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An AWE Implementation for Electromagnetic FEM Analysis

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Abstract

Although full wave electromagnetic systems are large and cumbersome to solve, typically only a few parameters, such as input impedance, S parameters, and far field pattern, are needed by the designer or analyst. A reduced order modeling of these parameters is therefore an important consideration in minimizing the CPU requirements. The Asymptotic Waveform Evaluation(AWE) method is one approach to construct a reduced order model of the input impedance or other useful electromagnetic parameters. We demonstrate its application and validity when used in conjunction with the finite element method to simulate full wave electromagnetic problems.

1 Introduction

The method of Asymptotic Waveform Evaluation (AWE) provides a reduced-order model of a linear system and has already been successfully used in VLSI and circuit analysis to approximate the transfer function associated with a given set of ports/variables in circuit networks [1, 2, 3]. The basic idea of the method is to develop an approximate transfer function of a given linear system from a limited set of spectral solutions. Typically, a Padè expansion of the transfer function is postulated whose coefficients are then determined

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by matching the Padé representation to the available spectral solutions of the complete system.

In this paper we investigate the suitability of the AWE method for approximating the response of a given parameter in full wave simulations of radiation or scattering problems in electromagnetics. Of particular interest is the use of AWE for evaluating the input impedance of the antenna over a given bandwidth from a knowledge of the full wave solution at a few (even a single) frequency points. Also, the method can be used to fill in a backscattering pattern with respect to frequency using a few data samples of that pattern. Below we first describe the recasting of the FEM system for application of the AWE. We then proceed to describe the AWE method and demonstrate its application, accuracy and efficiency in computing the input impedance of a shielded microstrip stub.

2 Theory of Asymptotic Waveform Evaluation

2.1 FEM System

The application of the finite element method to full wave electromagnetic solutions amounts to generating a linear system of equations by extremizing the functional [4]

$$\mathcal{F} = < \nabla \times \mathbf{E}, \overline{\overline{\alpha}} \cdot \nabla \times \mathbf{E} > -k^2 < \mathbf{E}, \overline{\overline{\beta}} \cdot \mathbf{E} > + k\mathbf{b.t.} \tag{1}$$

where $<,>$ denotes an inner-product and $\mathbf{b.t.}$ is a possible boundary term whose specific form is not required for this discussion. Also, the dyadics $\overline{\overline{\alpha}}$ and $\overline{\overline{\beta}}$ are material related coefficients, $k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$ is the wavenumber and ω is the corresponding operating frequency with c being the speed of light. A discretized form of (1) incorporating the appropriate boundary conditions is [5]

$$\left(\mathbf{A}_0 + k\mathbf{A}_1 + k^2\mathbf{A}_2\right)\{X\} = \{f\} \tag{2}$$

where $\mathbf{A}_i$ denote the usual square (sparse) matrices and $\{f\}$ is a column matrix describing the specific excitation.

Clearly (2) can be solved using direct or iterative methods for a given value of the wavenumber. Even though $\mathbf{A}_i$ is sparse, the solution of the system (2) is computationally intensive and must be repeated for each k to obtain a frequency response. Also, certain analyses and designs may require both temporal and frequency responses placing additional computational burdens and a repeated solution of (2) is not an efficient approach in generating these responses. An application of AWE to achieve an approximation to these responses is an attractive alternative. Below we formulate AWE in conjunction with the finite element method (2), for modeling antenna and microwave circuit problems. For these problems, the excitation column $\{f\}$ is a linear function of the wavenumber and can therefore be stated as

$$\{f\} = k\{f_1\} \tag{3}$$

with $\{f_1\}$ being independent of frequency. This observation will be specifically used in our subsequent presentation.

2.2 Asymptotic Waveform Evaluation

To describe the basic idea of AWE in conjunction with the FEM, we begin by first expanding the solution $\{X\}$ in a Taylor series about k_0 as

$$\begin{aligned}\{X\} = \{X_0\} + (k-k_0)\{X_1\} + (k-k_0)^2\{X_2\} + \ldots \\ +(k-k_0)^l\{X_l\} + \mathcal{O}\left\{(k-k_0)^{l+1}\right\}\end{aligned} \tag{4}$$

where $\{X_0\}$ is the solution of (2) corresponding to the wavenumber k_0. By introducing this expansion into (2) and equating equal powers of k in conjunction with (3), after some manipulations, we find that

$$\begin{aligned}\{X_0\} &= k_0\tilde{\mathbf{A}}_0^{-1}\{f_1\} \\ \{X_1\} &= \tilde{\mathbf{A}}_0^{-1}\left[\{f_1\} - \mathbf{A}_1\{X_0\} - 2k_0\mathbf{A}_2\{X_0\}\right] \\ \{X_2\} &= -\tilde{\mathbf{A}}_0^{-1}\left[\mathbf{A}_1\{X_1\} + \mathbf{A}_2(\{X_0\} + 2k_0\{X_1\})\right] \\ \ldots &= \ldots\ldots \\ \{X_l\} &= -\tilde{\mathbf{A}}_0^{-1}\left[\mathbf{A}_1\{X_{l-1}\} + \mathbf{A}_2(\{X_{l-2}\} + 2k_0\{X_{l-1}\})\right]\end{aligned} \tag{5}$$

with

$$\tilde{\mathbf{A}}_0 = \mathbf{A}_0 + k_0\mathbf{A}_1 + k_0^2\mathbf{A}_2 \tag{6}$$

Expressions (5) are referred to as the system moments whereas (6) is the system at the prescribed wavenumber (k_0). Although an explicit inversion of $\mathbf{A}_0^{-1}$ may be needed as indicated in (5), this inversion is used repeatedly and can thus be stored out-of-core for the implementation of AWE. Also, given that for input impedance computations we are typically interested in the field value at one location of the computational domain, only a single entry of $\{X_l(k)\}$ need be considered, say (the pth entry) $X_l^p(k)$. The above moments can then be reduced to scalar form and the expansions (5) become a scalar representation of $X_l^p(k)$ about the corresponding solution at k_0. To yield a more convergent expression, we can instead revert to a Padè expansion which is a conventional rational function.

For transient analysis the Padè expansion can be cast by partial fraction decomposition [3, 6] into

$$X_q^p(k) = X_{q0}^p + \sum_{i=1}^{q} \frac{r_i}{k - k_0 - k_i} \tag{7}$$

where X_{q0} is the limiting value as k tends to infinity. This is a qth order representation and is suitable for time/frequency domain transformation. As can be realized, the residues and poles (r_i and $k_0 + k_i$) in (7) correspond to those of the original physical system and play important roles in the accuracy of the approximation. As can be expected a higher order expansion with more zeros and poles can provide an improved approximation. The accuracy of AWE relies on the prediction of the dominant residues and poles located closest to k_0 in a complex plane. Its key advantage is that for many practical electromagnetic problems only a few poles and zeros are needed for a sufficiently accurate representation.

For a hybrid finite element – boundary integral system, the implementation of AWE is more involved because the fully populated boundary integral submatrix of the system has a more complex dependence on frequency. In this case we may instead approximate the full submatrix with a spectral expansion of the exponential boundary integral kernel to facilitate the extraction of the system moments. This approach does increase the complexity in implementing AWE. However, AWE still remains far more efficient in terms of CPU requirements when compared to the conventional approach of repeating the system solution at each frequency.

3 Numerical Implementation

As an application of AWE to a full wave electromagnetic simulation, we consider the evaluation of the input impedance for a microstrip stub shielded in a metallic rectangular cavity as shown in figure 1. The stub's input impedance is a strong function of frequency from 1–3 GHz and this example is therefore a good demonstration of AWE's capability.

The shielded cavity is $2.38cm \times 6.00cm \times 1.06cm$ in size and the microstrip stub resides on a $0.35cm$ thick substrate having a dielectric constant of 3.2. The stub is $0.79cm$ wide and $\lambda/2$ long at 1.785 GHz and we note that the back wall of the cavity is terminated by a metal-backed artificial absorber having relative constants of $\epsilon_r = (3.2, -3.2)$ and $\mu_r = (1.0, -1.0)$.

As a reference solution, the frequency response of the shielded stub was first computed from 1 to 3 GHz at 40 MHz intervals (50 points) using a full wave finite element solution. To demonstrate the efficacy and accuracy of AWE we chose a single input impedance solution at 1.78 GHz in conjunction with the 4th order and 8th order AWE in (7) to approximate the system response. Clearly the chosen number of poles or order of the expansion leads to different accuracies. As seen in Figure 2, the 4th order AWE representation is in agreement with the real and reactive parts of the reference input impedance solution over a 56% and 33% bandwidth, respectively. Surprisingly, the 8th order AWE representation recovers the reference solution over the entire 1-3 GHz band for both the real and reactive parts of the impedance. However, the CPU requirements for the 4th and 8th order approximations are nearly the same except for a few more matrix–vector products needed for the higher order expansion. The number of these operations are of the same order as that of the AWE expansion and are much smaller than the size of the original numerical system.

We conclude that the AWE representation is an extremely useful addition to electromagnetic simulation codes and packages for computing wideband frequency responses using only a few samples of the system solution.

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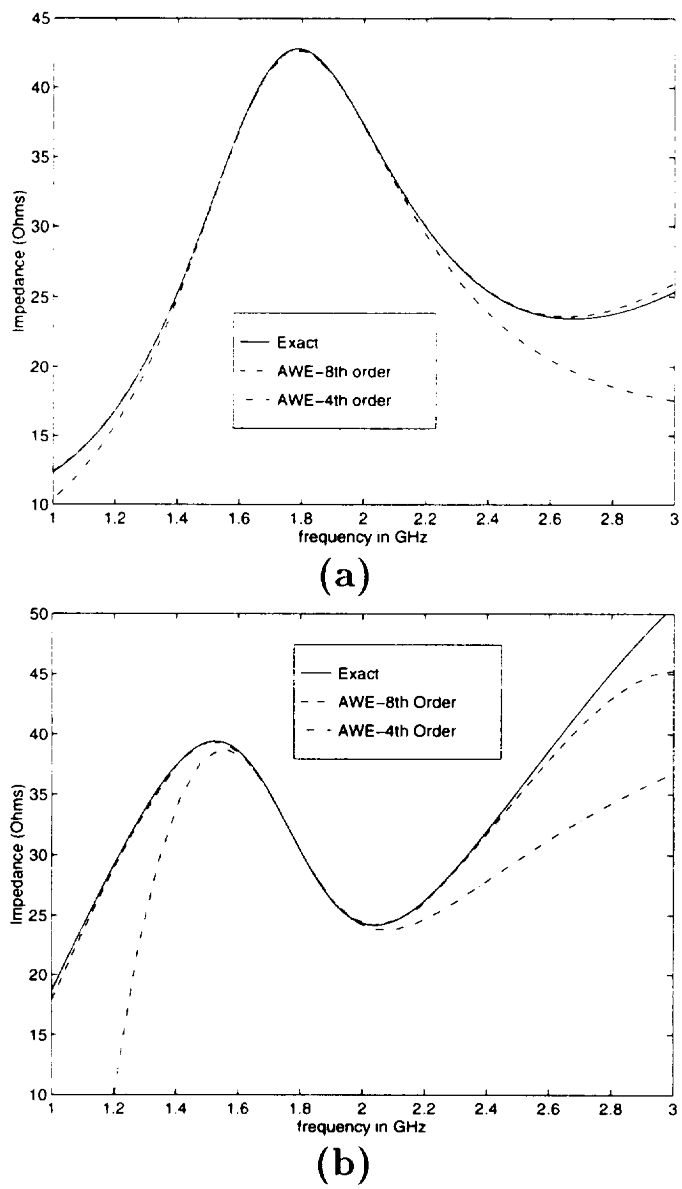

Figure 3: Results of the 4th and 8th order AWE implementation using a single point expansion at 1.78 GHz. (a) Real part and (b) imaginary part of the Input impedance