

NASA Contractor Report 198344

ICASE Report No. 96-42

ICASE

ENERGY SPECTRUM IN THE DISSIPATION RANGE OF FLUID TURBULENCE

**D.O. Martinez, S. Chen, G.D. Doolen,
R.H. Kraichnan, L.-P. Wang, and Y. Zhou**

*NASA Contract No. NAS1-19480
May 1996*

*Institute for Computer Applications in Science and Engineering
NASA Langley Research Center
Hampton, VA 23681-0001*

Operated by Universities Space Research Association



*National Aeronautics and
Space Administration*

*Langley Research Center
Hampton, Virginia 23681-0001*

Energy Spectrum in the Dissipation Range of Fluid Turbulence*

*D. O. Martínez^{1,2,3}, S. Chen^{2,1}, G. D. Doolen¹,
R. H. Kraichnan⁴, L.-P. Wang³, and Y. Zhou⁵*

¹Theoretical Division and Center for Nonlinear Studies
Los Alamos National Laboratory, Los Alamos, NM 87545

²IBM Research Division, T.J. Watson Research Center
P.O. Box 218, Yorktown Heights, NY 10598

³Department of Mechanical Engineering
University of Delaware, Newark, Delaware 19716-3140

⁴369 Montezuma No. 108, Santa Fe, New Mexico 87501

⁵ICASE, NASA, Langley Research Center, Hampton, Virginia 23861

Abstract

High resolution, direct numerical simulations of three-dimensional incompressible Navier-Stokes equations are carried out to study the energy spectrum in the dissipation range. An energy spectrum of the form $A(k/k_d)^\alpha \exp[-\beta k/k_d]$ is confirmed. The possible values of the parameters α and β , as well as their dependence on Reynolds numbers and length scales, are investigated, showing good agreement with recent theoretical predictions. A “bottleneck”-type effect is reported at $k/k_d \sim 4$, exhibiting a possible transition from near-dissipation to far-dissipation.

*This work was partially supported by the DOE at Los Alamos and the National Aeronautics and Space Administration under NASA Contract No. NAS1-19480 while one of the authors (Y.Z.) was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23681.

I. Introduction

The study of the statistical properties of fluid turbulence at small scales is very difficult from both experimental and numerical points of view. In the former, the signal at these scales is exceedingly weak and consequently it is easily contaminated by the background noise that is naturally present in experiments. Resolution of inertial-range and dissipation-range scales at high Reynolds numbers by direct numerical simulations (DNS) requires computational capabilities far beyond those available today.

In the last four decades, several functional forms for the energy spectrum in the dissipation and far-dissipation ranges have been proposed based on heuristic arguments. The energy spectrum in isotropic turbulence can be written as $E(k) = \epsilon^{2/3} k^{-5/3} F(k/k_d)$, consistent with Kolmogorov's ideas (Kolmogorov 1941). In the expression above, $E(k) = 4\pi k^2 \langle 1/2 |\hat{u}(k)|^2 \rangle$, where $\hat{u}(k)$ is the Fourier transform of the velocity field; $k_d = \epsilon^{1/4} \nu^{-3/4}$ is the Kolmogorov dissipation wave number, that represents the scale at which the relative importance of the inertial and dissipative terms is comparable; ϵ is the mean rate of energy dissipation per unit mass, and ν is the kinematic viscosity. $\langle \rangle$ denotes an ensemble average. For scales associated with wavenumbers much smaller than k_d , the energy spectrum becomes essentially independent of the viscosity ν , and $F(k/k_d)$ is in that limit the Kolmogorov constant C_K . For wavenumbers $k > k_d$, dissipation is important and the function F has a strong dependence on the viscosity through k_d .

A widely proposed form for $E(k)$ in the dissipation range is $E(k) \sim (k/k_d)^\alpha \exp(-\beta(k/k_d)^n)$. There has been considerable debate as to what the values of the parameters α , β , and n are. Townsend (1951) and Novikov (1961) suggested $n = 2$, based on a Taylor expansion of the velocity field at scales much smaller than $\eta \cong 1/k_d$, the dissipation length scale. This same value is preferred by Smith and Reynolds in a more recent work (Smith & Reynolds 1991) in which they compare the skewness for the longitudinal velocity derivative obtained from experiments, with the values provided by $E(k) \sim (k/k_d)^\alpha \exp(-\beta(k/k_d)^n)$ with $n = 1, 4/3$ and 2 , and $\alpha = -5/3$. They conclude that $n = 2$ is the best fit, and that $n = 1$ is a poor model. Sanada (1992) proposed a spectrum with $n = 1$, but with a sharp change at a transitional wavenumber. His model fit the spectrum

from simulations and yielded a skewness of the velocity derivative in agreement with experimental values. Manley (1992) argued that the measurements used by Smith and Reynolds did not extend much beyond the Kolmogorov scale η , and accordingly he introduced a cutoff wavenumber for the computation of the skewness. With this correction, $n = 1$ is a better fit for the experimental data than $n = 2$. Kraichnan's direct interaction approximation (DIA) (Kraichnan 1959) predicts $n = 1$ for the far-dissipation range; Orszag (1973) argues $n \leq 1$ and Foias *et al.* conclude $n \geq 1$ (1990) from their analysis. Sreenivasan (1985) found experimentally $n = 1$ (for $0.5 < k/k_d < 1.5$), and Chen *et al.* (1993) observed that the same value would best fit the energy spectrum, in a highly resolved DNS at a relatively low Reynolds number. Thus, the value of $n = 1$ seems to be established from theories, experiments, and numerical simulations. In the rest of this report, we will concentrate on an energy spectrum of the form

$$E(k) \sim (k/k_d)^\alpha \exp(-\beta(k/k_d)). \quad (1)$$

There are only few studies for determining α and β . Some phenomenological models (McComb 1990) suggest $\alpha = -5/3$, the value that has the attractive property of being consistent with Kolmogorov's scaling for lower wavenumbers. On the other hand, both DIA (Kraichnan 1959) and the quasi-normal theories (Orszag 1973) predict $\alpha = 3$. The recent numerical studies by Kida *et al.* (1992) with data for $k/k_d < 4$ found that $\alpha < 0$. Kida *et al.* argue that the negative value for α is connected to the fact that in the dissipation range, the energy transfer is dominated by nonlocal triads. See also the study by Zhou (1994). Domaradzki (1992) obtains $\alpha = -2$, deduced from a scaling ansatz for energy transfer. On the other hand, Chen *et al.* (1993) have confirmed the prediction of the DIA and the EDQNM for $4 < k/k_d < 10$, obtaining $\alpha = 3.3$ and $\beta = 7.1$. Since the Reynolds numbers have been so low, some concerns still remain. Using very large wind tunnel experiments, recently, Saddoughi & Veeravalli (1994) found $\beta = 5.2$, for $0.2 < k/k_d < 1$, at $R_\lambda \approx 600$, in accordance with other works (see e.g. Sanada 1992, Kida *et al.* 1992, Kida & Murakami 1990).

II. Analysis

In this note, we concentrate on the determination of α and β , and their possible dependence on the Reynolds number and wavelength, from numerical simulations. A series of direct numerical simulations of incompressible Navier-Stokes equations has been carried out in a periodic box with sides 2π . We use a pseudo-spectral code on the CM-5 parallel supercomputer at Los Alamos National Laboratory. The resolution ranges from 256^3 (maximum wavenumber $k_{max} = 128$), to 512^3 ($k_{max} = 256$). The system is initialized with a random velocity field with Gaussian statistics. These Fourier velocity amplitudes give an energy spectrum of $k^4 \exp[-(k/k_0)^2]$, with $k_0 \sim 5$. If the system is then left to evolve, nonlinear interactions will redistribute the energy among all available modes. This familiar process delivers energy (in a statistical sense) from larger scales to smaller scales, until the excited wavenumbers are high enough for the viscosity to efficiently dissipate energy. In order to achieve a steady state, energy is pumped into the system by stirring it at the large scales. Specifically, the mode amplitudes for $k = 1$ and 2 (the largest scales) are forced to values consistent with Kolmogorov's $-5/3$ law for the energy spectrum. Once statistically steady values of R_λ and dissipation per unit mass, ϵ , were reached, several snapshots of the energy spectrum, spanning several eddy-turnover characteristic times, were averaged and analyzed.

In Table I, the simulations are sorted by increasing $R_\lambda = v_0 \lambda / \nu$, the Taylor microscale Reynolds number, where v_0 is the mean square velocity, and λ is the Taylor microscale. The table also displays information about how well the dissipation range is resolved (column under " k_{max}/k_d "). Notice that although for some of the simulations k_{max} is of the order of k_d , for six of the simulations $k/k_d \gtrsim 5$, i.e., fairly deep into the dissipation range. In particular, the third entry in the Table ($R_\lambda = 38$) corresponds to a double-precision, 512^3 DNS, which constitutes to the best knowledge of the authors the memory-wise most demanding turbulence simulation to date.

To investigate the possible values for α and β , the quantity, $d \ln E(k) / d \ln k$, as a function of k/k_d , was computed from the energy spectrum, for each of the runs. If the functional form suggested in (1) is correct, then $d \ln E(k) / d \ln k$ should be close to a straight line. The slope, β/k_d , and the intersection of the line with the vertical axis (at $d \ln E(k) / d \ln k = \alpha$) were then determined

by means of a least-squares fit to restricted ranges of k/k_d .

Figure 1 shows $d \ln E(k)/d \ln k$ vs k/k_d for the run that best resolves the dissipation range (run 3 in Table I). Two different slopes are found for the lower and higher wavenumber regions, the latter one being steeper. The transition occurs at $k/k_d \sim 4$. These are common features for the simulations that penetrate deep into the dissipation range, namely, the two-slope behavior for $d \ln E(k)/d \ln k$ vs. k/k_d , and the transition between the two slopes at $k/k_d \sim 4$. It is possible that this shift in slope indicates a change in the nature of the balance of the triadic interactions. We have calculated the transfer energy function $T(k|p, q)$ that measures the amount of energy transferred to wavenumber k , due to interactions with wavenumbers p and q . A preliminary study of $T(k|p, q)$ has not proven to be very revealing.

A bump at $k/k_d \sim 4$ in Fig. 1 accompanies the occurrence of a change in the energy slope. Similar bumps at much smaller k/k_d have been ascribed to a “bottleneck” effect (Lohse & Müller-Groeling 1995) in which triadic interactions that can cascade energy out of the region are inhibited because the spectrum falls off rapidly as k/k_d increases. The energy pileup in our study seems to be of a weaker nature than the one reported by Lohse and Müller-Groeling. In that work, the second-order derivative of the energy with respect to k , is non-monotonic. In our study, the third derivative is the non-monotonic one. A completely satisfying explanation of this effect is more difficult since there is not a clearly conserved quantity associated with the dissipation range, whereas, in the inertial range, the energy flux is independent of the wavenumber, i.e., it is conserved. It could be conjectured through these observations that the sharp change in the local slope of the energy spectrum, will result in an energy pileup close to the transition point, due to the change of the nature of the balance of the triadic interactions.

From the energy spectrum shown in Fig. 1 we computed α and β as a function of k/k_d . The result is displayed in Figure 2. Panels a) and b) show α and β , respectively, as a function of k/k_d for run 3 in Table I. These values were obtained as the local α and β (i.e., local intersection with the y-axis and local slope) of $d \ln E/d \ln k$. The values for α and β assigned to a particular wavenumber, were computed as a least-squares fit to the energy spectrum in the range $[k/k_d - 0.9, k/k_d + 0.9]$. For α we observe a tendency to be negative for lower k and positive for higher wavenumbers. The

plot in panel b) shows that β is fairly constant at a value between five and six, except at $k/k_d \sim 4$, where it drops to about four. For both parameters we observe that a plateau is reached for $k/k_d < 2$ and $k/k_d > 8$. We note that this plateau is not expected to extend to the inertial range scales since that would imply that the energy spectrum would have the same functional form in the dissipation and in the inertial ranges.

Figure 3 shows the dependence of the coefficients α and β on R_λ . Panel a) displays α vs R_λ . Points indicated by the “ \times ” symbol, correspond to measurements done for $k/k_d < 3$ for all runs in Table I; the diamonds correspond to $k/k_d > 6$ for runs 1 through 4. For α , all values obtained from the range $k/k_d < 3$ are consistently negative, within the range -1 to -2 , in the neighbourhood of the Kolmogorov’s value $-5/3$. The four points obtained in the more dissipative range, are positive, in the range $3 - 6$. This two-value behavior of α is a preliminary result. It might be possible that the region of positive values for α can be extended beyond $R_\lambda \sim 40 - 50$ if these values are evaluated from computations that better resolve the dissipation range. In other words, it is possible that the negative values for α appear because they are being computed from the near-dissipation range.

For β , the values measured in the range $k/k_d > 6$ (diamonds), are consistently higher than those measured in the range $k/k_d < 3$. Recently, Sirovich *et al.* derived the energy spectrum (for both inertial range and dissipation range) from the third order structure function (Sirovich, Smith & Yakhot 1994). The value of β obtained in that work is a function of the skewness and the dissipation length. The skewness is believed to be independent of R_λ (Kerr 1985), and thus it is likely that the behavior of β observed in Fig. 2b) is consistent with the derivation in Sirovich *et al.* (1994).

III. Conclusion

These values for α and β are consistent with those reported by Chen *et al.* (1993) obtained with a DNS with a dissipation range resolution comparable to run number three in Table I, but with a lower Reynolds number. The results presented here, and particularly in reference to Fig. 3, might indicate that the exponential functional form (1) set forth for the energy spectrum in the dissipation range, may not be the most appropriate guess for the near-dissipation range, i.e.,

$k/k_d < 4$. Solidly establishing these parameters as a function of R_λ will require next-generation computer capabilities. At this point we are faced with the dilemma of having to choose between performing DNS with relatively high values for R_λ or having a relatively well-resolved dissipation range ($k_{max}/k_d \gg 1$) at a relatively low Reynolds number. This study shows a definite trend, and it should be taken as an effort to shed some light on these issues by using state-of-the-art DNS.

Computations were performed on the CM5 at the Advanced Computing Laboratory at Los Alamos, and on the SP-1 and SP-2 at the IBM Watson Research Center. Discussions with N. Cao, D. Lohse, Z.-S. She, and V. Yakhot are gratefully acknowledged.

REFERENCES

- [1] CHEN S., DOOLEN G.D., HERRING J.R., KRAICHNAN R.H., ORSZAG S.A., & SHE Z.S. 1993 Phys. Rev. Lett. **70**, 3051.
- [2] DOMARADZKI J. A. 1992 Phys. Fluids A, **4**, 2037.
- [3] FOIAS C., MANLEY O., & SIROVICH L. 1990 Phys. Fluids A **2**, 464.
- [4] KIDA S., KRAICHNAN R.H., ROGALLO R.S., WALEFFE F., & ZHOU Y. 1992 in Proceedings of the Summer Program 1992, Center for Turbulence Research, Stanford University and NASA/AMES Research Center.
- [5] KIDA S. & MURAKAMI Y. 1987 Phys. Fluids A, **30**, 2030; KERR R. 1990 J. Fluid Mech. **211**, 309.
- [6] KERR R. 1985 J. Fluid Mech. **153**, 31.
- [7] KOLMOGOROV A.N. 1941 C. R. Acad. Sci. URSS **30** 301.
- [8] KRAICHNAN R.H. 1959 J. Fluid Mech. **5**, 497.
- [9] LOHSE D. & A. MÜLLER-GROELING A. 1995 Phys. Rev. Lett. **74**, 1747.
- [10] MANLEY O.P. 1992 Phys. Fluids A **4**, 1320.
- [11] MCCOMB W.D. 1990 Chapter 2, Clarendon, Oxford.
- [12] NOVIKOV E.A. 1961 Dokl. Akad. Nauk. SSR **139**, 331.
- [13] ORSZAG S.A. 1973 in Fluid Dynamics 1973, Les Houches Summer School of Theoretical Physics, Gordon and Breach, 237.
- [14] SADDUGHI S.G. & VEERAVALLI S.V. 1994 J. Fluid Mech. **268**, 333.
- [15] SANADA T. 1992 Phys. Fluids A **4**, 1086.
- [16] SIROVICH L., SMITH L., & YAKHOT V. 1994 Phys. Rev. Lett. **72**, 344.

- [17] SMITH L.M. & REYNOLDS W.C. 1991 Phys. Fluids A **3**, 992.
- [18] SREENIVASAN K.R. 1985 J. Fluid Mech. **151**, 81.
- [19] TOWNSEND A.A. 1951 Proc. R. Soc. London Ser. A **208**, 534.
- [20] ZHOU Y. 1993 Phys. Fluids A, **5**, 2511.

TABLES

<i>Run</i>	R_λ	k_d	k_{max}/k_d	<i>Resolution</i>
1	20.9	9.7	10	256
2	33.0	13.9	8	256
3	38.0	16.6	12	512
4	39.1	16.7	7	256
5	48.4	19.9	5.8	256
6	52.9	23.6	4.7	256
7	67.9	33.4	3.3	256
8	101.9	55.7	4.4	512

TABLE I. Direct numerical simulations on a $2\pi^3$ periodic domain. Resolution ranges are 128^3 , 256^3 , and 512^3 . The third entry corresponds to a 512^3 resolution, double precision run.

FIGURES

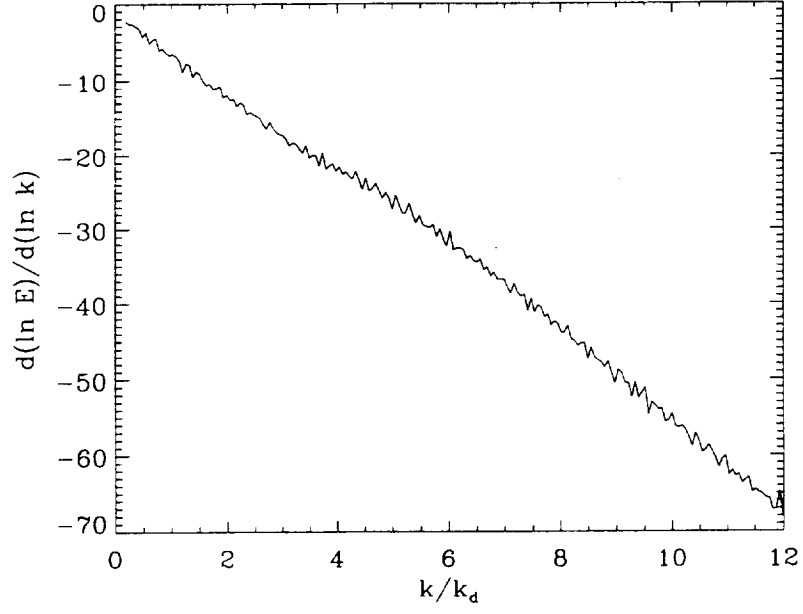


FIG. 1. $d \ln E / d \ln k$ vs. k / k_d for run 3 in Table I. Two slopes are observed in the low and high wavenumber range. The energy spectrum $E(k)$ is computed by averaging the energy spectrum over several eddy-turnover times after the system has reached a steady state.

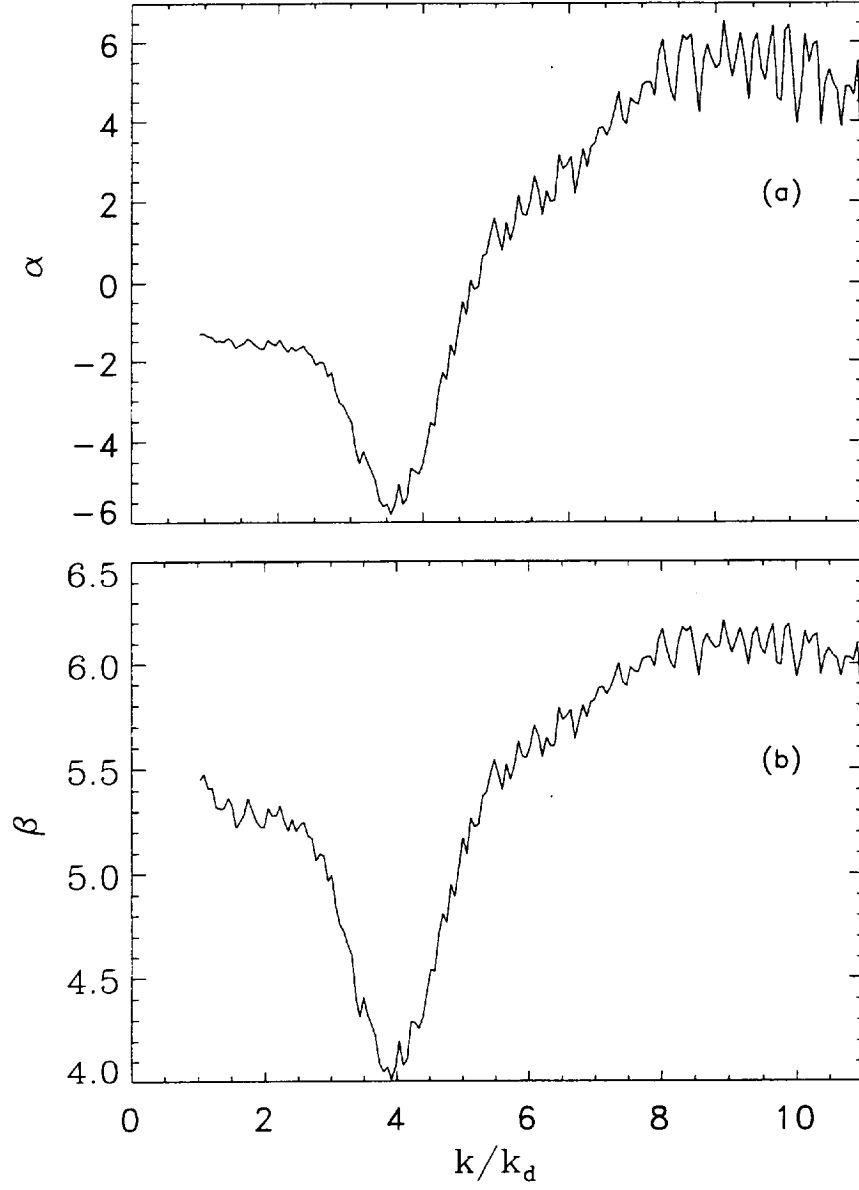


FIG. 2. Local values of α and β as a function of k/k_d , calculated for run 3 in Table I.

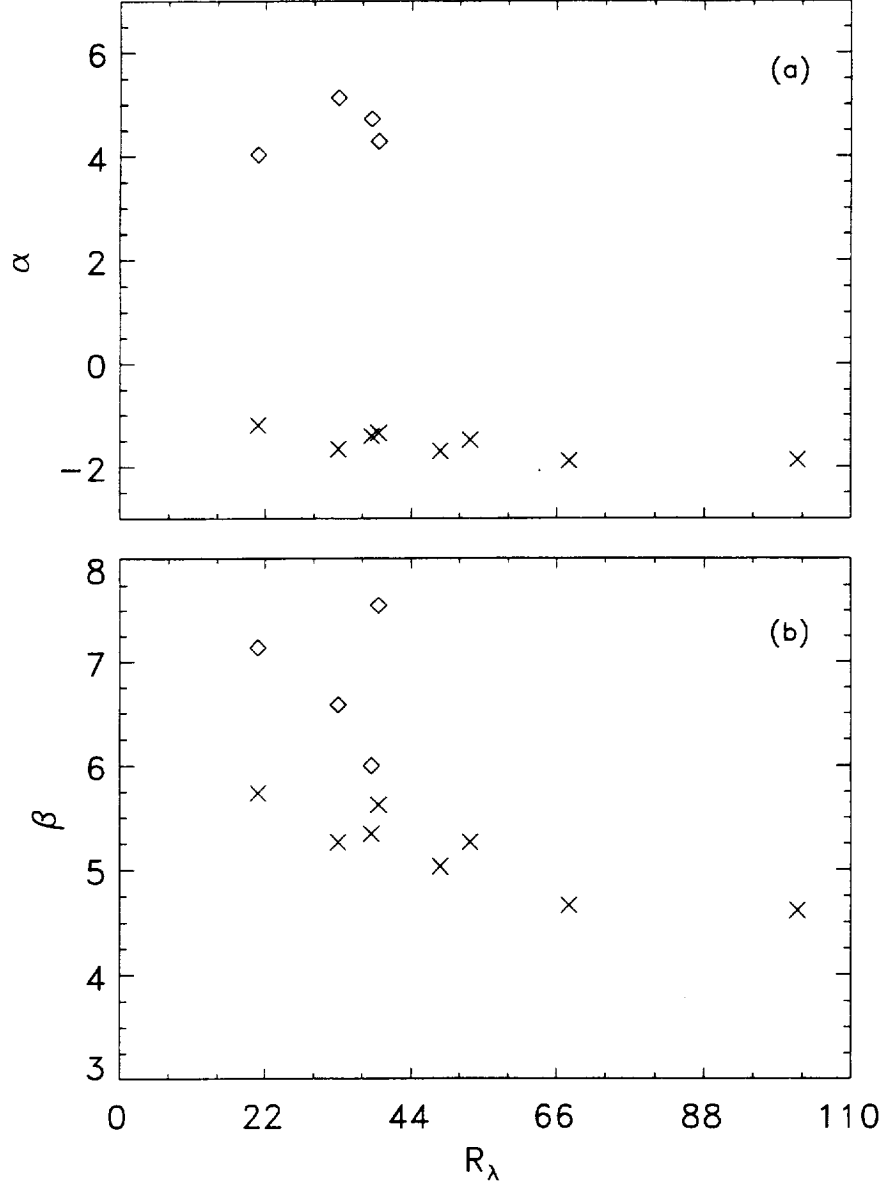


FIG. 3. Parameters α a), and β b), as a function of R_λ . These values are calculated by performing a least-squares fit to $d \ln E / d \ln k$ vs. k/k_d (see Fig. 1). Points indicated by the “ \times ” symbol, correspond to measurements done in the range $k/k_d < 3$ for all runs in Table I; the diamonds correspond to $k/k_d > 6$ for runs 1 through 4.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE May 1996	3. REPORT TYPE AND DATES COVERED Contractor Report		
4. TITLE AND SUBTITLE ENERGY SPECTRUM IN THE DISSIPATION RANGE OF FLUID TURBULENCE		5. FUNDING NUMBERS C NAS1-19480 WU 505-90-52-01		
6. AUTHOR(S) D.O. Martinez, S. Chen, G.D. Doolen, R.H. Kraichnan, L.-P. Wang, and Y. Zhou				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23681-0001		8. PERFORMING ORGANIZATION REPORT NUMBER ICASE Report No. 96-42		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Langley Research Center Hampton, VA 23681-0001		10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA CR-198344 ICASE Report No. 96-42		
11. SUPPLEMENTARY NOTES Langley Technical Monitor: Dennis M. Bushnell Final Report Submitted to the Journal of Plasma Physics.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Category 34		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) High resolution, direct numerical simulations of the three-dimensional incompressible Navier-Stokes equations are carried out to study the energy spectrum in the dissipation range. An energy spectrum of the form $A(k/k_d)^\alpha \exp[-\beta k/k_d]$ is confirmed. The possible values of the parameters α and β , as well as their dependence on Reynolds numbers and length scales, are investigated, showing good agreement with recent theoretical predictions. A "bottleneck"-type effect is reported at $k/k_d \sim 4$, exhibiting a possible transition from near-dissipation to far-dissipation.				
14. SUBJECT TERMS Turbulence; Energy Spectra; Far-dissipation Range			15. NUMBER OF PAGES 14	
			16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT	