + 2[\underline{\hat{p}}\times] + 2\,\underline{\hat{p}}\,\underline{\hat{p}}^T\right]\delta\underline{p}\end{aligned} \tag{35}$$

From

$$A(\underline{p}) = A(\delta\underline{p})\,A(\underline{\hat{p}}) \tag{36}$$

it follows that

$$L \equiv \left.\frac{\partial \underline{h}_k}{\partial \underline{p}}\right|_{\underline{x}_k=\underline{\hat{x}}_k} = \left.\frac{\partial}{\partial \underline{p}} A(\delta\underline{p})A(\underline{\hat{p}})\underline{B}_I\right|_{\underline{\hat{p}}} \tag{37}$$

and using the fact that for small $\delta\underline{p}$

$$A(\delta\underline{p}) \approx I_{3\times3} - 4[\delta\underline{p}\times] \tag{38}$$

Equation (37) can now be evaluated using the chain rule to yield

$$\begin{aligned}L &= 4\left[A(\underline{\hat{p}})\underline{B}_I\times\right]\left\{\left(1-\underline{\hat{p}}^T\underline{\hat{p}}\right)I_{3\times3} + 2[\underline{\hat{p}}\times] + 2\,\underline{\hat{p}}\,\underline{\hat{p}}^T\right\}^{-1}\Big|_{t_k} \\ &= \frac{4}{\left(1+\underline{\hat{p}}^T\underline{\hat{p}}\right)^2}\left[A(\underline{\hat{p}})\underline{B}_I\times\right]\left\{\left(1-\underline{\hat{p}}^T\underline{\hat{p}}\right)I_{3\times3} - 2[\underline{\hat{p}}\times] + 2\,\underline{\hat{p}}\,\underline{\hat{p}}^T\right\}\Big|_{t_k}\end{aligned} \tag{39}$$

which is in a simpler form as compared with Equation (32). In fact, these equations are identical, thereby proving the equivalence between a multiplicative and an additive approach for the MRP in the Kalman filter. Also, the matrix in Equation (39) has at most rank two, which reflects the fact that the observation vector contains no information about rotations around an axis specified by that vector at each measurement point. The extended Kalman filter equations for attitude estimation are summarized by

$$\dot{\hat{\underline{x}}} = \underline{f}(\hat{\underline{x}}, t) \tag{40a}$$

$$\dot{P} = \hat{F}\,P + P^T \hat{F} + \hat{G}\,Q\,\hat{G}^T \tag{40b}$$

$$\hat{\underline{x}}_k(+) = \hat{\underline{x}}_k(-) + K_k\left[\underline{z}_k - \underline{h}_k\left(\hat{\underline{p}}_k(-)\right)\right] \tag{40c}$$

$$P_k(+) = \left[I_{6\times6} - K_k\,H_k\left(\hat{\underline{x}}_k(-)\right)\right]P_k(-) \tag{40d}$$

$$K_k = P_k(-)\,H_k^T\left[H_k\,P_k(-)\,H_k^T + R\right]^{-1} \tag{40e}$$

where

$$\hat{G} = \begin{bmatrix} \hat{G}_{11} & \vdots & 0_{3\times3} \\ \cdots & \vdots & \cdots \\ 0_{3\times3} & \vdots & I_{3\times3} \end{bmatrix} \tag{41}$$

Spacecraft Simulation and Results

A simulation study is performed using the Tropical Rainfall Measurement Mission (TRMM) spacecraft orbit parameters. The TRMM spacecraft (see Figure 1) is due to be launched in 1997 with a nominal mission life of 42 months. The main objectives of this mission include: (i) to obtain multi-year measurements of tropical and subtropical rainfall, (ii) to understand how interactions between the sea, air, and land masses produce changes in global rainfall and climate, and (iii) to help improve the modeling of tropical rainfall processes and their influence on global circulation. The simulated spacecraft has a near circular orbit at 350 km. The nominal mission mode requires a rotation once per orbit (i.e., 236 deg/hr) about the spacecraft's y-axis while holding the remaining axis rotations near zero. The attitude sensors used in the simulation include a three-axis magnetometer (TAM) and two digital sun sensors (DSSs).

The magnetic field reference is modeled using a 10th order International Geomagnetic Reference Field (IGRF) model. TAM sensor noise is modeled by a Gaussian white-noise process with a mean of zero and a standard deviation of 0.5 mG. The two DSSs each have a field of view of about $50°\times50°$, and combine to provide sun measurements for about 2/3 of a complete orbit. Figure 2 shows the availability of the sun sensor as a function of time. The DSS sensor noise is also modeled by a Gaussian white-noise process with a mean of zero and a standard deviation of 0.05°. The gyro "measurements" are simulated using Equations (16) and (17), with a gyro noise standard deviation of 0.062 deg/hr, a ramp noise standard deviation of 0.235 deg/hr/hr, and an initial drift of 0.1 deg/hr on each axis.

A plot of the estimated MRP for a typical simulation run using the extended Kalman filter is shown in Figure 3. Since, the rotation does not exceed 360° a discrete jump to the origin is not required. A plot of the corresponding gyro-bias estimates is shown in Figure 4. Plots of the attitude covariance and gyro-bias covariances are shown in Figures 5 and 6, respectively. The increase in the attitude covariance (at approximately the second and fifth hour) is due to the fact that the rotation approaches 360° as shown in Figure 7 (i.e., the fourth quaternion component is close to 1). A plot of the roll, pitch, and yaw attitude errors is shown in Figure 8. From these figures, it is clear that the extended Kalman filter developed in this paper is able to accurately estimate the attitude and gyro-biases of the simulated spacecraft, and achieves the same degree of accuracy as the quaternion-based Kalman filter (see [9]).

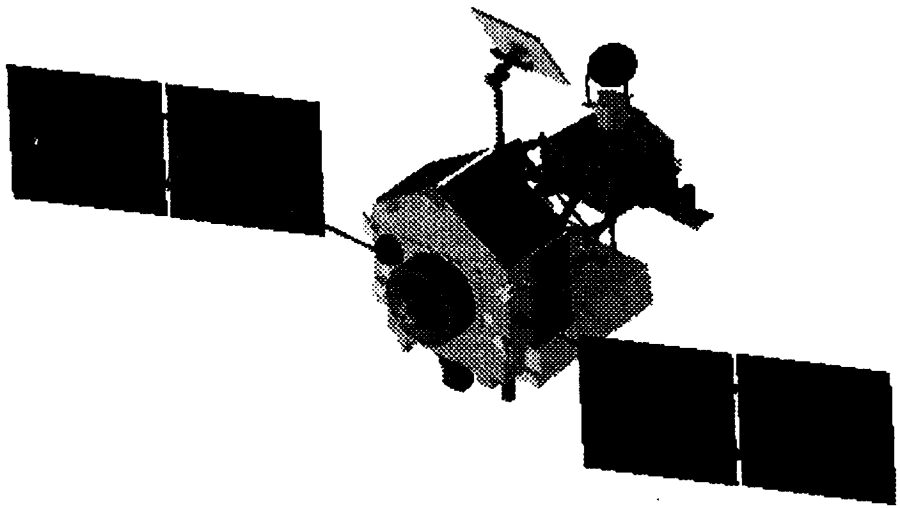

Figure 1 TRMM Spacecraft

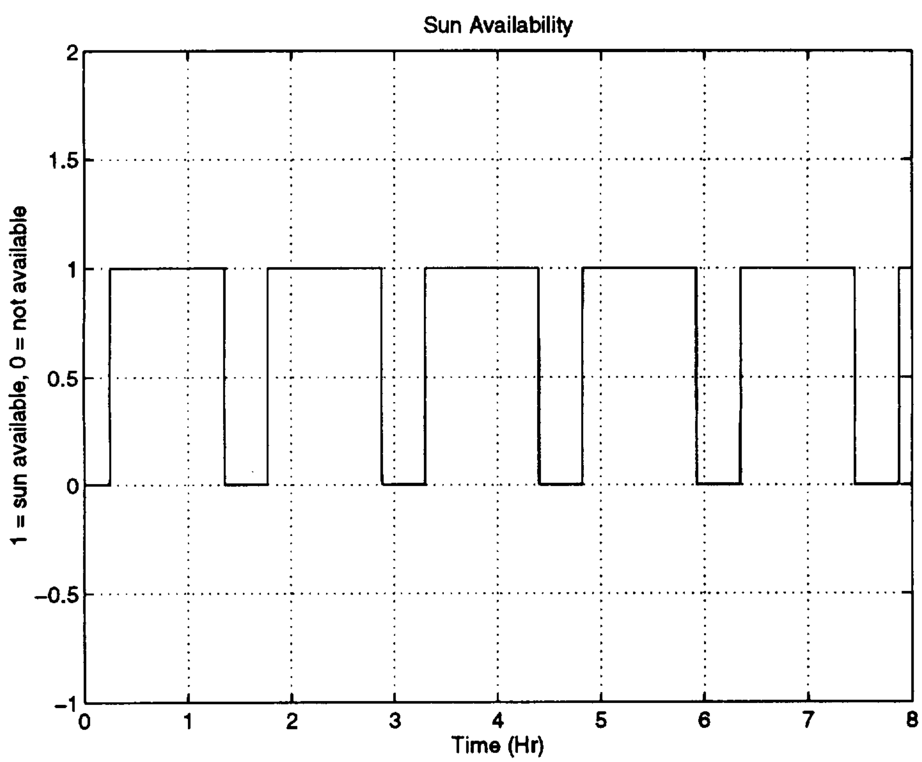

Figure 2 Plot of the Sun Availability

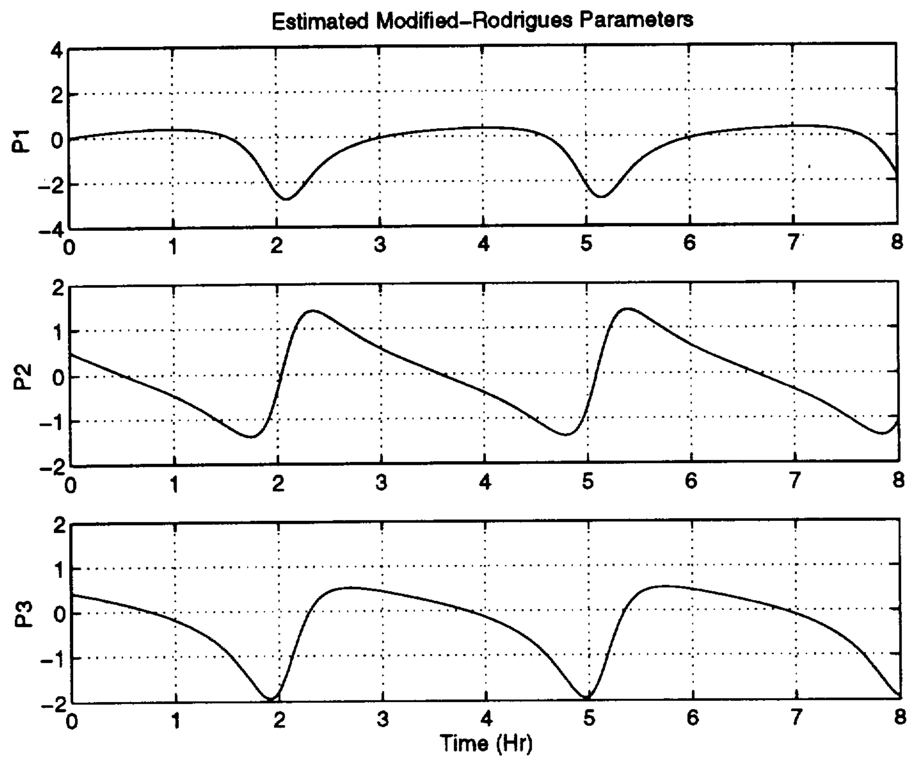

Figure 3 Plot of Kalman Filter MRP Estimates

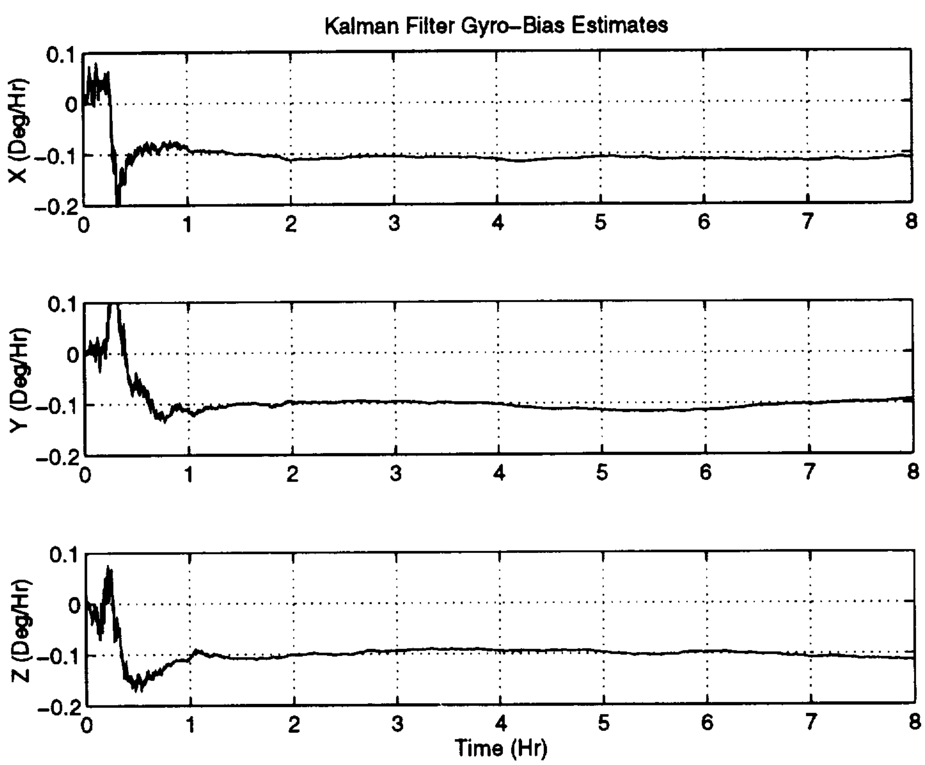

Figure 4 Plot of Kalman Filter Gyro-Bias Estimates

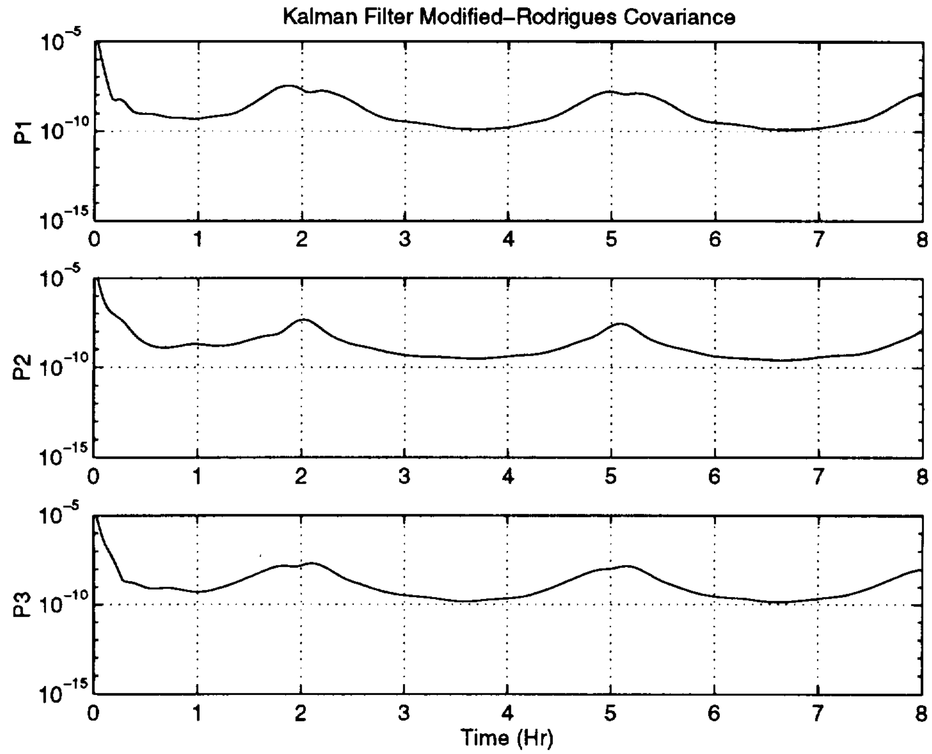

Figure 5 Plot of Attitude Covariances

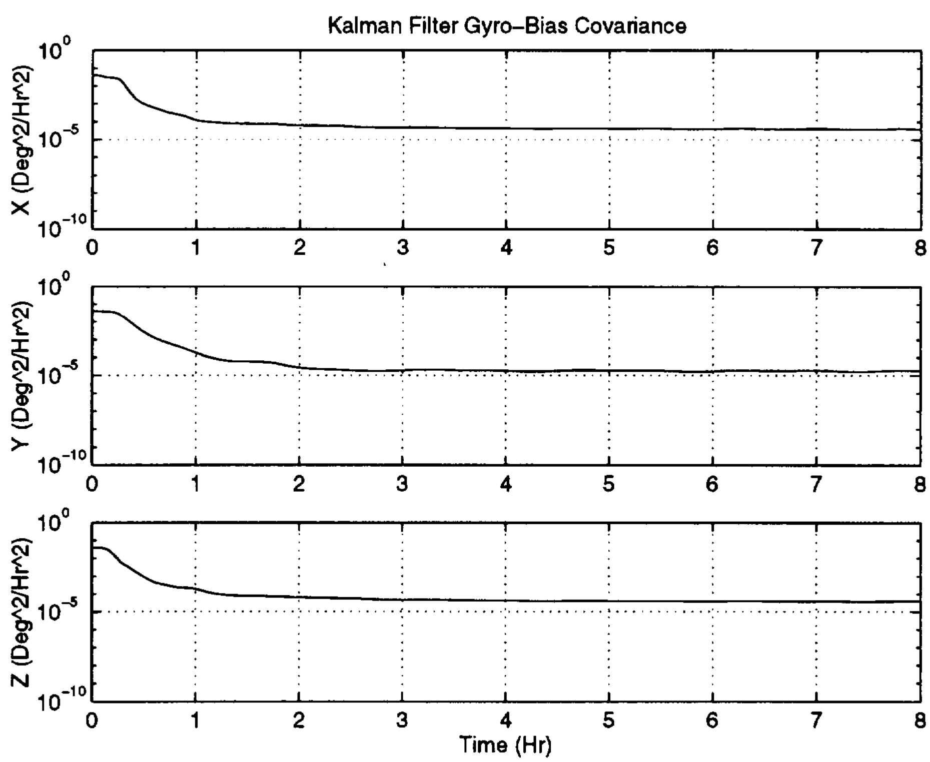

Figure 6 Plot of the Gyro-Bias Covariances

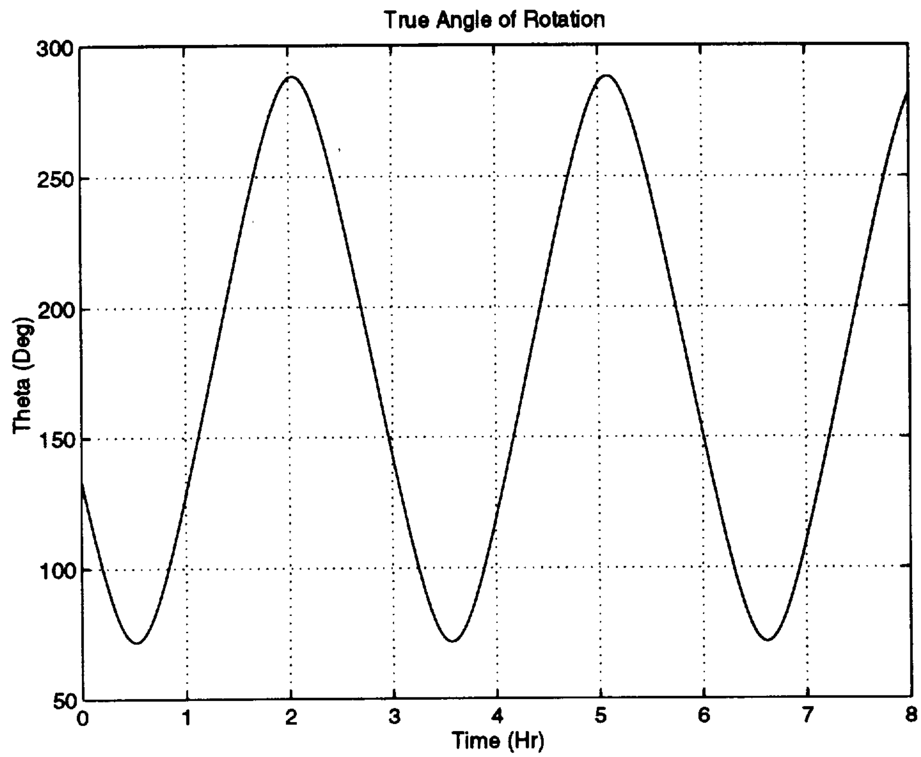

Figure 7 Plot of True Rotation Angle

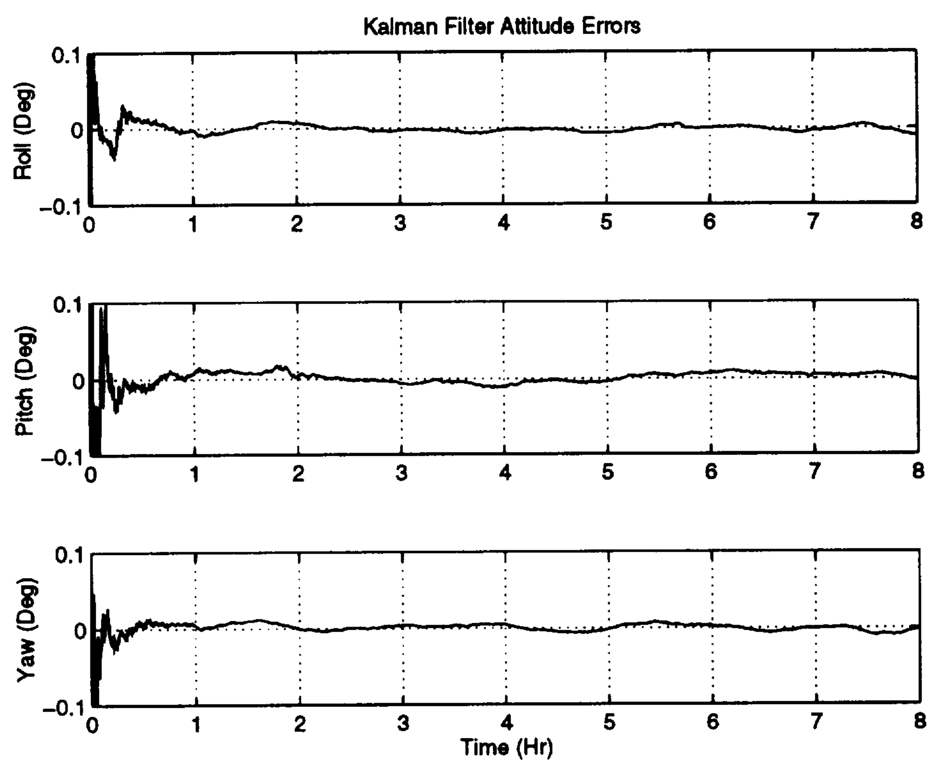

Figure 8 Plot of Attitude Error Trajectories

Conclusions

In this paper, a Kalman filter was developed for attitude estimation using the modified Rodrigues parameters. Conceptually, the computational requirements for the new algorithm are comparable to the quaternion-based Kalman filter. However, the formulation shown in this paper avoided the normalization constraint associated with the quaternion representation. Therefore, methods to maintain a singular covariance matrix using the quaternion representation in the Kalman filter have been eliminated. However, a singularity exists for 360 degree rotations. This may be avoided by allowing for a discrete jump to the origin when the rotation approaches the singularity. Simulation results indicate that the new algorithm was able to accurately estimate for the spacecraft attitude and the gyro-biases.

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References

[1] Lefferts, E.J., Markley, F.L., and Shuster, M.D., "Kalman Filtering for Spacecraft Attitude Estimation," *Journal of Guidance, Control and Dynamics*, Vol. 5, No. 5, Sept.-Oct. 1982, pp. 417-429.

[2] Ramirez, H.S., and Dwyer, T.A.W., "Variable Structure Control of Spacecraft Reorientation Maneuvers," *Proceedings of AIAA Guidance, Navigation, and Control Conference*, Williamsburg, VA, Aug. 1986, AIAA Paper #86-1987, pp. 88-96.

[3] Slotine, J.-J.E., and Di Benedetto, M.D., "Hamiltonian Adaptive Control of Spacecraft," *IEEE Transactions on Automatic Control*, Vol. 35, No. 7, July 1990, pp. 848-852.

[4] Shuster, M.D., "A Survey of Attitude Representations," *The Journal of the Astronautical Sciences*, Vol. 41, No. 4, Oct.-Dec. 1993, pp. 439-517.

[5] Marandi, S.R., and Modi, V.J., "A Preferred Coordinate System and Associated Orientation Representation in Attitude Dynamics," *Acta Astronautica*, Vol. 15, No. 11, Nov. 1987, pp. 833-843.

[6] Tsiotras, P. "Recent Results on the Attitude Control Problem," *Journal of Guidance, Control and Dynamics*, in review.

[7] Schaub, H., Junkins, J.L., and Robinett, R.D., "Adaptive External Torque Estimation by Means of Tracking a Lyapunov Function," *Proceedings of the AAS/AIAA Space Flight Mechanics Meeting*, Austin, TX, Feb. 1996, AAS Paper #96-172.

[8] Crassidis, J.L., and Markley, F.L., "Sliding Mode Control Using Modified Rodrigues Parameters," *Journal of Guidance, Control and Dynamics*, in review.

[9] Crassidis, J.L., Andrews, S.F., Markley, F.L., and Ha, K., "Contingency Designs for Attitude Determination of TRMM," *Proceedings of the Flight Mechanics/Estimation Theory Symposium*, NASA-Goddard Space Flight Center, Greenbelt, MD, 1995, pp. 419-433.