Investigation of Models and Estimation Techniques for GPS Attitude Determination

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ABSTRACT

Much work has been done in the Flight Dynamics Analysis Branch (FDAB) in developing algorithms to meet the new and growing field of attitude determination using the Global Positioning System (GPS) constellation of satellites. Flight Dynamics has the responsibility to investigate any new technology and incorporate the innovations in the attitude ground support systems developed to support future missions. The work presented here is an investigative analysis that will produce the needed adaptation to allow the Flight Dynamics Support System (FDSS) to ingest GPS phase measurements and produce observation measurements compatible with the FDSS.

A simulator was developed to produce the necessary measurement data to test the models developed for the different estimation techniques used by Flight Dynamics. This paper will give an overview of the current modeling capabilities of the simulator, models and algorithms for the adaptation of GPS measurement data, and results from each of the estimation techniques. The paper will also outline future analysis efforts to evaluate the simulator and models against inflight GPS measurement data.

Background

Originally the GPS constellation was conceived to produce accurate position information for ground, air and space based systems. This information would be available to anyone who possessed a GPS receiver, on a continuous basis. With the advance of technology that produced low cost and lightweight receivers, arose a new application of the GPS constellation; attitude determination. It was discovered that with a pair of GPS antenna a user can determine a phase difference between like signals of that antenna pair. This process is commonly known as the interferometric principle, and has been used in the Minitrack system in the early days of space flight orbit determination. This principle is illustrated by Figure 1 below, which shows the relationship between wavelength (a function of phase difference) and the wavefront angle.

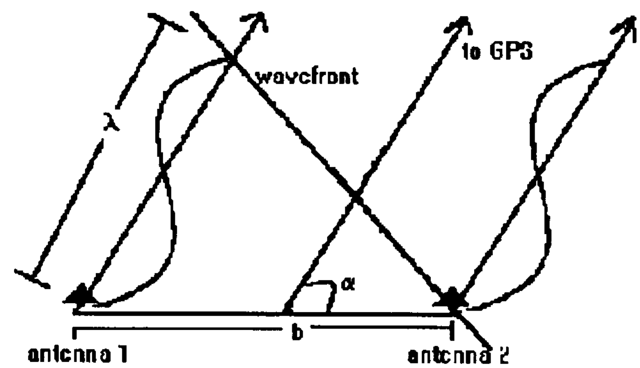

Figure 1. Baseline phase/angle relationship

As a center of expertise for attitude determination and calibration, the FDD began to investigate this new technology to determine it's capabilities. This investigation begins with a fundamental equation which governs the phase difference computation. The fundamental equation can be determined from Figure 1 and is given by:

$$\cos \alpha = (n + k\phi) (\lambda / b) \qquad \text{(Equation 1)}$$

where

- α is the angle between the baseline and line of sight to the GPS spacecraft
- n is the integer number of cycles in the phase difference between receivers
- ϕ is the decimal part of the phase difference received from the GPS signal
- k is a scale factor which depends on ϕ's units
- λ is the wavelength of the GPS signal (GPS has two frequencies,
  L1 at 1575.42 MHz., and
  L2 at 1227.6 MHz.
  The wavelengths are
  0.19042541 meters and 0.24437928 meters, respectively)
- b is the baseline length

If we were to rewrite equation 1 as

$$n + k\phi = (b/\lambda)\cos\alpha \quad \text{(Equation 2)}$$

we can determine the integer limits for a given baseline length. To see this let the baseline length be 1 meter, which is what is used for all analysis presented here. Then let $\alpha = 0$ and use the L1 frequency, for which $\lambda = 0.1904$ meters. Solving this equation we get $n + k\phi = 5.25$. So we know as the GPS spacecraft enters the field of view and traverses from 0 to 180 degrees, then the integer component of the phase difference, in units of wavelengths, will range from +5 to -5.

If we again rewrite equation 1 as

$$\alpha = \mathrm{acos}[(n + k\phi)((\lambda / b)] \quad \text{(Equation 3)}$$

let $\phi = 0$, and let n range from + 5 to -5, we can create a table of angle ranges for each integer, again based on a 1 meter baseline. Figure 2 gives the table of angle ranges for the 1 meter baseline.

Angular Range (deg)	Integer Part of Phase
180.00 - 162.20	-5
162.19 - 139.61	-4
139.60 - 124.84	-3
124.83 - 112.38	-2
112.37 - 100.98	-1
100.97 - 90.01	-0
90.00 - 79.02	+0
79.01 - 67.61	1
67.60 - 55.16	2
55.15 - 40.38	3
40.37 - 17.80	4
17.79 - 0.00	5

Figure 2. Angular Range for a 1 meter baseline

From this range table we can determine how the phase difference would look like as it ranges through the field of view of the baseline sensor. The measured phase difference is determined by comparing in the electronics the signals from both antennae of a baseline and shifting on until both signals are in phase. Thus the most that can be detected is just under one wavelength difference. This produces a plot that looks like Figure 3 for the 1 meter baseline.

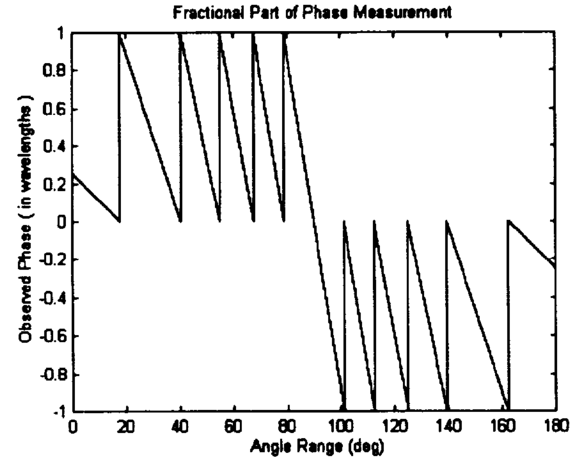

Figure 3. Phase measurement for 1 m. baseline

The lack of integer information is the well documented problem of integer ambiguity. There are several methods that can be used for the initial determination of the integer values. The most straightforward method involves a search method over the integer values using the table in Figure 2 to fit the visible GPS observations to the correct integer. This can be used on the ground for off-line processing because of the high power computers and the fact that the process is not a real-time process. After the initial integers are determined, then the phase difference measurement can be monitored to track when the integer value should change, as is illustrated by Figure 3. Other methods will be discussed when we talk about the extended Kalman Filter later.

Still this is only one bit of the information needed to compute the desired observation vector. FD ground attitude determination software makes use of time tagged observation vectors in BCS and reference vectors in GCI to determine the attitude solution. With the use of another baseline, preferably orthogonal to the first, the line of sight vector from the user spacecraft to a GPS space vehicle (SV) can be determined.

Knowing this cosine of the angle and that from another baseline, it is possible to determine the observation vector of the visible GPS SV. The angle determined by one of the baselines describes a cone around the baseline vector and likewise for the second baseline. Where the two cones intersect (see Figure 4) are the two possible solutions. Knowing the normal vector to the two baseline's plane can reconcile which is the true solution. Paired with a known reference vector of the GPS SV at that time, the analyst can determine the attitude using several well known and established attitude estimation techniques employed within the FDOA.

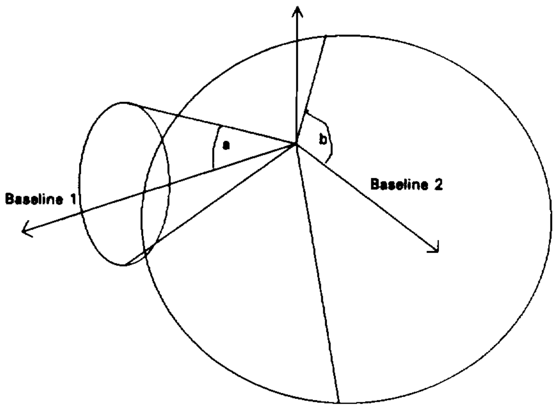

Figure 4. Observation vector resolution

Figure 4 shows the geometry of the two orthogonal baselines and the intersection of the two cones. Using Equation 1 we can relate the direction cosines to the phase differences as

$$\cos\alpha = (n_1 + k\phi_1) (\lambda/b) \qquad \text{(Equation 1a)}$$

$$\cos\beta = (n_2 + k\phi_2) (\lambda/b) \qquad \text{(Equation 1b)}$$

$$\cos\gamma = [1 -\cos^2\alpha - \cos^2\beta]^{1/2} \qquad \text{(Equation 4)}$$

These define a unit vector in the receiver coordinate system defined by the two orthogonal receiver baselines fixed in the spacecraft and, therefore, the body coordinate system frame. That is

$$x_B = [M]\ x_R$$

where $x_R = [x_r\ y_r\ z_r]$ transposed, the observation vector in receiver coordinates

x_B is the observation vector in BCS

[M] is the transformation matrix from the receiver to BCS

and

$$x_r = \cos\alpha$$
$$y_r = \cos\beta$$
$$z_r = \cos\gamma$$

Error Sources

If the integer ambiguity in Equation 1 were the only parameter that needed to be computed, then the matter of attitude determination would be straightforward and no calibration would be necessary. However, as all engineers know there is some uncertainty in every measurement taken, and it's these uncertainties that need to be characterized and/or compensated for. Figure 5 shows a graphical representation of the difference between the observed GPS measurement and what is the truth.

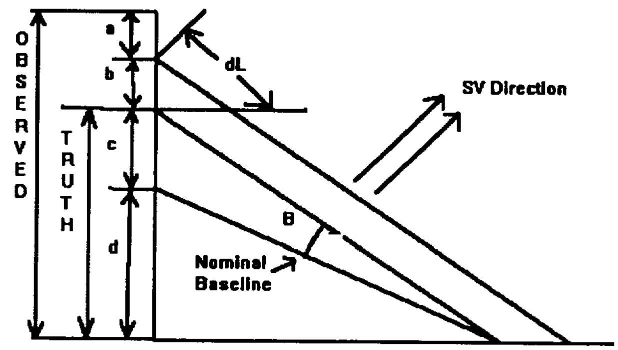

Figure 5. Components of Observed and Truth

It can be seen that the true measurement is the nominal quantity d, that is what you would expect if the system were perfect, added with a quantity associated with any misalignments. What is observed are the additional two components, a and b. The component a is a bias associated with the electronics and is different for every GPS antenna. It represents a time bias in the system. The component b is associated with the unknown length of the baseline. Although these two parameters can be measured quite accurately here on the ground (self survey mode), when in space the thermal and other environmental perturbations effects change the known values. Likewise the alignment of the sensors can be determined very accurately before launch, but the vibrations do to launch shock will result in some displacement. This may necessitate a postlaunch calibration to determine alignment and or placement of each antenna. These additional parameters change the fundamental equation to:

$$\cos\alpha = (n + k\phi + \text{noise} + \text{line bias})(\lambda / [M](b + db)) \qquad \text{(Equation 5)}$$

In this equation only the noise cannot be determined as a systematic error and taken out by determining the correct compensation.

Multipath

Both solutions also lack the modeling of multipath, which can be a large source of error. Multipath is essentially the reflection of a GPS SV signal off a surface on the spacecraft and received basically as an echo of the original signal. The echo obviously has the same identification as the original signal but has a different phase shift of the wavelength, giving erroneous measurements if it were not weeded out as the false signal. Spacecraft engineers can greatly reduce this source of error by mounting the antennas on booms away from the main body, or flush with spacecraft surfaces and strategically placed to reduce signal reflection.

Prediction Utility

In order to enhance the analysis process of GPS attitude determination algorithms and techniques for specific missions, it was necessary to produce a tool that would give accurate predictions for the GPS constellation as viewed by the user spacecraft. The utility was developed as an analysis tool on an IBM compatible PC using Microsoft FORTRAN and executing under the DOS operating system. The prediction tool allows the user to input parameters to fit the simulation. The setup used for all analysis and predictions is:

Earth pointing mission (+Z BCS is nadir)
Semi-major axis = 6728.83 km
Altitude = 350.8 km
Eccentricity = 0.001
Inclination = 28.5 deg.
RA of Asc. Node = 90.0 deg.
Mean Anomaly = 0.0 deg.
Arg. of Perigee = 0.0 deg.

The boresight of each antenna point in the anti-nadir direction, or in vector form

boresight vector = [0.0 0.0 -1.0] transpose, BCS

The predictions and analysis are all done at a step size of 10 seconds and a total simulation/prediction time of 1000 steps. This is about 1.6 orbits.

Internally the utility models the 24 GPS SV constellation by storing their Keplerian elements and epoch time, and using a simple two-body propagator. The simulation produces two kinds of ouput data. The first is a time ordered history file of GPS spacecraft visible to the user's antenna baseline and the second produces a statistical analysis of the simulation. The statistics and parameters outputted are:

- report of simulation user supplied input parameters selected
- Acquisition and loss of signal for each GPS SV based on line of sight and beam width mask
- total time each GPS spacecraft is visible to the antenna baseline
- percentage of simulation time that each GPS spacecraft is visible to the antenna baseline
- the total number of GPS observations
- density distribution of GPS spacecraft (count of how many times n number of spacecraft are visible to the antenna baseline at any simulation step)
- maximum and minimum amount of continuous time for each event of the density distribution described above
- maximum and minimum continuous visibility time for each GPS spacecraft.

The two output datasets are written to DOS ASCII files (alphanumeric, readable format) and can be edited and printed. The data can easily be input to a plotting package for a more graphical representation. Figures 6 and 7 give two examples of the statistics for a half cone angle for each antenna of 90 degrees. This showed that the antenna baseline system would see a total of 9024 GPS observations.

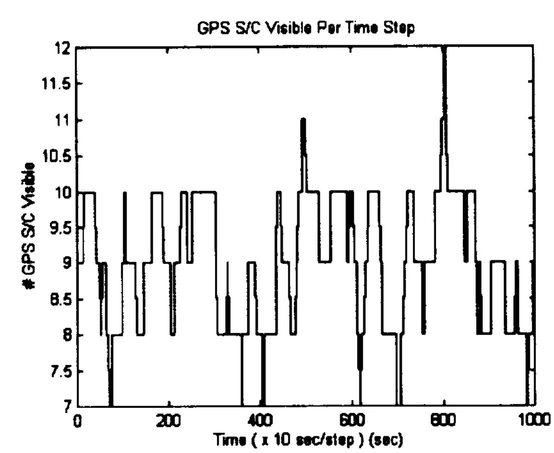

Figure 6. Distribution for half cone of 90 deg.

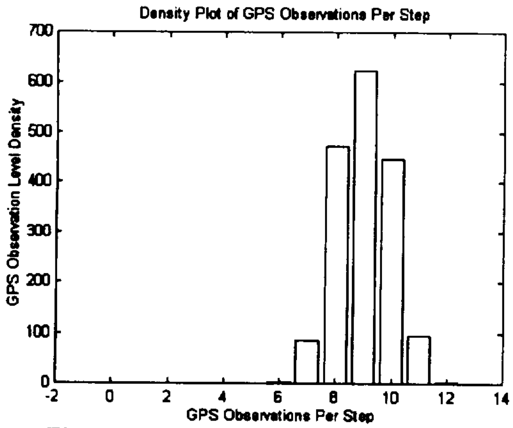

Figure 7. Density for half cone of 90 deg.

Repeating this same test case setup, except we will use only the main lobes of the antenna pattern, which changes the half cone angle for each antenna to be 32 degrees. Figures 8 and 9 show the same distribution and density plots for this setup. The total number of GPS observations in this case is considerably less, 920 observations.

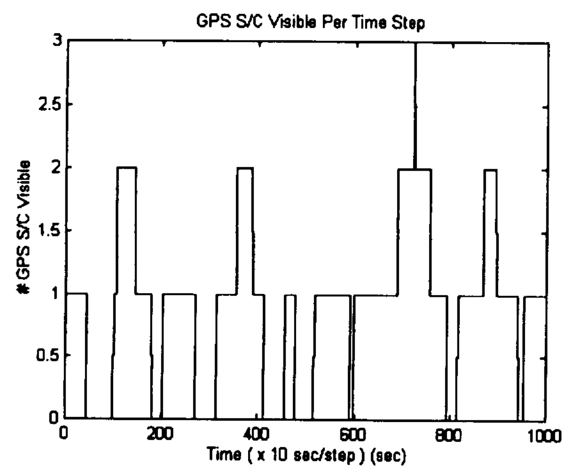

Figure 8. Distribution for half cone of 32 deg.

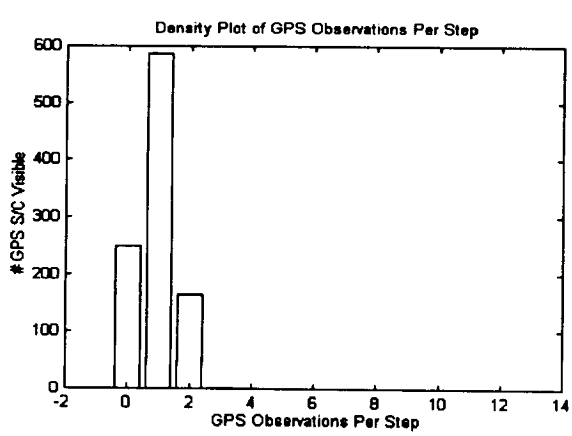

Figure 9. Density for half cone of 32 deg.

Estimation Simulator

The uncertainties in the phase measurements make it necessary to employ estimation techniques to determine attitude and/or each of the parameters listed in the error budget above. To this end an estimation simulator was developed to investigate new algorithms, and to test the GPS attitude determination capabilities. The simulator essentially models a given spacecraft's ephemeris and dynamics, and uses the above equations to produce the observed phase difference. The processing of the phase difference employs a selection algorithm and methods for resolving the integer ambiguity (several methods have been examined for this simulator).

A couple of methods were used successfully, but the method that was used for this analysis involves using the tables of integers and angle ranges generated earlier for a one meter baseline. The method is simply a search through all the possible integer values (for a one meter baseline there are only 11, values from -5 to +5) and matching the angular separation to within some tolerance using the angular separation of the reference vectors after they have been transformed to the nominal BCS coordinate frame. This can be done at every time point or once the initial integers are found they can be updated by monitoring the change in phase measurements. The first method is a good way to go for non real-time estimation. It simply is easier to implement. But for a real-time attitude estimation where computer time is at a premium, it is more efficient to initialize the integers and then monitor for changes. Once the integer phase has been determined it is simple to compute the observation vectors, as defined by equations listed earlier. The resulting observation vectors are paired with reference vectors and form the input data for an extended Kalman Filter and a single frame estimator. The simulator provides a means to vary modeling and algorithms to investigate the affect.

Figure 10 gives a plot of what the simulated true attitude is for the test case scenario. The scenario is an earth pointing 1 rotation per orbit (RPO) spacecraft. The dynamics also has a small noise characteristic which produces the small amount of jitter in the plot. This is probably a very smooth case as compared to actual spacecraft attitude behavior, but it serves as a basis for further analysis.

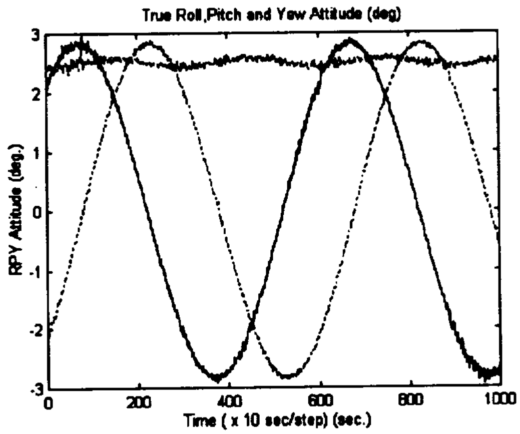

Figure 10. Plot of True RPY Attitude

Figure 11 illustrates what the simulator would ouput for a GPS SV that transverses the entire angle range from 0 to 180 degrees with the addition of attitude errors and noise. This plot has a measurment noise of 0.1 wavelengths or about 2 cm. It demonstrates some things that need to be considered for when monitoring of phase changes is used for updating the integers. First as can be seen the phase difference will change integer values without getting close to 1.0 or 0.0 because of the noise. This has to be considered, as the wrong choice of the integer can add an error as much as 18.0 degrees in the observation vector computation. What is not seen here but does happen is sometimes the integer oscillates between two integers for a brief time before moving on. This has to do with attitude motion as much as the noise.

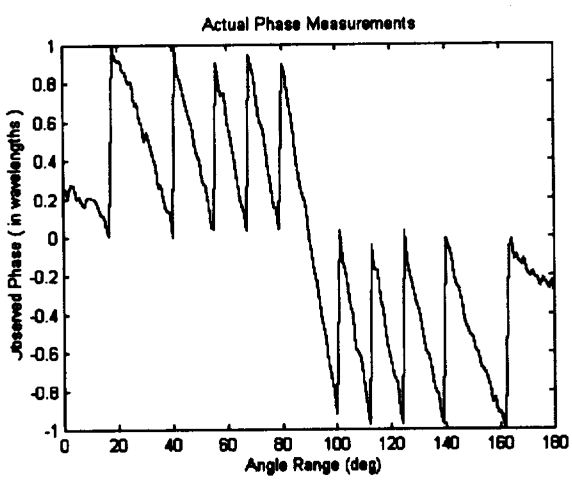

Figure 11. Plot of Actual Measured Phase

The estimation techniques that are used in this paper, an extended Kalman Filter and a single frame estimator (QUEST), will look at two cases which represents the best and worse case scenarios as far as noise on the phase measurements. They are the 0.1 wavelength (2 cm) case and the 0.01 wavelength (0.2 cm) case. Estimates have been made that the measurement noise can be reduced to about 0.5 cm. (reference 2). Thus the use of 0.2 cm and 2.0 cm. certainly represents the best and worse case scenarios. For both cases it is assumed that the the location of the antennas and the time bias have been determined so as not to affect the solution. Both cases also use the a hemispherical antenna pattern, which is to say a half cone of 90 degrees for the antenna field of view. In actual use the half cone of 32 degrees may be used because of the higher noise characteristics for observations in the higher angle region, or the side lobes of the antenna pattern. The affect of signal to noise ratio on observation depending on their location in the main or side lobes will be investigated in subsequent analysis. The worse case scenario will use the higher noise characteristic, but will apply it to all observations. Thus the expected in-flight accuracy will be somewhere between the worse and best case scenarios.

Estimation Models

The first estimation technique is the extended Kalman Filter. Originally a basic Kalman Filter was used and produced good results. However after implementing an extended Kalman Filter the results were much improved. This simply has to do with adding some knowledge to the system about the expected trajectory. This additional knowledge simply evaluates the measurement matrix and the dynamics, or state transition, matrix based upon the last estimate of the state. In the case of the extended Kalman Filter the state consists of errors or deltas away from the a priori attitude at each step. Once an estimate of the error at a time step is made then the attitude error is updated based on the state deltas, the measurement and state dynamics matrices are recomputed using this new updated state and the filter is reset for the next time step. The math specifications for equations that are specific to the extended Kalman Filter are:

$$\Delta\dot{x}(t) = \left[\frac{\partial f}{\partial x}\right]_{x=x^*} \cdot \Delta x(t) + u(t)$$

$$[z - h(x^*,t)] = \left[\frac{\partial h}{\partial x}\right]_{x=x^*} \cdot \Delta x(t) + v(t)$$

Trajectories are evaluated along current estimate of the updated attitude error. This is found by taking the deltas at this time step and adding them to the previous error estimate, or

$$x_i = x_{i-1} + \Delta x_i$$

The reader is directed to reference 3 for a detailed discussion on the extended Kalman filter.

Figures 12, 13 and 14 show the attitude error (true minus the estimated attitude) and the statistics for the worse case scenario. It has a lot of structure to the plot, but a upper and lower bound is around 0.5 degrees.

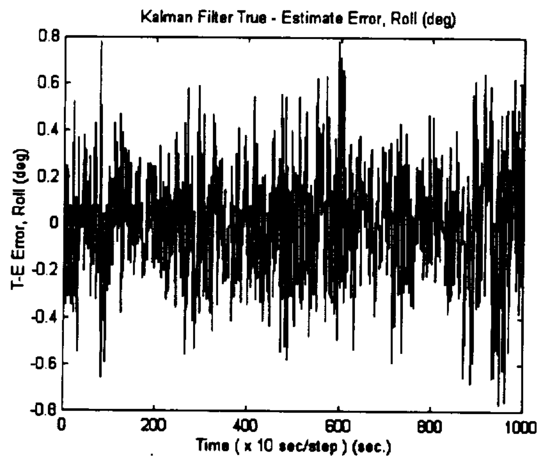

Figure 12. Roll Error for worse case

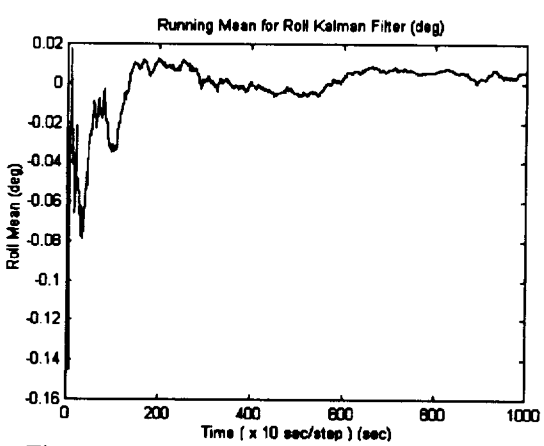

Figure 13. Roll running mean for worse case

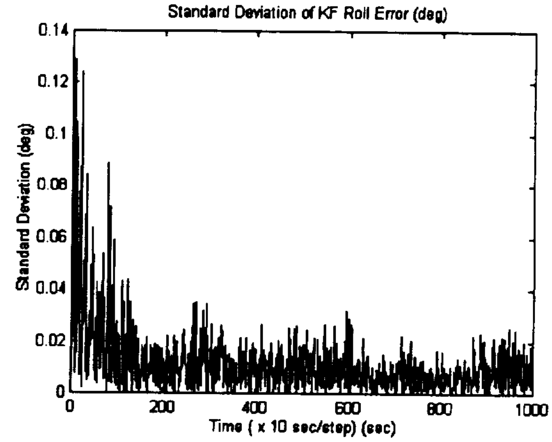

Figure 14. Roll running std. dev. for worse case

Figures 15, 16 and 17 show the results for the best case scenario. Figure 15 shows a bound around 0.1 degrees.

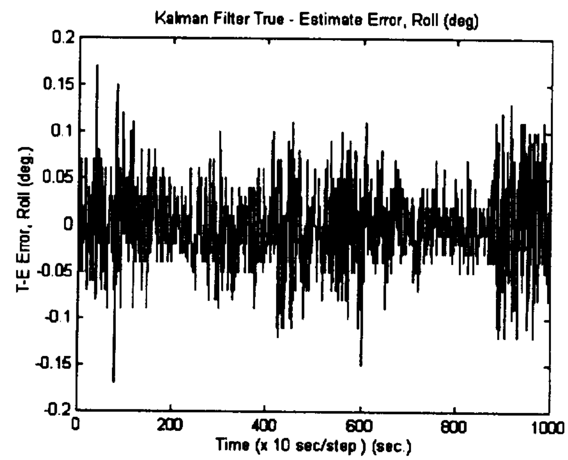

Figure 15. Roll Error for best case

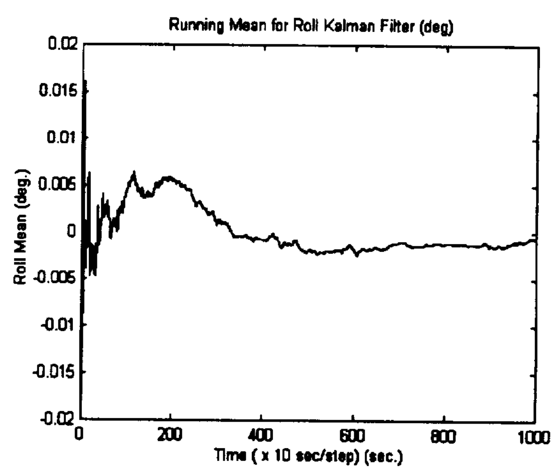

Figure 16. Roll running mean for best case

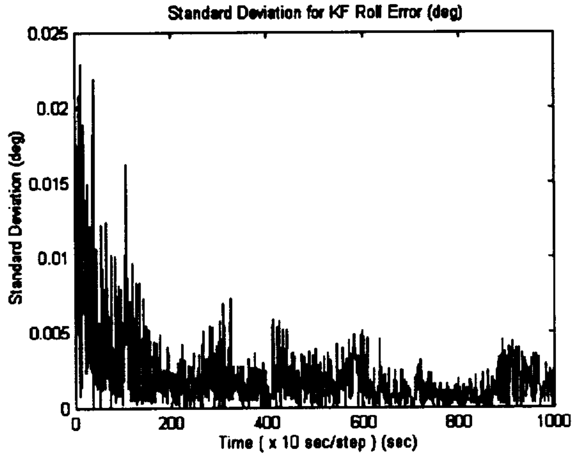

Figure 17. Roll running std. dev. for best case

It is obvious that both show convergence. The best interpretation for this case study is that one can expect to achieve somewhere between 0.2 degrees and 0.5 degrees accuracy depending on the measurement noise and how accurately the noise is compensated for in the filter. In both of these cases perfect knowledge of the measurement noise characteristics was known and compensated for in the filter's state measurement noise covariance matrix.

Single Frame Estimation (QUEST)

The QUEST modeling likewise has demonstrated that a less sophisticated method can still achieve accuracies of less than one degree using GPS measurement data. The reader is referred to another source (see reference 1) for a detailed description of the QUEST attitude determination algorithm. Here again the method for determining the integer ambiguity was the search method employed by extended Kalman Filter. The monitoring method was examined also and produced the same results, however it was more convenient to compute the integers again at each step and without a timing constraint it presented no problems.

A unique problem exists for the single frame solution that the filter does not have, simply because it processes one observation at a time. The single frame method, however, needs at least two observation vectors to determine an attitude and as a further constraint they must not be collinear. The best solution would be to find three observation vectors that are orthogonal to each other, or as close to this configuration as possible. This describes the geometric selection problem for the single frame solution.

Three test cases were run to demonstrate the importance of employing a selection scheme. Figure 18 shows the results of the case where all observation vectors were used. Figure 19 shows the case where the first four observation vectors were used. And Figure 20 shows the results when the selection algorithm was employed. In all three cases the best case scenario was used, which translates to almost having perfect knowledge of the system.

The selection algorithm used is based on the statement made earlier of finding three observation vectors (actually using the reference vectors) that as close as possible form an orthogonal triad. This is simply done by looking at all combinations of three observation vectors and use the group that has the smallest sum of the dot products. Assuming that a maximum of 12 observations are visible at any one time, and choosing three at a time from this, there are 220 groupings to search. This isn't bad and in fact takes very little time because of the simplicity of the algorithm.

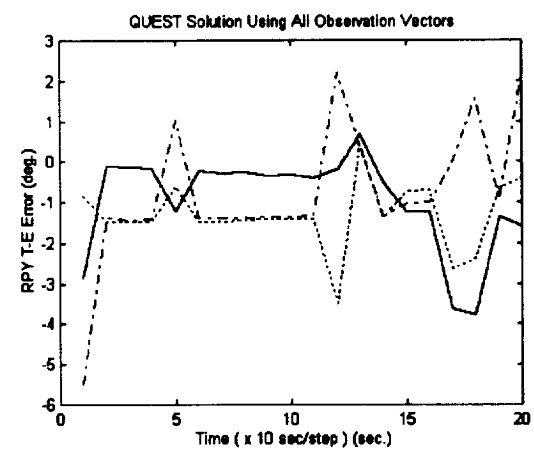

Figure 18. Single Frame Error; all observations

It's obvious that the selection algorithm produces the best solution, with using any four (in this case the first four) being the second best method. The reasoning behind this can be interpreted as being over observed. That is the other observations add more uncertainty to the estimation. The residual spikes, at this time, have no resolution, with the data and estimation algorithm having been verified for correctness.

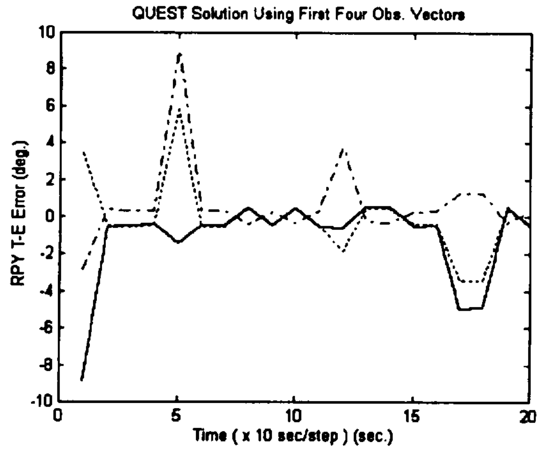

Figure 19. Single Frame Error; first 4 observe.

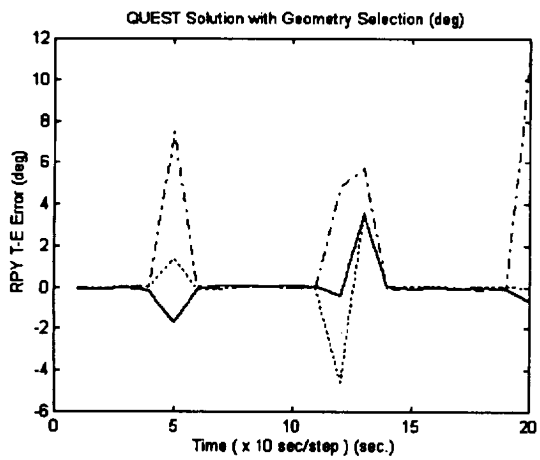

Figure 20. Single Frame Error; with selection

One more case was run for the single frame estimation method. This was using the geometry selection algorithm, but using the worse case scenario. Figure 21 illustrates the results.

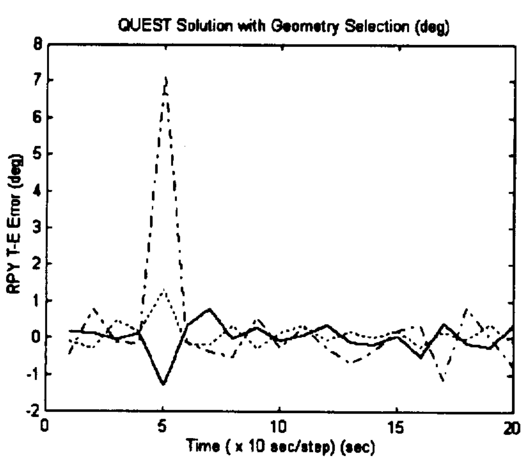

Figure 21. Single Frame Error, worse case

From this analysis of the worse case and best case, and using a selection algorithm, the QUEST method for attitude determination can produce an accuracy between 0.5 and 1.0 degrees.

Future Analysis

There are many future items to be implemented and considered connected with this analysis. They are listed below according to function or system.

Estimation Simulator Enhancements

- Expansion of user input parameters such as:
  1) allow varying of baseline length
  2) allow varying placement of antennas
  3) allow varying number of antennas
  4) model boom and uncertainties due to deflection of boom
  5) model main and side lobes in antenna pattern for differing noise characteristics
  6) implement P-code for investigation of a more accurate measurement

Extended KF

- extend state to include gyro and/or antenna biases
- add misalignments to state for calibration

Single frame solutions

- continue to look at geometric considerations and selection process
- look at REQUEST implementation

Processing of Actual Inflight GPS Data

- have acquired Crista-SPAS data
- looking to use Spartan/GADACS data

Conclusions

The Kalman Filter has demonstrated that it is possible to get better than 0.5 degrees per axis in determining attitude for a one meter baseline. And likewise it is possible to get better than one degree from a single frame attitude solution using a geometry selection algorithm. All of these analyses were done with a 90 degree half cone angle field of view for each antenna, that is both main and side lobes of the antenna pattern. Further analysis needs to be done using only the main lobe and analysis which uses both lobes but implements a better noise characterization based on the angle from the antenna boresight. And of course the processing of GPS phase measurements from on-orbit spacecraft will be done to validate the algorithms used so far.

The main purppose of this paper is to demonstrate that the GPS phase measurements can be adapted to the existing ground attitude determination software. With the use of the cones method for resolving the line of sight vector to the observed GPS SV and also with new methods for the integer ambiguity resolution it is definitely possible to use the existing method of processing time tagged observation and reference vector pairs.

Although the field of attitude determination using GPS is still young, this study has shown that it is possible to adapt the GPS measurements to the existing design of FD ground attitude determination systems. Still, there is much yet to be done for future analysis in order for GPS to be routinely accepted as an alternative to more expensive sensor configurations.

<u>References</u>

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2. C. Cohen, & B. Parkinson; "Expanding the Performance Envelope of GPS Based Attitude Determination"; Proc. Int. Tech. Mtg., ION GPS, Albuquerque, NM, Sept. 1991.

3. A. Gelb, (editor); " Applied Optimal Estimation"; M.I.T. Press; 1989