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Reducing On-Board Computer Propagation Errors Due to Omitted Geopotential Terms By Judicious Selection of Uploaded State Vector

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Several future, and some current missions, use an on-board computer (OBC) force model that is very limited. The OBC geopotential force model typically includes only the J_2, J_3, J_4, $C_{2,2}$ and $S_{2,2}$ terms to model non-spherical Earth gravitational effects. The Tropical Rainfall Measuring Mission (TRMM), Wide-Field Infrared Explorer (WIRE), Transition Region and Coronal Explorer (TRACE), Submillimeter Wave Astronomy Satellite (SWAS), and X-ray Timing Explorer (XTE) all plan to use this geopotential force model on-board. The Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX) is already flying this geopotential force model. Past analysis has shown that one of the leading sources of error in the OBC propagated ephemerides is the omission of the higher order geopotential terms. However, these same analyses have shown a wide range of accuracies for the OBC ephemerides.

Analysis was performed, using EUVE state vectors, that showed that the EUVE four day OBC propagated ephemerides varied in accuracy from 200 m. to 45 km. depending on the initial vector used to start the propagation. The vectors used in the study were from a single EUVE orbit at one minute intervals in the ephemeris. Since each vector propagated practically the same path as the others, the differences seen had to be due to differences in the initial state vector only.

An algorithm was developed that will optimize the epoch of the uploaded state vector. Proper selection can reduce the previous errors of anywhere from 200 m. to 45 km. to generally less than one km. over four days of propagation. This would enable flight projects to minimize state vector uploads to the spacecraft. Additionally, this method is superior to other methods, in that no additional orbit estimations need to be done. The definitive ephemeris generated on the ground can be used, as long as the proper epoch is chosen. This algorithm can easily be coded in software that would pick the epoch, within a specified time range, that would minimize the OBC propagation error.

This technique should greatly improve the accuracy of the OBC propagations on-board future spacecraft such as TRMM, WIRE, TRACE, SWAS, and XTE without increasing complexity in the ground processing.

INTRODUCTION

Several future, and some current missions, use an on-board computer (OBC) propagator known as the *Landis propagator*. These missions include TRMM, WIRE, TRACE, SWAS, XTE, and SAMPEX. This propagator is a simple two-body propagator with additional force models added for atmospheric drag and nonspherical Earth harmonics. The geopotential model uses only the zonal harmonics J_2, J_3, and J_4 and sectorial harmonics $C_{2,2}$ and $S_{2,2}$. The modeling in the propagator is limited by OBC memory and CPU constraints. Ideally, each mission using this propagator would want the generated ephemeris to closely match the ground determined ephemeris which uses more advanced modeling. The error in the ground generated ephemeris is dominated by uncertainties in the daily f10.7 solar flux level. This error would be found in any ground generated or on-board ephemeris, regardless of other modeling, unless solar flux prediction techniques improve. The leading error in the difference between the ground generated ephemeris and the OBC ephemeris is the geopotential model. It is this error that is examined in this paper.

All accuracy numbers quoted in this paper are three sigma.

FORCE MODELING

The *Landis Propagator* was developed by Peter Hui of Advanced Technology Research Corporation and modified slightly by Dr. Landis Markley of the Goddard Space Flight Center[1]. The propagator was developed to provide simple on-board ephemeris propagation.

A gravitational potential model for a nonspherical body is commonly represented by a finite series of associated legendre polynomials (ALPs). The size of such a model is determined by the maximum degree and order of the ALPs included in the expansion series. The gravitational potential of the Earth can be expressed by the following spherical harmonic representation[2]:

$$V(r,\phi,\lambda)=\frac{\mu}{r}+\frac{\mu}{r}\sum_{n=2}^{\infty}C_n^0\left(\frac{R_E}{r}\right)^n P_n^0(\sin\phi)+\frac{\mu}{r}\sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_E}{r}\right)^n P_n^m(\sin\phi)\left(S_n^m\sin m\lambda+C_n^m\cos m\lambda\right)$$

where

V = gravitational potential

μ = Earth's GM

R_E = equatorial radius of the Earth

P_n^m = ALP of degree n and order m

S_n^m, C_n^m = harmonic coefficients

r = magnitude of radius vector

ϕ = geocentric latitude

λ = geocentric longitude

The current geopotential model used by the Flight Dynamics Facility (FDF) is the Joint Gravity Model (JGM)-2. This geopotential model has degree and order 50 and is considered the most accurate geopotential model available. An ephemeris propagated with this full geopotential model would be considered the most accurate available. The on-board ephemeris accuracies quoted later in this paper are compared to a truth ephemeris propagated with the following force modeling:

- JGM-2 degree and order 50
- atmospheric drag with the Jacchia-Roberts model
- solar radiation
- Earth tides
- Sun and Moon point masses

The *Landis Propagator* has the following force modeling:

• Earth nonspherical gravitational effects:	zonal harmonics J_2, J_3, and J_4 sectorial harmonics $C_{2,2}$ and $S_{2,2}$
• Earth atmosphere:	modified Jacchia-Roberts '71 model
• solar radiation pressure:	not modeled
• Earth tides:	not modeled
• noncentral point masses:	not modeled

Covariance analysis of the OBC propagator error was performed using the error model shown in Table 1.

TABLE 1: OBC Error Model

Parameter	Uncertainty
Earth nonspherical gravitation effects	100% of the harmonic coefficients excluded in the OBC model
Earth atmosphere	none
solar radiation	100% of C_r
Earth tides	100% of second Love number
noncentral point masses	100% of GM of Sun and Moon

The Earth atmosphere error contribution was not included because the atmospheric models used on the ground and on the OBC are very similar and any inherent error in the modeling would appear in both propagations.

The covariance analysis was performed using state vectors from the Extreme UltraViolet Explorer (EUVE). The results of the covariance analysis indicate that, for the EUVE mission orbit, the above error model contributed approximately the errors shown in Table 2 to the OBC ephemeris after four days:

TABLE 2: Covariance Analysis Results

Parameter	Position Error (m.)
Earth nonspherical gravitation effects	200 to 44600
solar radiation	40
Earth tides	55
Moon point mass	350
Sun point mass	50

The omission of the additional geopotential terms is clearly the dominate source of error in the OBC propagation compared with the ground ephemeris. The range of the geopotential errors is explained later in this paper.

PAST RESULTS

The initial purpose of this analysis was to investigate the true errors introduced by using a severely truncated geopotential model for onboard propagation and to explain the differences seen in past results.

Premission analyses were performed for both TRMM and XTE. Ref. [3] stated that the XTE OBC propagation would degrade to 60 km. in accuracy in 5.4 to 9.3 days. However, this analysis incorporated f10.7 solar flux uncertainties in the prediction accuracy. Since the predicted uncertainties are dominated by uncertainties in both the f10.7 solar flux and the geopotential, the errors quoted in this report could not be attributed solely to the geopotential. Ref. [4] stated TDRS OBC accuracies for prediction onboard XTE. The seven day predicted accuracy was three km. This result was not relevant because of the small geopotential effect on geosynchronous orbits. Ref. [5] stated TRMM OBC accuracies at both beginning of life (BOL) and end of life (EOL). The 32 hour prediction accuracy was 22 km. for BOL and 32 km. for EOL. These results however were based upon orbits with different semi-major axes and also included f10.7 solar flux uncertainties. Ref. [6], by A. Schanzle, stated XTE OBC accuracies for five different arcs based upon five different epochs. The five epochs were each one day apart. F10.7 solar flux uncertainties were not considered. The four day prediction accuracy ranged from 1.2 to 43.0 km. The leading error contribution to the prediction error was the geopotential which confirmed the covariance analysis mentioned in the last section. It was this report that gave the first documented results of large variations in the OBC predicted ephemeris accuracy.

Schanzle had speculated that "... the large variation in the magnitude of the propagation differences noted in the [software] results is a consequence of the epoch semi-major axis relative to its average value. If this is the case, then errors in an OBC-generated trajectory may be minimized by uplinking a state vector to the OBC that occurs at a time when the semi-major axis is close to its average value." The correlation between OBC propagation accuracy and the difference between the osculating semi-major axis and the mean semi-major axis is shown in Figure 1. The correlation coefficient between the two was 0.75 but the sample was very small. Schanzle made two additional simulations later that are also shown in Figure 1. These two additional points did not confirm his theory and, in fact, brought the correlation coefficient down to 0.35.

ANALYSIS

The first step was to perform covariance analysis using different epochs but at a much closer interval than the one day interval for the samples in Ref. [6]. The goal was to determine some sort of pattern from the seemingly random OBC propagation errors. Two different spacecraft orbits were chosen: SAMPEX and EUVE. In each case, an operational mission vector was propagated for four hours using full force modeling. Then, covariance analysis was performed on a four day OBC propagation using an initial vector every 20 minutes from the four hour ephemeris. The results are shown in Figures 2 and 3.

The SAMPEX OBC errors after four days ranged from 1.1 to 49.3 km. The EUVE OBC errors after four days ranged from 2.5 to 40.0 km. Neither graph shows a smooth pattern of how the OBC propagation error changes with the epoch. It is clear, however, that the OBC error predictions vary even with epochs from the same orbit. This, despite the fact, that the epochs were just minutes apart and that they covered almost the identical trajectory in their propagations over the four days. This indicated that there was some characteristic of the initial starting vector that contributed greatly to the OBC propagation error, even over several days. The unknown OBC errors between each of the points on these graphs needed to be determined to investigate how the OBC errors change.

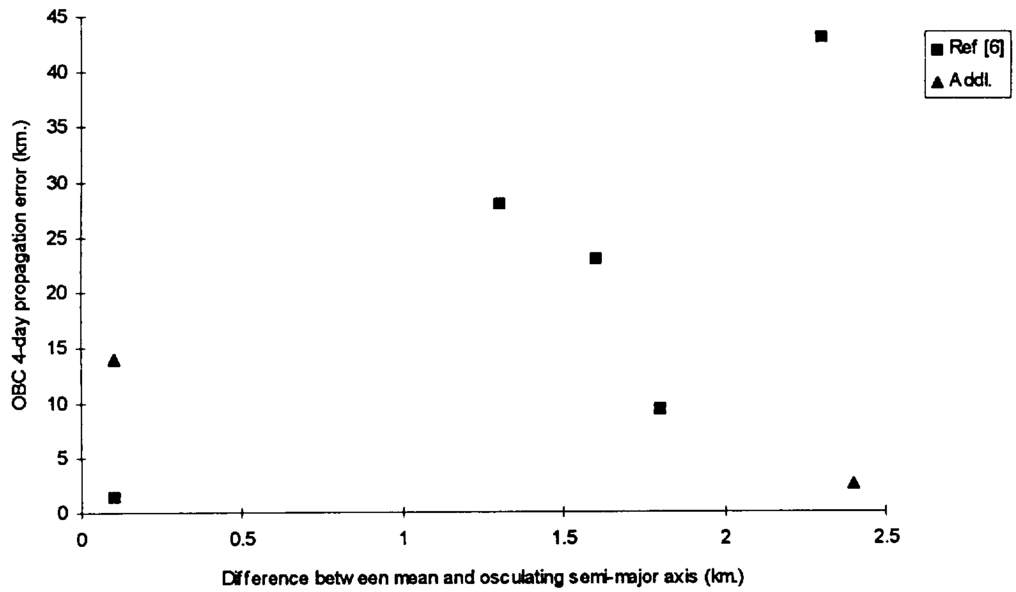

FIGURE 1: Schanzle's OBC Propagation Error vs. Semi-major Axis

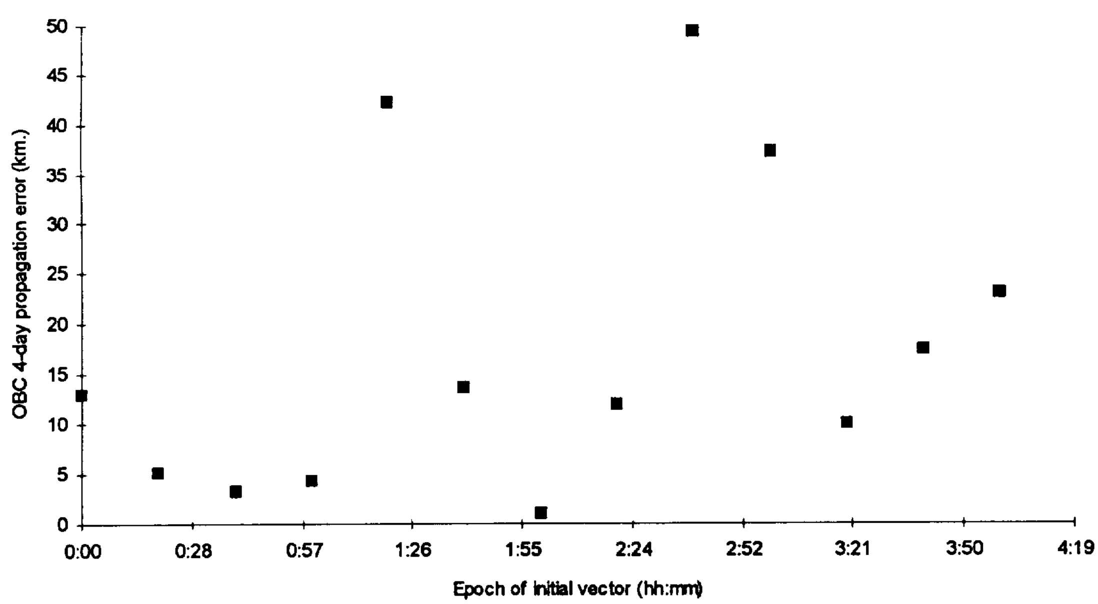

FIGURE 2: SAMPEX 4-day OBC Propagation Errors

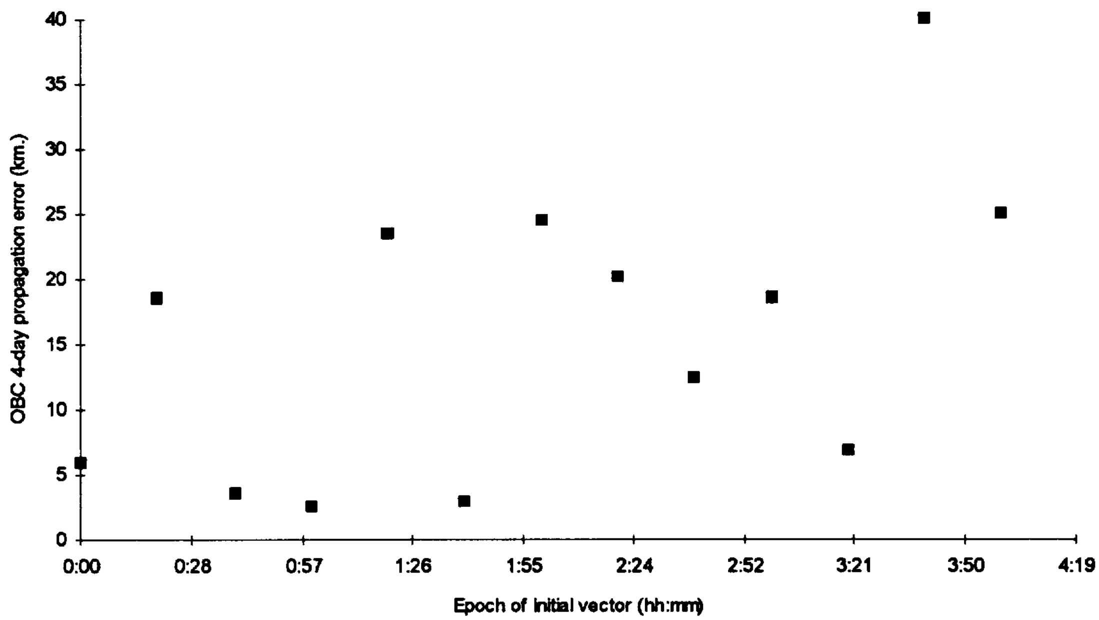

FIGURE 3: EUVE 4-day OBC Propagation Errors

The EUVE mission orbit was chosen and covariance analysis of a four day OBC propagation was done using EUVE vectors every one minute for an entire EUVE orbit of 95 minutes. The results are shown in Figure 4.

The pattern is finally clearly seen, and seems to exhibit a smoothly changing error that reverses itself abruptly at zero, which may indicate that the error has changed sign. The OBC propagation errors ranged from 0.2 to 44.6 km. The problem was now reduced to finding some characteristic of the initial vector that exhibited some correlation with this OBC propagation error pattern.

To further investigate Schanzle's theory, the EUVE OBC propagation error was compared to the difference between the mean and the osculating semi-major axis, shown in Figure 5. The correlation coefficient between the two was only -0.12.

The OBC propagation error was far too complex to be correlated to the altitude of the spacecraft around the orbit (correlation coefficient of 0.16 with altitude). The OBC propagation error also did not correlate with the eccentricity (correlation coefficient of 0.11) or the argument of latitude (0.30).

The analysis was then concentrated on the gravitational potential at the epoch. The two ephemerides being compared, the ground generated ephemeris and the OBC generated ephemeris, had two different gravitational potentials at the epoch. Therefore, they also had different gravitational forces and energy at epoch. The OBC propagation errors were compared to various potentials, gravities, and energies in various directions as shown in Table 3. The symbols g50, G50, V50, and E50 represent the gravitation acceleration magnitude, acceleration vector, potential, and total energy, respectively, of the full geopotential model up to degree and order 50, as is used in the ground generated ephemeris. The symbols gOBC, GOBC, VOBC, and EOBC represent the gravitational acceleration magnitude, acceleration vector, potential, and total energy, respectively, of the OBC truncated geopotential model. The arguments: x, y, z, r, i, c, e, and n, represent directions: inertial Cartesian x, y, and z, radial, intrack, crosstrack, east , and north.

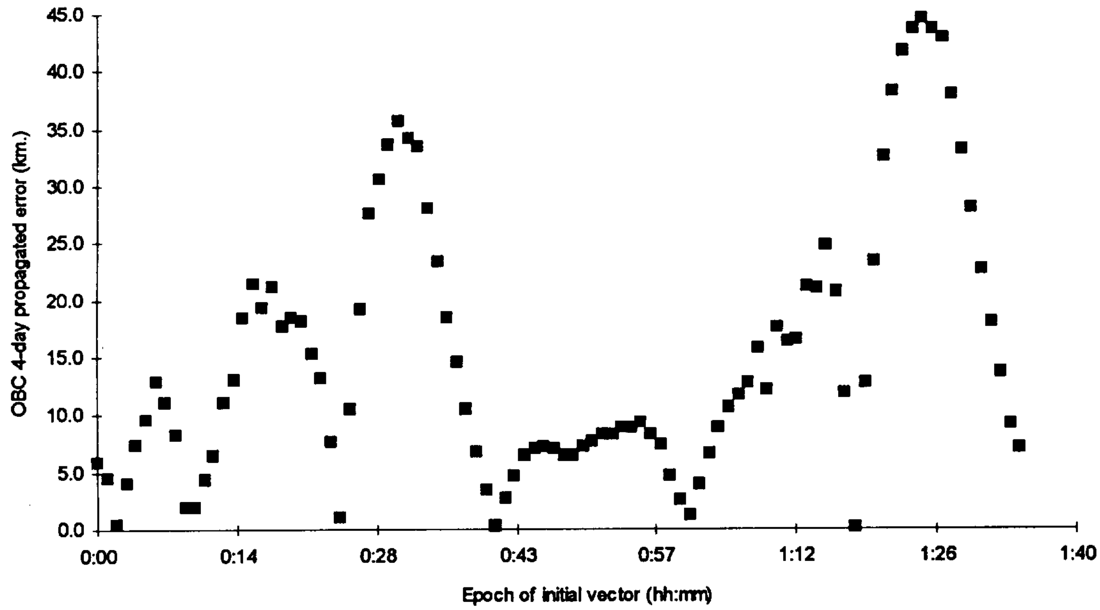

FIGURE 4: EUVE 4-day OBC Propagation Errors At One Minute Intervals

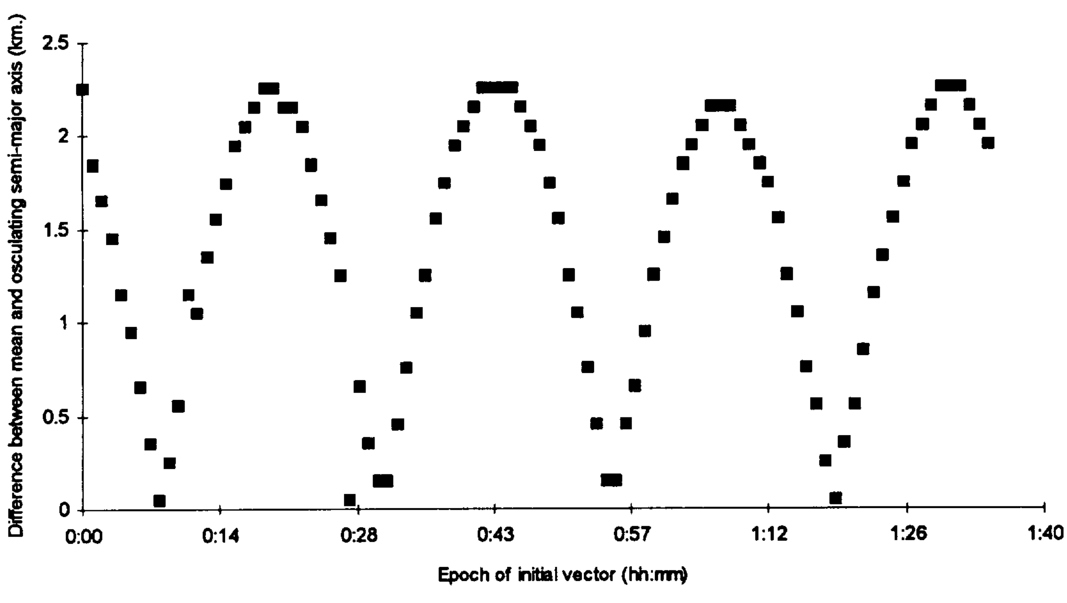

FIGURE 5: EUVE Osculating Semi-major Axis

TABLE 1: Correlation Coefficients

Characteristic	Correlation Coefficient	Characteristic	Correlation Coefficient
g50	-0.19	Δ(gOBC(r)-g50(r))	0.01
gOBC	-0.18	gOBC(i)-g50(i)	0.00
|g50-gOBC|	0.79	| gOBC(i)-g50(i)|	-0.01
Δ|g50-gOBC|	0.08	Δ(gOBC(i)-g50(i))	0.00
|G50-GOBC|	0.52	gOBC(c)-g50(c)	0.17
Δ|G50-GOBC|	-0.03	| gOBC(c)-g50(c)|	-0.31
gOBC(x)-g50(x)	-0.57	Δ(gOBC(c)-g50(c))	0.06
| gOBC(x)-g50(x)|	0.49	gOBC(e)-g50(e)	0.00
Δ(gOBC(x)-g50(x))	0.15	| gOBC(e)-g50(e)|	-0.09
gOBC(y)-g50(y)	-0.20	Δ(gOBC(e)-g50(e))	0.07
| gOBC(y)-g50(y)|	0.11	gOBC(n)-g50(n)	0.16
Δ(gOBC(y)-g50(y))	-0.32	| gOBC(n)-g50(n)|	-0.09
gOBC(z)-g50(z)	0.26	Δ(gOBC(n)-g50(n))	0.16
| gOBC(z)-g50(z)|	0.25	VOBC-V50	0.23
Δ(gOBC(z)-g50(z))	-0.27	|VOBC-V50|	0.96
gOBC(r)-g50(r)	-0.15	EOBC-E50	-0.20
| gOBC(r)-g50(r)|	0.79		

Three characteristics of the epoch show marked correlation with the OBC propagated errors:

- the differences in the magnitude of the gravitational acceleration vectors between the full 50 by 50 model and the OBC model
- the differences in the radial acceleration vector between the full 50 by 50 model and the OBC model
- the differences in the gravitational potential between the full 50 by 50 model and the OBC model

The potential differences are extremely well correlated with a correlation coefficient of 0.96! The three characteristics are shown together in Figure 6 over the full 95 minutes of the EUVE orbit.

It can be seen from this graph that the differences in the magnitude of the acceleration vectors is due mostly to the differences in the radial direction.

Since the potential differences showed the highest correlation, they are shown together with the OBC propagation errors in Figure 7.

The differences in the gravitational potential between the full and the OBC fields is a clear indicator of the OBC propagation accuracy.

ALTERNATIVE TECHNIQUES

Alternative techniques have been suggested to improve OBC propagation accuracy. Roger Hart, Flight Dynamics Division, Goddard Space Flight Center, has developed and demonstrated a technique that also eliminates the OBC propagation error due to the truncated geopotential field. The technique is as follows:

1. Generate a predicted ephemeris based upon an orbit estimation using full force modeling.

2. Convert the state vectors in the predicted ephemeris into tracking measurements.
3. Use the simulated tracking measurements in an orbit estimation using the OBC truncated geopotential model.
4. Generate a definitive ephemeris from the orbit estimation state.
5. Generate the Extended Precision Vector (EPV) from the definitive ephemeris.

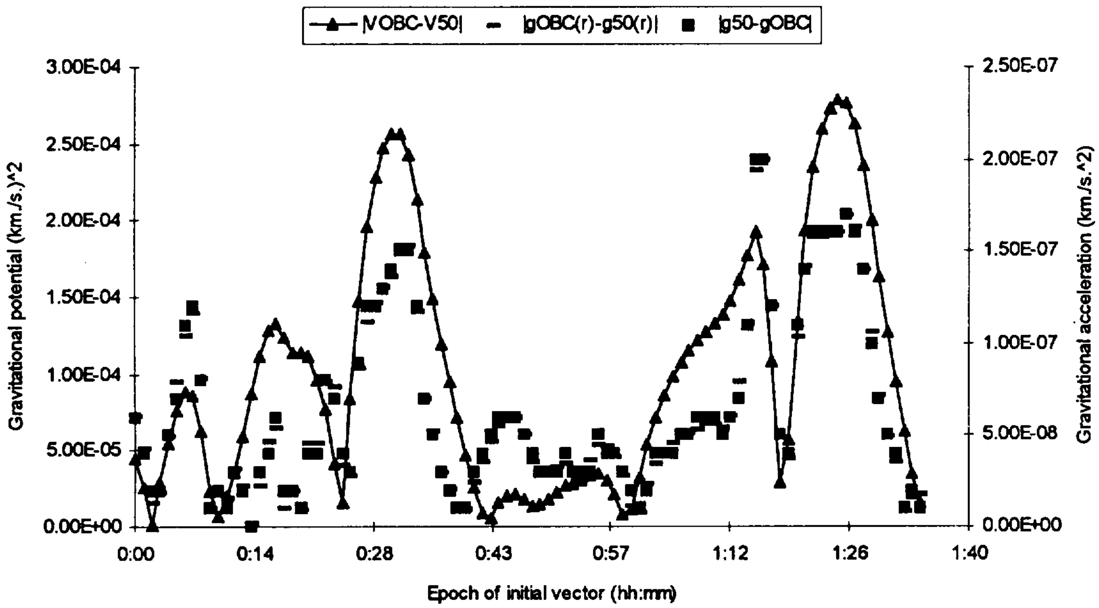

FIGURE 6: Potential & Acceleration Differences Between Full & OBC Geopotential Models

The algorithm uses an EPV generated using the same force modeling that is on-board the OBC. Thus, the EPV will propagate the same as the definitive ephemeris generated on the ground. This definitive ephemeris is forced to fit the ground generated predicted ephemeris using full force modeling. So the OBC propagation will be generally similar to the ground generated predicted ephemeris for the span of the ephemeris.

The drawback to this method is the complexity of the procedure. Three additional steps are added to the current procedure: simulating the tracking measurements, performing a batch orbit estimation, and generating the definitive ephemeris. None of the above steps is trivial. Experiments using this technique have been generally favorable though.

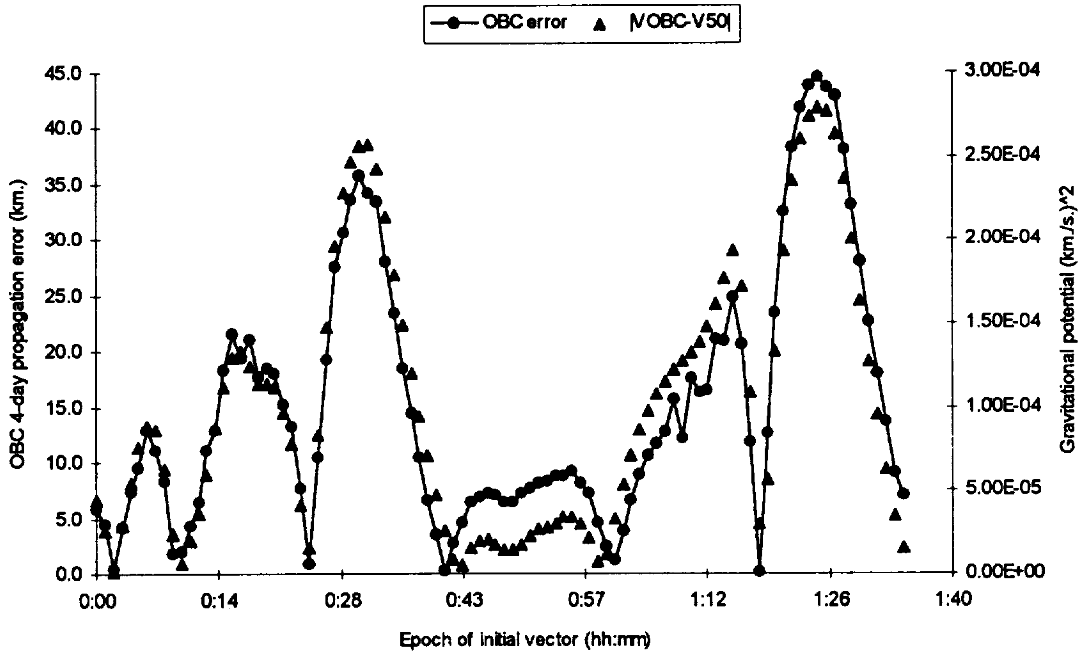

FIGURE 7: Comparison Between Gravitational Potential and OBC Propagation Errors

IMPACT

The impact of this finding is that each of the missions using this OBC geopotential force model can greatly improve the accuracy of the OBC propagation by a simple selection of the epoch of the uploaded state vector.

EPVs are uploaded to the spacecraft for SAMPEX, which is the only current mission using the *Landis propagator*. An additional piece of EPV optimization software, approximately 150 lines of code, would be needed to select the optimum epoch between user defined boundaries. The flowchart for a prototype of this software is shown in Figure 8. The minimum potential difference is found between user defined boundaries and the epoch of this minimum is output. The software checks each state vector in the definitive ephemeris, so if the definitive ephemeris is at 60 second intervals, then the optimized epoch will be chosen to the minute.

Currently for SAMPEX, EPVs at 00:00 Greenwich Mean Time (GMT) on the day of the orbit determination are uploaded to the spacecraft. To take advantage of the improvements detailed in this paper, EPVs with epochs from approximately 23:15 GMT to 00:45 GMT would have to be uploaded. This would give an entire orbit of varying potential differences and should ensure propagation accuracies of less than one km. after four days.

Current FDF ground operations, including orbit determination, predicted ephemeris propagation, coordinate transformation, and EPV generation, are highly automated. The use of this EPV optimization software would not impact that automation. The data flow for FDF ground operations is shown in Figure 9.

The EPV optimization would have to be individually tailored to meet the specific needs of each mission.

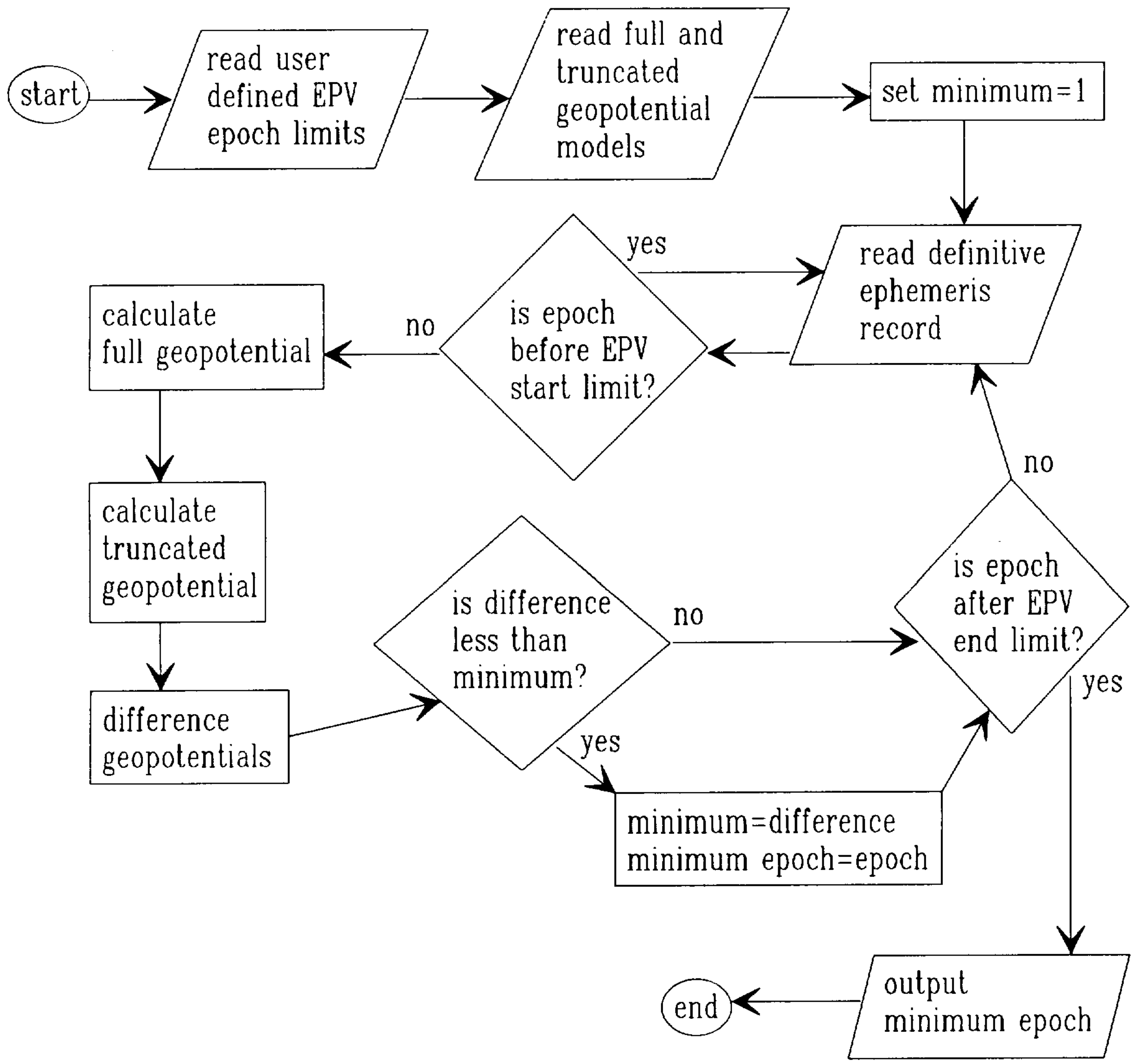

FIGURE 8: EPV Optimization Software Flowchart

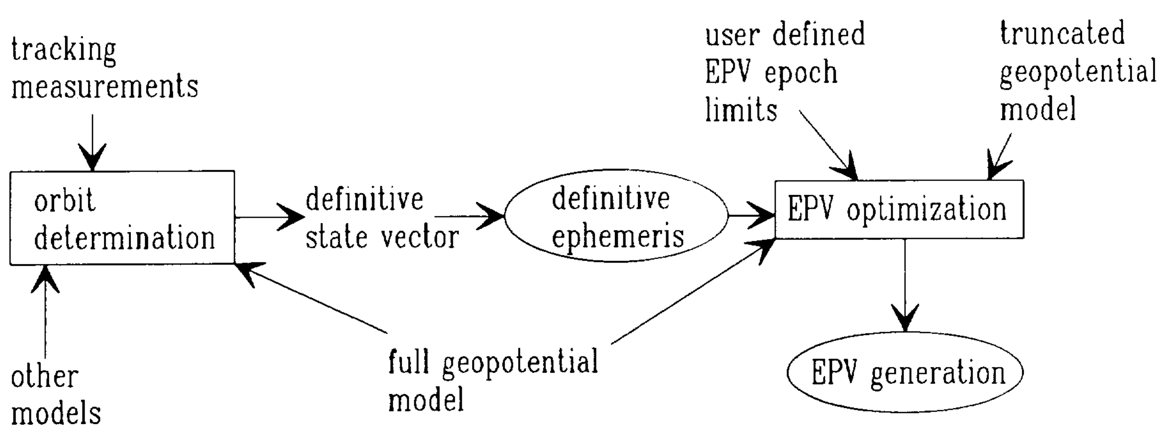

FIGURE 9: Data Flow for FDF Ground Operations

CONCLUSIONS

OBC propagation errors due to gravitational potential effects can be greatly reduced by simple judicious selection of the epoch of the uploaded state vector. The OBC propagation accuracy is highly correlated to the difference in gravitational potential at epoch between the full geopotential model and the truncated OBC model. Simple software can be added to the automated generation of the uploaded EPVs that will select the optimum epoch to use. Use of this procedure can increase OBC propagation accuracy by up to 1000% when compared to the ground generated ephemeris. Generally, a four day OBC propagation can be optimized to less than one km. in accuracy.

ACKNOWLEDGMENTS

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