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Accuracy Studies of a Magnetometer-Only Attitude-and-Rate-Determination System*

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Abstract

A personal computer-based system was recently prototyped that uses measurements from a three-axis magnetometer (TAM) to estimate the attitude and rates of a spacecraft using no *a priori* knowledge of the spacecraft's state. Past studies using in-flight data from the Solar, Anomalous, and Magnetospheric Particles Explorer focused on the robustness of the system and demonstrated that attitude and rate estimates could be obtained accurately to 1.5 degrees (deg) and 0.01 deg per second (deg/sec), respectively, despite limitations in the data and in the accuracies of the truth models. This paper studies the accuracy of the Kalman filter in the system using several orbits of in-flight Earth Radiation Budget Satellite (ERBS) data and attitude and rate truth models obtained from high-precision sensors to demonstrate the practical capabilities.

This paper shows the following:

- Using telemetered TAM data, attitude accuracies of 0.2 to 0.4 deg and rate accuracies of 0.002 to 0.005 deg/sec (within ERBS attitude control requirements of 1 deg and 0.005 deg/sec) can be obtained with minimal tuning of the filter.
- Replacing the TAM data in the telemetry with simulated TAM data yields corresponding accuracies of 0.1 to 0.2 deg and 0.002 to 0.005 deg/sec, thus demonstrating that the filter's accuracy can be significantly enhanced by further calibrating the TAM.

Factors affecting the filter's accuracy and techniques for tuning the system's Kalman filter are also presented in this paper.

1. Introduction

Contingency algorithms for determining the attitude and rates of a spacecraft using only a three-axis magnetometer (TAM) and using no *a priori* knowledge of the spacecraft's state have been of interest recently in view of situations such as the control anomaly on the Earth Radiation Budget Satellite (ERBS) in 1987 (Reference 1) and the control system failure on the Relay Mirror Experiment (RME) satellite (Reference 2). It has recently been shown (Reference 3) using in-flight data from the Solar, Anomalous, and Magnetospheric Particles Explorer (SAMPEX) mission that an optimal combination of two different algorithms—the Deterministic Attitude Determination From Magnetometer-Only Data (DADMOD) (Reference 2) and the Real-Time Sequential Filter (RTSF) (References 4 and 5)—solves the problem by maintaining both accuracy and robustness. The concept was subsequently automated by prototyping a personal computer (PC)-based system (Reference 6) referred to here as TAMONLY for the Flight Dynamics Division (FDD) at the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center (GSFC).

The working principle of TAMONLY involves using DADMOD to obtain coarse estimates of the epoch state of the spacecraft and then using this state as the initial state for the RTSF's finer estimates. The previous study of TAMONLY (Reference 6) involved extensive tests using a variety of SAMPEX in-flight data and demonstrated that accuracies of 1.5 degrees (deg) in attitude and 0.01 deg per second (deg/sec) in spacecraft rates could be obtained.

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Note that these accuracies are within the attitude requirements for SAMPEX (2 deg). These accuracy estimates, however, are only coarse because of the following factors:

- The most accurate sensor onboard SAMPEX is the Sun sensor, which is accurate to 0.5 deg.
- SAMPEX does not have gyros so that the truth model for the rates was obtained by differentiating the attitude truth model data.
- Short spans of data (< 3000 sec) were used.
- The TAM was not calibrated.
- The TAM is used to determine the pitch angle of the attitude truth model.

TAMONLY was also used to study the above-mentioned ERBS control anomaly, and similar accuracies were obtained (Reference 7). Here again, accurate truth models were absent and only coarse accuracy estimates could be obtained during periods when gyro data were unsaturated in the B-Dot control mode of the spacecraft.

This paper attempts to remedy the situation by focusing on accuracy evaluation of the RTSF in TAMONLY using ERBS in-flight data. The data are from a period when the spacecraft was in its nominal mission mode soon after its launch in 1984; during this period, accurate truth models are available for the attitude via Earth-sensor and fine-Sun-sensor measurements, and accurate gyro data provide the truth model for the rates.

Section 2 of this paper describes the Kalman filter, and Section 3 describes ERBS and the characteristics of the data used in this study. A discussion of the criteria used to evaluate the RTSF is given in Section 4. The results obtained using telemetered TAM data and the results obtained using simulated TAM data are given in Sections 5 and 6, respectively. Section 7 presents a summary of the conclusions of the study.

2. Description of the Kalman Filter

In view of space considerations, only details relevant to the tuning of the RTSF are presented here. A full mathematical description of the RTSF has been provided elsewhere (References 4 and 5).

The RTSF's state vector $\vec{X}$ comprises of the four components of the attitude quaternion, $\vec{q}$, and the corrections, $\vec{b}$, to the spacecraft's rates, $\vec{\omega}$:

$$\vec{X} = \left[\vec{q}^{\,T} \ \vec{b}^{\,T}\right]^T \tag{1}$$

(Note that the components of $\vec{b}$ and $\vec{\omega}$ are resolved along the spacecraft's x, y, z axes.)

The RTSF uses sensor data to estimate $\vec{q}$ as well as $\vec{b}$, with $\vec{b}$ being estimated kinematically in the same manner as gyro biases for a gyro-based spacecraft; i.e., by attributing differences between the measured and propagated attitudes to errors in $\vec{\omega}$. The $\vec{b}$ estimates are then used to correct $\vec{\omega}$, and these corrected rates are used as initial conditions to propagate Euler's equation to the next measurement time. The propagation of $\vec{b}$ is modeled via a first-order Markov model:

$$\frac{d\vec{b}}{dt} = -\frac{\vec{b}}{\tau} + \vec{\eta}_b \tag{2}$$

where $\vec{\eta}_b$ is a white noise term, and τ is a finite time constant. A suitable value for τ is the time between measurements –5 seconds for the SAMPEX data and 16 seconds for the ERBS data used here. (In contrast, the same model, when used for gyro bias estimation, requires a τ of several hours.)

The rates are assumed to contain a white noise component, $\vec{\eta}_a$, and are propagated using Euler's equation after accounting for the angular momentum contributed by the wheels, and the total external torques acting on the spacecraft. TAMONLY currently models the gravity-gradient torque and the magnetic control torque acting on the spacecraft. (The aerodynamic drag torque and the radiation pressure torque have been intentionally omitted to reduce the amount of spacecraft modeling required. The RTSF relies on the rate-corrections, $\vec{b}$, to compensate for the small effects of these torques.)

The covariance matrix, P, is propagated by numerically integrating the following equation:

$$\frac{dP}{dt} = F(\bar{\omega})P + P\,F^{T}(\bar{\omega}) + Q \qquad (3)$$

Here $F(\bar{\omega})$ is described in Reference 5; the quantity of interest is the 6×6 matrix Q that quantifies the propagation noise and is of the following diagonal form:

$$Q = \mathrm{diag}\,[\, Q_{ax}, Q_{ay}, Q_{az}, Q_{bx}, Q_{by}, Q_{bz} \,] \qquad (4)$$

Here Q_{ax}, Q_{ay}, and Q_{az} represent the strength of the noise term $\vec{\eta}_a$ and contribute to the growth of the attitude error covariances about the body X-, Y-, and Z-axes during propagation. Similarly, Q_{bx}, Q_{by}, and Q_{bz} represent the strength of the noise term η_b, and contribute to the growth of the error covariances of the rate corrections during propagation. These six elements of Q are collectively referred to here as the *Q-parameters* and rough estimates of their numerical values for ERBS are given in the next section. Another quantity that we must consider during tuning is σ_{TAM}, the strength of the white noise in the TAM measurements.

3. Description of ERBS and Data Characteristics

ERBS is an Earth (geodetic)-pointing three-axis stabilized spacecraft with nominal inclination 57 deg and a nominal altitude of 610 kilometers (References 8 and 9). The onboard attitude requirement of 1.0 deg is met by using a fine Sun sensor of accuracy 0.05 deg and a scan wheel-type Earth sensor of accuracy 0.02 deg. These high-precision sensors are complemented by a TAM onboard the spacecraft. Rate information is obtained from gyros that are accurate to 0.001 deg/sec. Attitude control is achieved through magnetic torquer bars (MTBs) and a momentum wheel that provides momentum bias along the pitch axis (negative orbit normal). A small angular momentum contribution also arises from the scan wheels in the Earth sensor assembly. ERBS also uses thrusters for its monthly 180-deg yaw maneuvers, but these were not activated during the data period discussed here.

The data used in this study are a 13-hour span of processed (adjusted) engineering data generated on the mainframe computers using the ERBS Attitude Ground Support System (AGSS) and covering the period 841225.221932 to 841226.105348. The adjustments comprised mainly bias determinations for the Sun and Earth sensors, the gyros, and the TAM. It is important to note here that the TAM was not calibrated for misalignments and MTB coupling. As shown in Reference 10, these calibrations can significantly alter the performance. The attitude history generated by the AGSS using the adjusted Earth and Sun sensor data is taken as the attitude truth model, and the adjusted gyro data are taken as the truth model for the rates. For the remainder of this paper, the times of events are described relative to the epoch of 841225.221932.

These data were then downloaded to a PC where a dataset having the TAMONLY input format was generated. The processed telemetry was input to TAMONLY at a period of 16 sec (equal to the availability of control torque data) and consisted of the following:

- Time
- Reference magnetic field
- Measured magnetic field
- Dipole moments of the MTBs
- Wheel speeds
- Position and velocity of the spacecraft

The profile of the truth models is presented in Figures 1a through 1c, where Figure 1a shows the true attitude, Figure 1b shows the true rates (where the rate components in the figure are denoted by "wx" "wy" "wz"), and Figure 1c shows the TAM residuals generated by the true attitude. Note the statistics of the data presented in Figures 1a through 1c. For example, it can be seen that the mean residuals are different from zero; in fact, these mean values had to be subtracted off from the TAM measurements input to TAMONLY to improve its performance slightly.

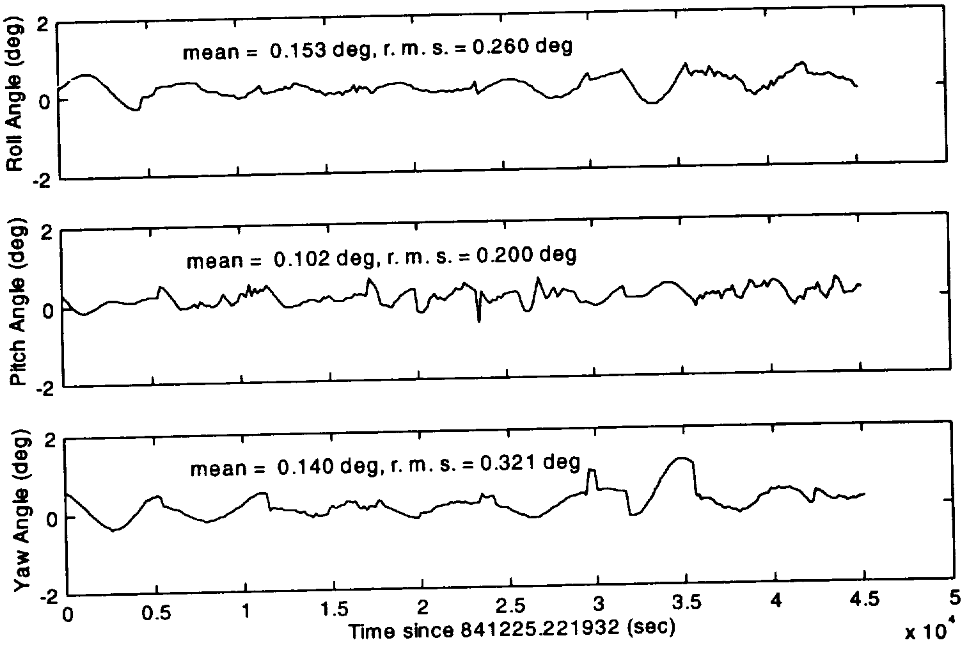

Figure 1a. Profile of the Attitude Truth Model Obtained Using Earth and Sun Sensors

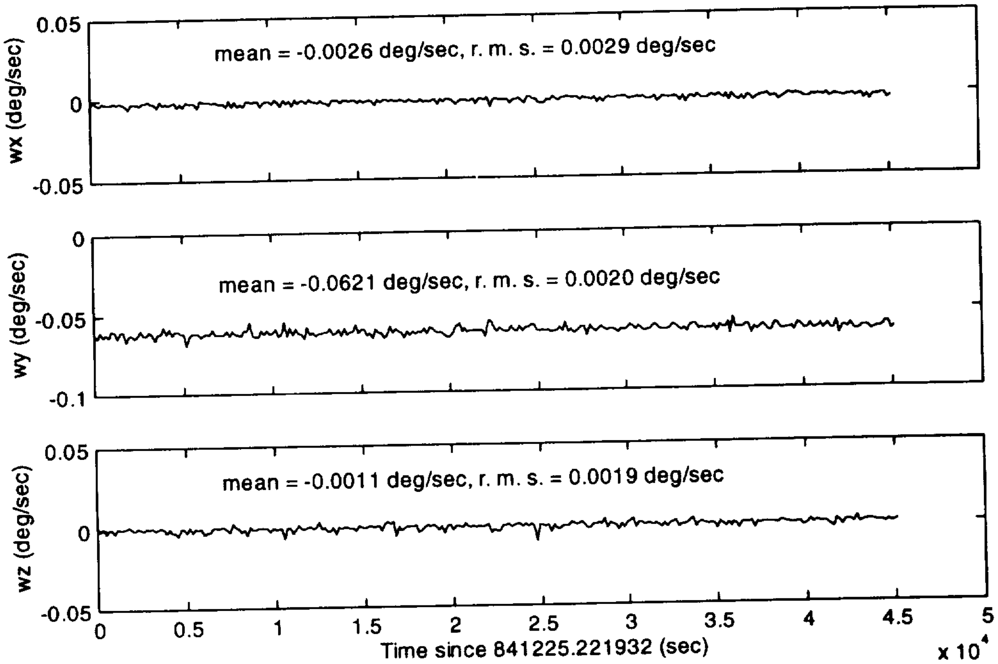

Figure 1b. Profile of the Rate Truth Model Obtained Using Gyros

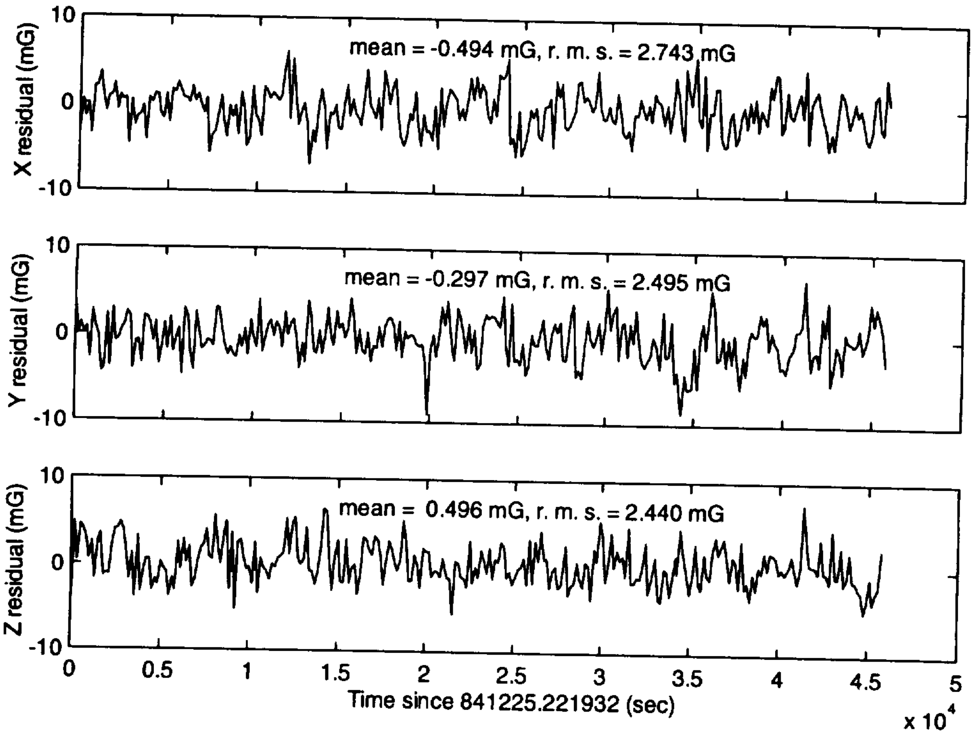

Figure 1c. TAM Residuals Obtained Using the Attitude Truth Model

During the course of this investigation, the optimal tuning parameters for the telemetered data (Section 5) indicated that calibration of the TAM might be in order. To see what effects a perfect TAM might have, simulated TAM data with no noise were generated using the true attitude and the reference magnetic field. These simulated TAM data were then used in place of the actual TAM measurements thus providing a much-needed degree of control over the study, and the results are described in Section 6. Note that only the TAM data were replaced; other quantities, such as the MTB and wheel data, were left untouched in this process.

The noise parameters were determined as follows by considering the noise in the data. The nominal value of σ_{TAM} was determined as 6.4 milligauss (mG), which is the discretization of the TAM data in the telemetry. Regarding the Q-parameters, it was found that the principal source of noise during propagation was the digitization in the telemetered wheel momenta, which was about 0.7 Newton-meter-second (N-m-s) along the Y-axis and 0.01 N-m-s along the Z-axis. In view of the telemetry period of 16 sec, this generated torques of the order of 0.04 N-m and 0.0006 N-m, respectively. It was assumed that the noise torque along the X-axis was of the same magnitude as that about the Z-axis. The squares of the corresponding uncertainties in the spacecraft rates are then

$$(\Delta\omega_y)^2 = 2.3 \times 10^{-8} \text{ radians}^2/\text{second}^2 \ (\text{rad}^2/\text{sec}^2) \tag{5}$$

$$(\Delta\omega_x)^2 = (\Delta\omega_z)^2 = 10^{-11} \text{ rad}^2/\text{sec}^2 \tag{6}$$

During the telemetry period, Δt, the attitude error covariances corresponding to the i^{th} axis grow by $(\Delta\omega_i)^2 (\Delta t)^2$ rad^2. Because, from Equation (3), $\boldsymbol{Q}$ is the rate of change of $\boldsymbol{P}$, this yields:

$$Q_{ai} = (\Delta\omega_i)^2 \, \Delta t \text{ rad}^2/\text{sec}, \ (i = x, y, z) \tag{7}$$

where Δt = 16 sec and Equations (5) and (6) are to be used for $(\Delta\omega_i)^2$.

The bottom three elements of Q are assigned numerical values by invoking the following relationship (Reference 11) between the steady-state covariance, p_∞ of an exponentially correlated random variable (such as $\vec{b}$) and its spectral density, q:

$$q = \frac{2p_\infty}{\tau} \tag{8}$$

It is assumed that the noise in $\vec{b}$ is of the same order of magnitude as that in the rates themselves, so that p_∞ was chosen equal to the squares of the rate uncertainties to yield

$$Q_{bi} = 2\,(\Delta\omega_i)^2 / \tau \ \text{rad}^2/\text{sec}^3 \ (i = x, y, z) \tag{9}$$

where $\tau = 16$ sec and the numerical values of $(\Delta\omega_i)^2$ are to be used from Equations (5) and (6).

From Equations (7) and (9), it can be seen that the Q-parameters can be parametrized by the rate uncertainties according to this scheme, and these uncertainties will be used in describing the results of tuning the RTSF. It should be noted that this study is limited to tuning the RTSF noise parameters to improve its performance, and that the errors contributed by imperfect models of the spacecraft's dynamics are ignored here. The latter contributions may, in fact, be sizable because a great number of error sources exist, such as spacecraft mass properties (inertia tensor), flexible modes of the spacecraft, misalignments of the wheel axes, scale factors for the wheel speeds, misalignments of the MTBs, and limitations of the numerical integration algorithms.

4. Accuracy Criteria

The following criteria were adopted to determine the accuracy of the RTSF as well as the optimality of the tuning parameters:

- ***Attitude differences***, which are the differences between the true attitude and RTSF estimates
- ***Rate differences***, which are the differences between the true rates and RTSF estimates
- ***TAM residuals***, which depend upon the accuracy of the propagation, the accuracy of the reference magnetic field, and the accuracy of the TAM. Accurate RTSF estimates are typified by TAM residuals of less than 10 mG.
- ***The RTSF's rate corrections,*** $\vec{b}$, which are not to be confused with the rate differences noted above. Accurate RTSF estimates are typified by rate-correction estimates of less than 2 deg/hour. In this study, the rate-correction estimates are used only qualitatively, i.e., they are used to confirm the convergence of the RTSF to the truth model.

DADMOD usually initializes the RTSF to within 5 deg of the true attitude and 0.05 deg/sec of the true rates. The results from the first 5000 sec of a run were omitted from the statistics to allow for filter convergence. Typical results of a TAMONLY run are presented in Figure 2a and 2b where results for the Y- and Z-axes are shown. In these figures, the rate components are denoted by "wy" and "wz," and the components of the RTSF's rate corrections are denoted by "by" and "bz."

5. Results Using In-Flight ERBS Data

About 25 runs were made using a wide range of the noise parameters. A notable (and welcome) feature was that a wide range of numerical values did not alter the accuracies substantially. A sample of the results is shown in Table 1. The first column of Table 1 denotes the numerical values of the noise parameters [σ_{TAM}, $(\Delta\omega_y)^2$, and $(\Delta\omega_z)^2$], together with a mnemonical description of the set. Against each set of tuning parameters, the mean and root-mean-square (r.m.s.) of the attitude differences, rate differences, and TAM residuals for each body axis are displayed. *Note that accuracies of the RTSF are taken to be the r.m.s. values of the differences from the truth model.*

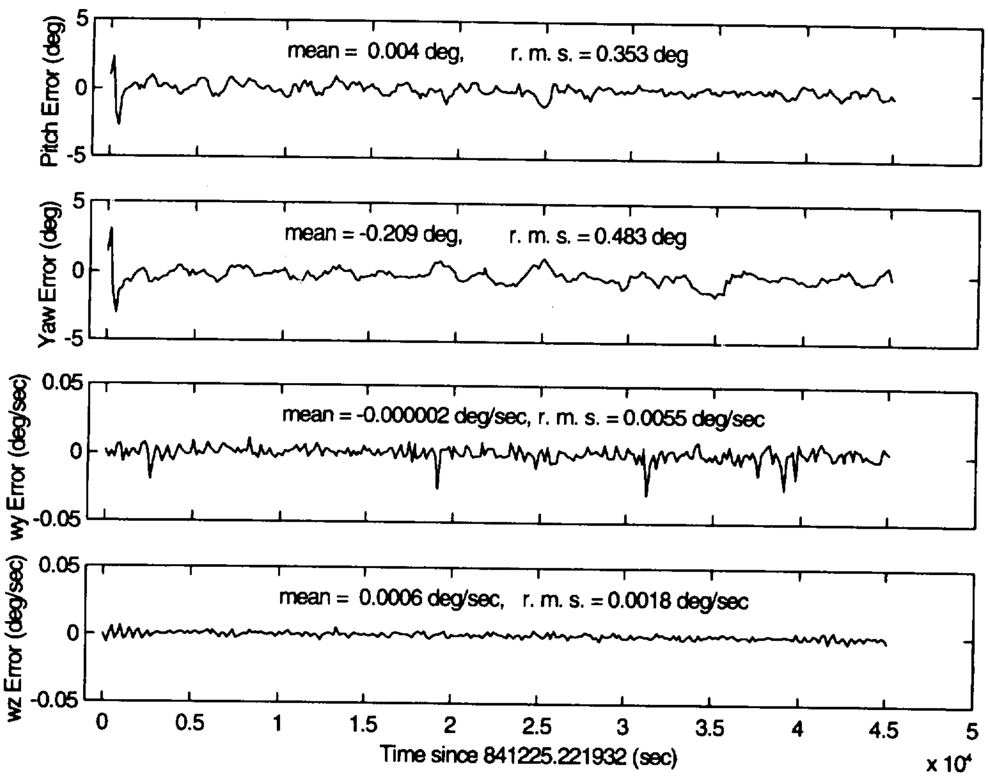

Figure 2a. Typical Attitude Results (Top Two Plots) and Rate Results (Bottom Two Plots) Obtained Using TAMONLY

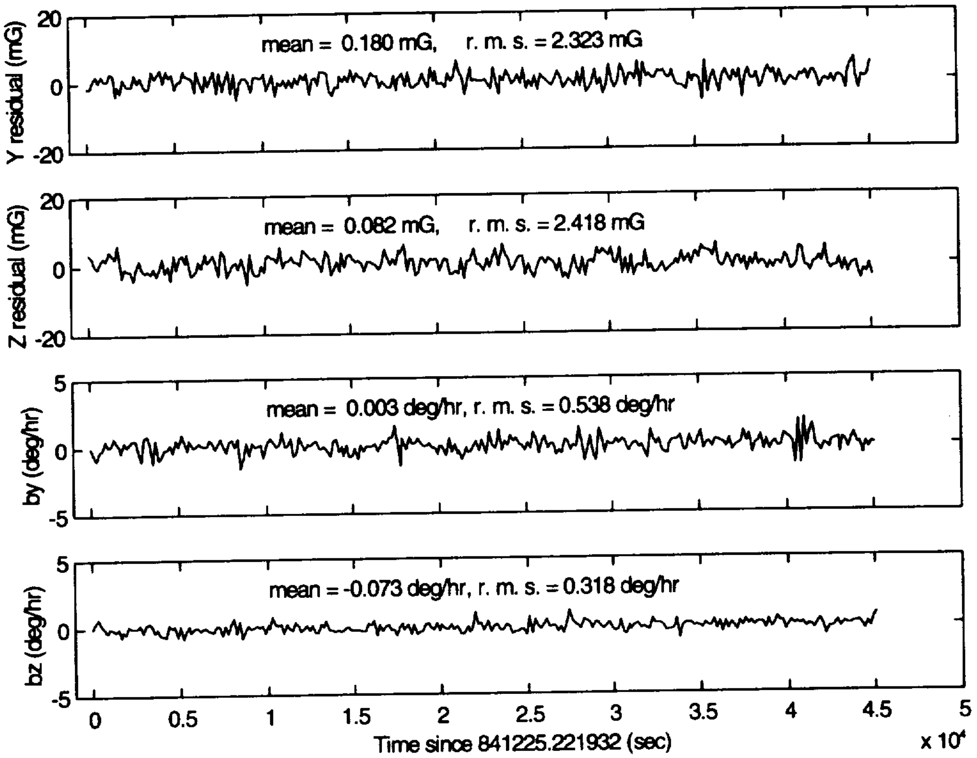

Figure 2b. TAM Residuals (Top Two Plots) and Rate-Correction Estimates (Bottom Two Plots) Obtained During the Run of Figure 2a

Table 1. Error Statistics and Tuning Using 40,200 Sec of Telemetered TAM Data

Description $[\sigma_{TAM}, (\Delta\omega_y)^2, (\Delta\omega_z)^2]$	Attitude Differences With Truth (deg) Mean / r.m.s.			Rate Differences With Truth (deg/sec) Mean / r.m.s.			TAM Residuals (mG) Mean / r.m.s.		
	x	y	z	x	y	z	x	y	z
Optimal $[40, 3\times10^{-8}, 3\times10^{-8}]$	–0.071 / 0.214	0.085 / 0.406	–0.080 / 0.434	0.0025 / 0.0030	0.00029 / 0.0054	0.00076 / 0.0020	–0.570 / 2.615	0.229 / 2.327	0.005 / 2.425
SAMPEX defaults $(10.4, 3 \times 10^{-8}, 3\times10^{-8})$	–0.063 / 0.287	0.031 / 0.395	–0.105 / 0.588	0.0026 / 0.0048	0.000033 / 0.0062	0.00076 / 0.0036	–0.140 / 2.756	0.355 / 2.642	0.089 / 2.554
Modeled Q for ERBS $(6.4, 2.3\times10^{-8}, 10^{-11})$	–0.428 / 0.508	0.032 / 0.451	–0.560 / 0.663	0.0026 / 0.0040	0.000083 / 0.0062	0.0005 / 0.0031	–0.457 / 2.784	–0.240 / 3.493	0.756 / 2.681
Large Q $(40, 10^{-6}, 10^{-6})$	–0.081 / 0.298	0.022 / 0.392	–0.147 / 0.529	0.0026 / 0.0043	0.000030 / 0.0067)	0.00077 / 0.0030	–0.137 / 2.812	0.351 / 2.587	0.092 / 2.564
Small Q $(3, 10^{-11},10^{-11})$	–0.275 / 0.383	0.063 / 0.851	–0.453 / 0.610	0.0026 / 0.0038	0.000088 / 0.0057	0.00050 / 0.0029	–0.617 / 2.952	–0.451 / 3.206	0.949 / 2.802

A number of useful deductions can be made by examining Table 1:

- The optimal (i.e., best) parameters yield higher accuracies (0.2 to 0.4 deg in attitude and 0.002 to 0.005 deg/sec in rates) than was concluded from past studies.
- The rate accuracies are relatively insensitive to tuning, indicating that they are dominated by the accuracy of the dynamical modeling (mass properties and torque models) rather than noise modeling.
- The optimal parameters yield TAM residuals comparable in quality with those of Figure 1c, which were obtained from the truth model.
- The large value of σ_{TAM} in the optimal parameters indicates a need for calibrating the TAM for misalignments and for the effects of the MTBs. This, in turn, may be necessitating the different numerical values for the Q parameters than what was roughly estimated in Section 3.

Another issue studied was whether taking averages over an integral number of orbits made a difference in the statistics. Two sets of averages were determined by dividing the timespan into sections spanning: (1) one orbit each (5864 sec) and (2) spanning 4000 sec each. Sample results are presented in Tables 2a and 2b; no significant orbital period effects were detected in the statistics.

6. Results Using Simulated TAM Data

As noted in Section 3, the TAM measurements in the telemetry were replaced with simulated TAM data and the results are shown in Table 3. A number of additional useful deductions can be made by comparing the optimal parameter results of Table 1 with those of Table 3:

- The optimal parameters of Table 1 understandably yield poorer results now, because the σ_{TAM} value of 40 mG is significantly different from the simulated value of 0 mG.
- The TAM residuals are significantly smaller than in Table 1, correctly reflecting the perfect sensor situation.
- The best results (third row of Table 3) are obtained when the modeled noise parameters for ERBS are used and result in accuracies of 0.1 to 0.2 deg in attitude. Note that, although $\sigma_{TAM} = 0$ ideally, a small nonzero value for σ_{TAM} is necessary in practice to avoid divergences. No further improvements in performance were detected for smaller values of σ_{TAM}.
- As in Table 1, the rate accuracies are relatively insensitive to the noise parameters, indicating that further improvements in accuracy must originate from dynamical models.

Table 2a. Error Statistics Over Orbital Period (Using Optimal Parameters of Table 1)

Time Span (seconds)	**Attitude Accuracies (deg)**		
	Roll (mean, r.m.s)	**Pitch (mean, r.m.s)**	**Yaw (mean, r.m.s)**
5008–10880	–0.047, 0.150	0.098, 0.470	–0.092, 0.350
16752–22640	–0.025, 0.207	–0.070, 0.345	–0.106, 0.424
34384–40256	–0.130, 0.223	–0.004, 0.233	–0.357, 0.631

Table 2b. Error Statistics Over 4000-Second Spans (Using Optimal Parameters of Table 1)

Time Span (seconds)	**Attitude Accuracies (deg)**		
	Roll (mean, r.m.s)	**Pitch (mean, r.m.s)**	**Yaw (mean, r.m.s)**
9008–13008	–0.044, 0.169	–0.025, 0.391	–0.165, 0.338
21008–25008	–0.043, 0.200	–0.171, 0.187	–0.329, 0.480
37008–41008	–0.213, 0.296	–0.008, 0.274	–0.282, 0.373

Table 3. Error Statistics and Tuning Using Simulated TAM Data

Description $[\sigma_{TAM}, (\Delta\omega_y)^2, (\Delta\omega_z)^2]$	**Attitude Differences With Truth (deg) Mean / r.m.s.**			**Rate Differences With Truth (deg/sec) Mean / r.m.s.**			**TAM Residuals (mG) Mean / r.m.s.**		
	x	y	z	x	y	z	x	y	z
Optimal Parameters of Table 1 $[40, 3\times10^{-8}, 3\times10^{-8}]$	–0.134 / 0.207	0.042/ 0.313	–0.260 / 0.373	0.0026 / 0.0029	0.00015 / 0.0055	0.00060 / 0.0016	–0.312 / 1.055	0.053 / 0.574	–0.301 / 0.774
Small σ_{TAM} $(10^{-2}, 3\times10^{-8}, 3\times10^{-8})$	–0.117 / 0.230	0.003 / 0.100	–0.190 / 0.282	0.0026 / 0.0031	0.00010 / 0.0054	0.00089 / 0.0019	0.129 / 0.500	0.062 / 0.269	–0.306 / 0.423
Modeled Parameters $(10^{-2}, 2.3\times10^{-8}, 10^{-11})$	–0.093 / 0.179	0.0004 / 0.088	–0.137 / 0.219	0.0026 / 0.0030	0.00008 / 0.0053	0.001 / 0.0019	0.117 / 0.495	0.046 / 0.255	–0.291 / 0.414

7. Conclusions

The principal conclusions derived from this study are as follows:

- Using telemetered TAM data, the accuracies (r.m.s. differences from the truth models) of TAMONLY were found to be

  0.2 to 0.4 deg for attitude

  0.002 to 0.005 deg/sec for rates

  Note that part of the uncertainties here could be due to the errors in the truth models (0.02 deg in roll and pitch angle estimates, 0.05 deg in yaw angle estimates, and 0.001 deg/sec in the gyro rate estimates). *Note that these accuracies are within the ERBS control requirements of 1.0 deg for attitude and 0.005 deg/sec for rates.*

- The results are *not* severely degraded if the default (SAMPEX) parameters are used. This is a very useful result, since tuning a Kalman filter is a manpower-intensive exercise.
- No significant orbital-period effects were seen in the error statistics.
- The simulated ERBS TAM data were useful by demonstrating the following:
  - Additional calibration of the TAM for misalignments and MTB coupling effects will extract better performance from the RTSF.
  - The modeled noise parameters indeed give better results when the TAM data are of high quality.
  - Very little improvement resulted in the r.m.s. values for rate differences, thus indicating that further improvements in accuracy must be generated by more accurate dynamics, i.e., through improved propagation algorithms and more accurate spacecraft and torque models.

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