APPROACH TO SYNCHRONIZATION CONTROL OF MAGNETIC BEARINGS USING FUZZY LOGIC

Li-Farn Yang[*]
Department of Mechanical Engineering
National Chung Cheng University
Chia-Yi, TAIWAN, R.O.C.

ABSTRACT

This paper presents a fuzzy-logic approach to the synthesis of synchronization control for a magnetically suspended rotor system. The synchronization control enables a whirling rotor to undergo synchronous motion along the magnetic bearing axes; thereby avoiding the gyroscopic effects that degrade the stability of rotor systems when spinning at high speed. The control system features a fuzzy controller acting on the magnetic bearing device, in which the fuzzy inference system is trained through fuzzy rules to minimize the differential errors between four bearing axes so that an error along one bearing axis can affect the overall control loop for the motion synchronization. Numerical simulations of synchronization control for the magnetically suspended rotor system are presented to show the effectiveness of the present approach.

INTRODUCTION

Active magnetic bearings, which permit non-contact suspension of levitated objects, have received increasing attention in recent years. The contactless nature of magnetic bearings brings many advantages over the conventional bearings, such as energy efficiency, low wear, longer life span, absence of lubrication and mechanical maintenance, and wide range of working temperature. Also, closed-loop control of magnetic bearings enables active vibration suppression and on-line control of bearing stiffness. Studies [1-3] performed so far indicate that magnetic bearings have been very effectively used for the suspension of high-speed rotating machines, essentially at speeds over 10,000 rpm. For instance, NASA has employed magnetic bearings to support the high-speed turbopumps for use in space shuttle main engines. On the other hand, magnetic bearings have been installed on momentum wheels and CMG devices at NASA [1-5] as the gyrotorquers for the attitude control and momentum arrangement of space structures. Other industrial applications of actively controlled magnetic bearings include high-speed centrifuges, magnetic spindles, vacuum pumps, micromachines, etc. [1-2].

Magnetic bearing dynamics are inherently nonlinear as the result of nonlinearities in the electromagnetic fields and force coupling effects among the various axes. These nonlinear characteristics require an increase in the complexity of modeling, estimation, and control of the

---

[*] Associate Professor, AIAA Member.

575

magnetic bearings. In addition, the control performance of magnetic bearings is very sensitive to unpredictable disturbances and the plant uncertainties, wherein the classical feedback control does not seem entirely effective and satisfactory for such purposes. Various control strategies have been studied for the adaptive control of magnetic bearings when disturbed with uncertain dynamics or unknown parameters. Some of them reviewed as references here are adaptive control [6], time delay control [7], sliding mode control [3], and learning control [8]. The investigation of these papers was aimed at using different approaches to adapt the respective controller to changes in the parameters of magnetic bearings so as to continuously strive to optimize the control performance. Once a controller is activated, the magnetically suspended rotor will experience the conical and translatory whirl modes when whirling about the central axis as shown in Fig. 1. The conical whirl mode gives rise to the unbalanced torque's twisting the rotor which in turn triggers the gyroscopic effects, and thereby effecting the instability of the rotor system if not properly controlled. However, the translatory whirl mode yields the synchronous motion along the radial directions, important for a spinning rotor especially for use in high-precision, fine-cutting process. The purpose of this paper is thus to explore the synchronization problem of the magnetically suspended rotor system from the viewpoint of fuzzy-logic control [9,10].
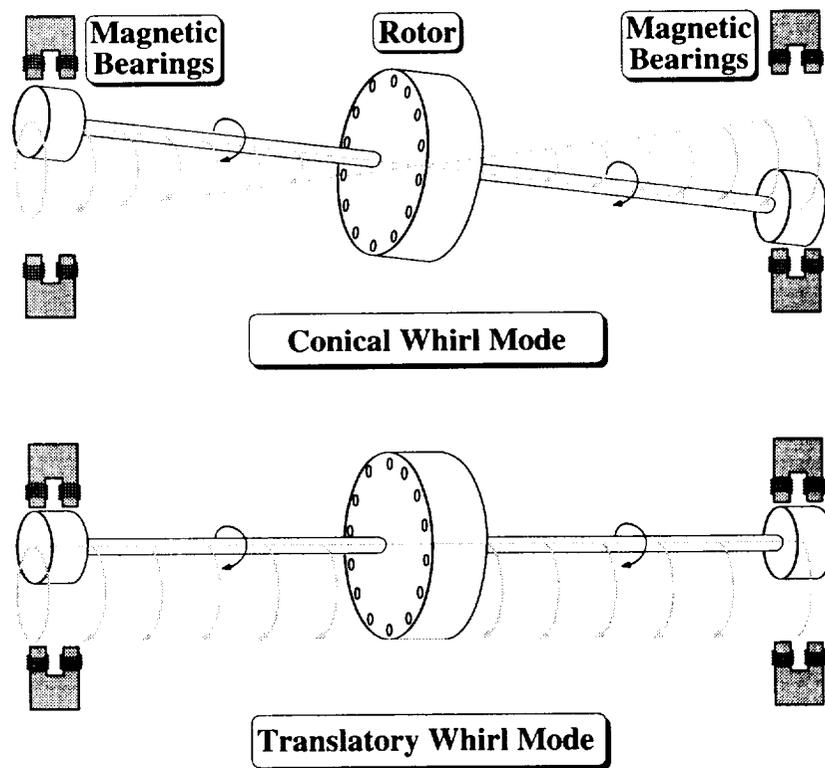
Figure 1: Whirl modes of a rigid rotor in magnetic bearings

In this paper, we introduce a fuzzy-logic scheme for the adaptive synchronization of the magnetic bearing system, which consists of a local disturbance compensator and a coupling controller for each bearing axis. The local compensator balances the rotor against disturbances, while the

coupling control responds to the synchronization error, i.e., the difference between the two motion errors along the bearing axes. The fuzzy inference system is trained through fuzzy rules for the synchronization control, in which each fuzzy controller fuzzifies the inputs by the trapezoidal membership function, applies an "AND" logic operator to handle multiple fuzzified inputs, shapes the consequent set by a fuzzy associated memory (FAM) matrix, and aggregates all outputs during defuzzifying. The defuzzification method is the centroid computation which involves the computation of centroid values of regions defined by overlapping membership functions.

The content of this paper will be outlined as follows: First, the description of the system dynamics is presented for a rotor system equipped with magnetic bearings. Then, the derivation of the fuzzy controller is detailed and applied to the synthesis of synchronization control for rejecting gyroscopic effects as well as disturbances from the rotor system. Finally, simulation results are presented for discussion.

SYSTEM DYNAMICS

In this section, the dynamics of a rotor system and magnetic bearings are studied and incorporated into a rotor-bearing system. This system is composed of a disk mounted on a rigid shaft which is actively positioned in the radial directions through the use of the magnetic bearings located at two shaft ends. This paper considers the utilization of radial magnetic bearings as an actuating device onboard the rotor system. Let's start with the description of rotor dynamics to serve as the background for the investigation of this paper.
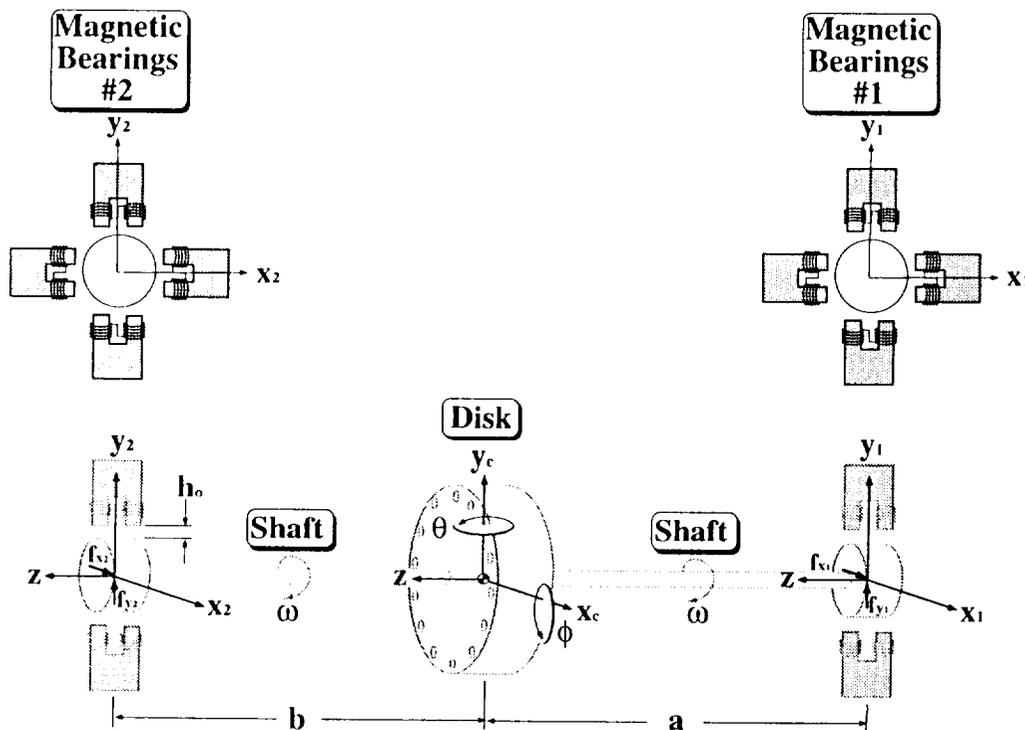
Figure 2: A rigid rotor system with actively controlled magnetic bearings

As demonstrated by Fig. 2, the rotor is modeled as a disk constantly spinning about the principal z-axis of the shaft while simultaneously suspended by two sets of magnetic bearings. Assume x_c and y_c denote the displacements of the center of mass of the rotor along the x and y directions as shown in Fig. 2, and ϕ and θ indicate (roll and pitch) angles of rotation about the x and y axes, respectively. These angles are normally assumed to be very small due to the restriction of narrow air gap within the magnetic bearings. The equations of motion of this rotor-bearing system can then be derived in the following matrix form.

$$
\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_{rr} & 0 \\ 0 & 0 & 0 & I_{rr} \end{bmatrix} \begin{bmatrix} \ddot{x}_c \\ \ddot{y}_c \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + I_{zz}\omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ a & -b & 0 & 0 \\ 0 & 0 & -a & b \end{bmatrix} \begin{bmatrix} f_{x1} \\ f_{x2} \\ f_{y1} \\ f_{y2} \end{bmatrix}
\tag{1}
$$

where m is the rotor mass, I_{rr} is the moment-of-inertia of the rotor in the radial direction, I_{zz} is the moment-of-inertia about the z-axis, and ω is the spinning rate of the rotor about the z-axis. The second term in Eq. (1) can readily be recognized as the gyroscopic momenta depending on the spinning rate ω.

As for the input in Eq. (1), the f_{xi}'s and f_{yi}'s represent the magnetic forces provided by two sets of magnetic bearings each made of an eight pole electromagnet. The electromagnets, when charged, are capable of inducing coupled magnetic forces along the x and y axes. The coupling effects of magnetic forces across two axes can be neglected without loss of generality so that the magnetic force along each axis is derived independently of the displacement and current along the other axis. Figure 2 demonstrates the magnetic bearings #1 and #2 located at distances a and b from the disk. Assume that the control current signals along the x-axis are denoted by i_{x1} and i_{x2} for the bearings #1 and #2, respectively, while those along the y-axis indicated by i_{y1} and i_{y2}, accordingly. Also, the displacements of the shaft from the bearing center are described by x_i and y_i at two bearing ends. Based on Maxwell's law, the magnetic forces due to the electromagnet along the x and y axes can be modeled by:

$$
f_{x1} = G\left[\frac{(I_o+i_{x_1})^2}{(h_o-x_1)^2} - \frac{(I_o-i_{x_1})^2}{(h_o+x_1)^2}\right], \quad f_{x2} = G\left[\frac{(I_o+i_{x_2})^2}{(h_o-x_2)^2} - \frac{(I_o-i_{x_2})^2}{(h_o+x_2)^2}\right],
$$
$$
f_{y1} = G\left[\frac{(I_1+i_{y_1})^2}{(h_o-y_1)^2} - \frac{(I_o-i_{y_1})^2}{(h_o+y_1)^2}\right] - \frac{b}{a+b}mg, \quad f_{y2} = G\left[\frac{(I_2+i_{y_2})^2}{(h_o-y_2)^2} - \frac{(I_o-i_{y_2})^2}{(h_o+y_2)^2}\right] - \frac{a}{a+b}mg
\tag{2}
$$

where h_0 is the nominal air gap at equilibrium and G is an electromagnet constant. The I_0 is a bias current in Eq. (2), whereas the bias currents I_1 and I_2 account for balancing the weight of the disk and they can be written by :

$$
I_1 = \frac{h_o}{\sqrt{G}}\sqrt{\frac{GI_o^2}{h_o^2} + (\frac{b}{a+b})mg} \quad \text{and} \quad I_2 = \frac{h_o}{\sqrt{G}}\sqrt{\frac{GI_o^2}{h_o^2} + (\frac{a}{a+b})mg}
\tag{3}
$$

Since the air gap is restricted to be very narrow, the magnetic forces in Eq. (2) can be linearized and arranged into a matrix form given by:

$$
\begin{bmatrix} f_{x_1} \\ f_{x_2} \\ f_{y_1} \\ f_{y_2} \end{bmatrix} = \frac{2GI_o}{h_o^2} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1+\frac{I_1}{I_o} & 0 \\ 0 & 0 & 0 & 1+\frac{I_2}{I_o} \end{bmatrix} \begin{bmatrix} i_{x_1} \\ i_{x_2} \\ i_{y_1} \\ i_{y_2} \end{bmatrix} + \frac{2GI_o^2}{h_o^3} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1+(\frac{I_1}{I_o})^2 & 0 \\ 0 & 0 & 0 & 1+(\frac{I_2}{I_o})^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}
\tag{4}
$$

which decouples control currents and displacements for the magnetic forces. Moreover, the following relations are generated from the geometry illustrated in Fig. 2.

$$
\begin{aligned}
x_c &= \frac{bx_1 + ax_2}{a+b} \\
y_c &= \frac{by_1 + ay_2}{a+b}
\end{aligned}
\quad \text{and} \quad
\begin{aligned}
\theta &= \tfrac{1}{a}(x_1 - x_c) \\
\phi &= \tfrac{1}{a}(y_c - y_1)
\end{aligned}
\tag{5}
$$

Substituting the disk variables (x_c , y_c , θ, ϕ) in Eq. (1) with those relative to the bearing axes yields:

$$
\ddot{\mathbf{x}} = \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{B}\mathbf{u}
\tag{6}
$$

where the state variable vector is given by $\mathbf{x} = [x_1 , x_2 , y_1 , y_2]^T$, and the input vector by $\mathbf{u} = [i_{x1} , i_{x2} , i_{y1} , i_{y2}]^T$. In Eq. (6), the damping, stiffness, and input matrices are:

$$
\mathbf{C} = \left(\frac{.w}{a+b}\right)\left(\frac{I_{zz}}{I_{rr}}\right) \begin{bmatrix} 0 & 0 & a & -a \\ 0 & 0 & -b & b \\ a & -a & 0 & 0 \\ -b & b & 0 & 0 \end{bmatrix},
$$

$$
\mathbf{K} = \frac{2G}{h_o^3} \begin{bmatrix} 2I_o^2(\frac{1}{m} + \frac{a^2}{I_{rr}}) & 2I_o^2(\frac{1}{m} - \frac{ab}{I_{rr}}) & 0 & 0 \\ 2I_o^2(\frac{1}{m} - \frac{ab}{I_{rr}}) & 2I_o^2(\frac{1}{m} + \frac{b^2}{I_{rr}}) & 0 & 0 \\ 0 & 0 & (I_o^2+I_1^2)(\frac{1}{m} + \frac{a^2}{I_{rr}}) & (I_o^2+I_2^2)(\frac{1}{m} - \frac{ab}{I_{rr}}) \\ 0 & 0 & (I_o^2+I_1^2)(\frac{1}{m} - \frac{ab}{I_{rr}}) & (I_o^2+I_2^2)(\frac{1}{m} + \frac{b^2}{I_{rr}}) \end{bmatrix},
$$

$$
\mathbf{B} = \frac{2G}{h_o^2} \begin{bmatrix} 2I_o(\frac{1}{m} + \frac{a^2}{I_{rr}}) & 2I_o(\frac{1}{m} - \frac{ab}{I_{rr}}) & 0 & 0 \\ 2I_o(\frac{1}{m} - \frac{ab}{I_{rr}}) & 2I_o(\frac{1}{m} + \frac{b^2}{I_{rr}}) & 0 & 0 \\ 0 & 0 & (I_o+I_1)(\frac{1}{m} + \frac{a^2}{I_{rr}}) & (I_o+I_2)(\frac{1}{m} - \frac{ab}{I_{rr}}) \\ 0 & 0 & (I_o+I_1)(\frac{1}{m} - \frac{ab}{I_{rr}}) & (I_o+I_2)(\frac{1}{m} + \frac{b^2}{I_{rr}}) \end{bmatrix}
$$

Equation (6) is the model of a rotor system equipped with magnetic bearings as necessary for the derivation of the fuzzy-based synchronization controller in the next section.

CONTROL DESIGN

In this paper, the control objective of the magnetic bearing system is to synchronize the motion of a suspended rotor during whirling, so as to insure the rotor system against the unbalance and gyrostatics. In doing so, we present a synchronization control concept that makes concurrent use of a fuzzy-logic control and a coupling control, and assesses its applicability to the magnetic control of the rotor system. An integrated cross-coupled, fuzzy-based controller is developed with the respective merits of each particular control included. First, it is known that fuzzy control is very effective in adapting nonlinear systems to the plant uncertainties and unpredictable disturbances. Then, adding the coupling control in conjunction with the fuzzy adaptation can enable the resulting control system to perform the motion synchronization for the multi-axis system.

Figure 3 shows the control block diagram of a fuzzy-based synchronization controller for the magnetically suspended rotor system. The fuzzy inference system consists of four local compensators that are cross-coupled to one another by using three coupling controllers to compromise the differential errors between the bearing axes. The local compensators are applied to the four bearing coordinates in balancing the rotor around the center of rotation. The coupling controllers link the $x_1 - x_2$, $y_1 - y_2$, and $x_1 - y_1$ axes, respectively; in response to the synchronization errors as defined by:

$$
\begin{aligned}
e_x(t) &= x_1(t) - x_2(t), & \dot{e}_x(t) &= \dot{x}_1(t) - \dot{x}_2(t), \\
e_y(t) &= y_1(t) - y_2(t), & \dot{e}_y(t) &= \dot{y}_1(t) - \dot{y}_2(t), \\
e_{xy}(t) &= x_1(t) - y_1(t), & \dot{e}_{xy}(t) &= \dot{x}_1(t) - \dot{y}_1(t)
\end{aligned}
\tag{7}
$$

which in fact represent the differences between the two motion errors along the bearing axes. The first two equations in Eq. (7) attempt to synchronize the motion of the rotor in parallel to the directions of $x_1 - x_2$ and $y_1 - y_2$ respectively, whereas the last equation cross-couples that between the $x_1 - y_2$ axes. It can be seen that each controller collects the information of displacement and velocity of the rotor as inputs to the fuzzy system that outputs a control current to the magnetic bearings. Also, note that the synchronization errors drive the two sets of local fuzzy controllers in opposite directions, which will result in fast removal of the synchronization errors.
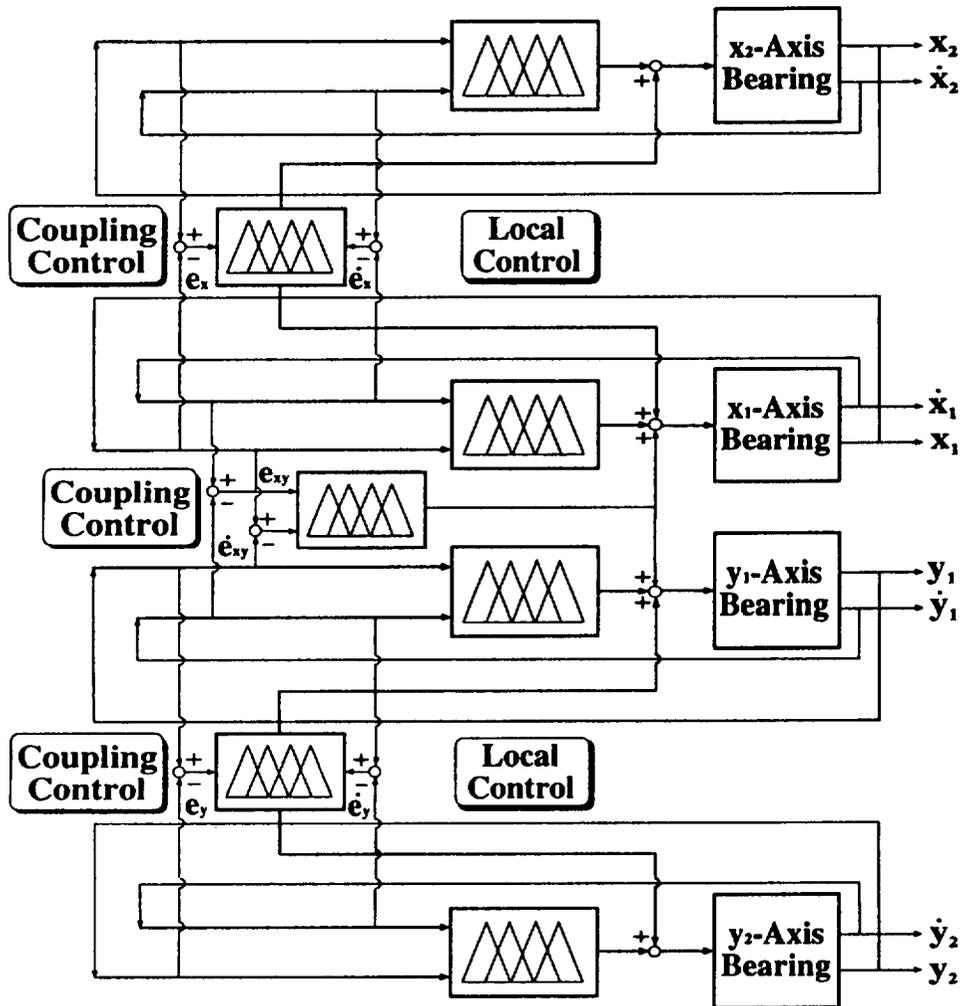
Figure 3: Control Block diagram of a fuzzy-based synchronization control

The fuzzy inference system is guided through the fuzzy rules in order to train the entire control system as for the motion synchronization. In this paper, the fuzzy controller fuzzifies the inputs by a trapezoidal membership function, applies "AND" logic operator to sort out multiple fuzzified inputs, shapes the consequent set by a FAM matrix, and aggregates all outputs during defuzzifying. Figure 4 demonstrates the membership functions and FAM matrices for the displacement and velocity of the rotor in each bearing axis. Three fuzzy sets that are specified for the input variables $(x_1, \dot{x}_1, e_x, \dot{e}_x,$ etc.) are "Positive" (P), "Zero" (ZE), and "Negative" (N), and five fuzzy sets are assigned for the output variables $(i_{x1}, i_{x2}, i_{y1}, i_{y2})$ including "Positive Large" (PL), "Positive Small" (PS), "Zero" (ZE), "Negative Small" (NS), and "Negative Large" (NL). Then, the fuzzy intersection (AND) operator is applied to aggregate two membership grades such that:

$$\mu_{P \cap N}(x_1, y_1) = \mu_P(x_1) \otimes \mu_N(y_1) = min[\mu_P(x_1), \mu_N(y_1)] \tag{8}$$

where $\mu P(x_1)$ stands for the value of a membership function for the input x_1 according to the fuzzy set "Positive". The outcome from Eq. (8) is used to weight each entry in the FAM matrix, for instance:

$$w_{PL}(x_1, \dot{x}_1) = \mu_N(x_1) \otimes \mu_N(\dot{x}_1) \tag{9}$$

for the first entry in the $x_1 - \dot{x}_1$ FAM matrix. As for the output defuzzified from the fuzzy system, the centroid computation is conducted to calculate the centroid values of regions defined by overlapping membership functions. For instance, for the $x_1 - \dot{x}_1$ fuzzy system the output is defuzzified by:
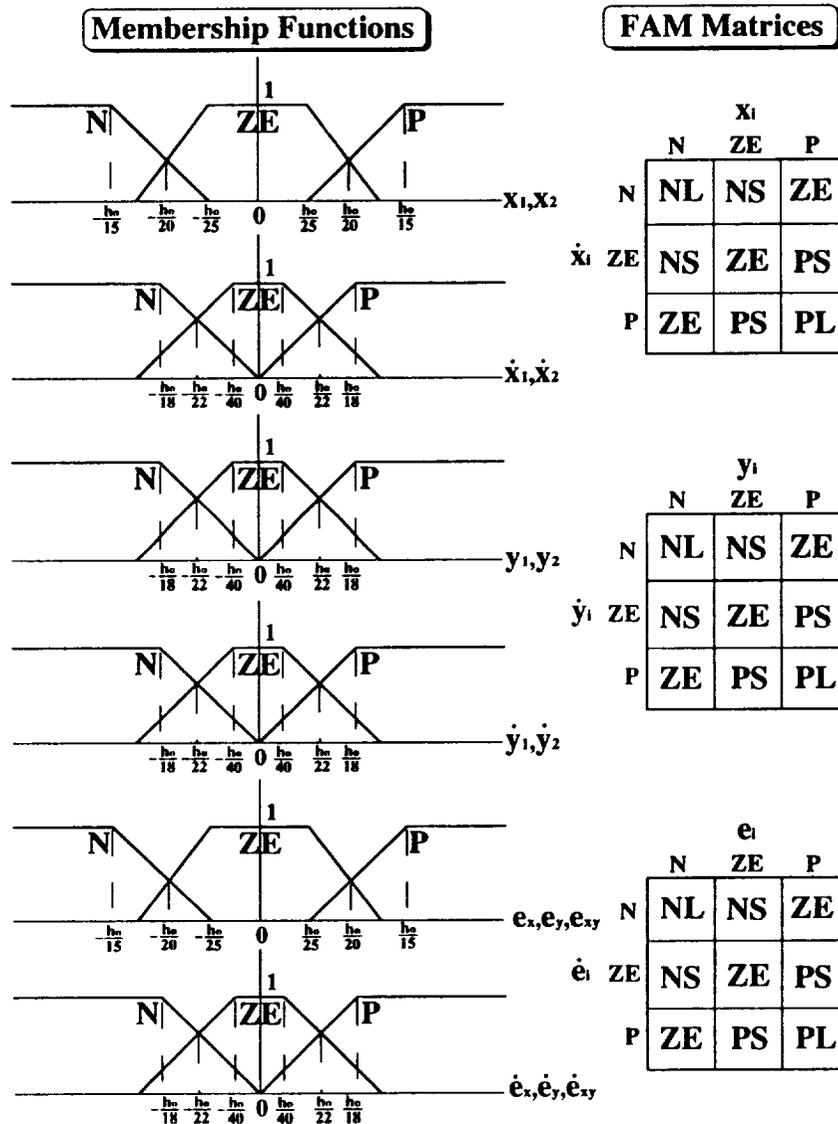
Figure 4: Input/output membership functions and FAM matrices

$$i_{x_1} = \frac{1}{\sum\limits_i w_i}(w_{PL} \cdot PL + w_{PS} \cdot PS + w_{ZE} \cdot ZE + w_{NS} \cdot NS + w_{NL} \cdot NL) \tag{10}$$

which provides the control current for the x_1 axis. In this way, the control currents for the other axes can be defuzzified from the respective fuzzy system. Note that the total control current for each bearing axis must include those from the coupling controllers, although they are not explicitly expressed here.

The cross-coupled, fuzzy-logic control synthesis is then accomplished in this section for the synchronization control design onboard the magnetic bearing system. A closed-loop system is then obtained for the implementation of the numerical simulations, which will be the subject in the following section.

SIMULATION RESULTS

The model parameters of the rotor system with magnetic bearings for simulations are summarized in Table 1. A fuzzy-based synchronization controller has been applied to the rotor-bearing system to implement 2-second simulations subject to: (i) a sequence of impulsive disturbances on the four axes of bearings #1 and #2, and (ii) the sudden change of speed on the spinning rotor, respectively.

Table 1: Model parameters of a rotor-bearing system

Disk:		Magnetic Bearings:	
m=0.75	Kg	G=1.2×10⁻⁴	N-m²/Amp²
I_{rr}=2.5×10⁻³	Kg/m⁴	h_o=1.5 ×10⁻³	m
I_{zz}=5.0×10⁻³	Kg/m⁴	I_o=1.0	Amp
e=0.01	m	I_1=1.0361	Amp
ω=100.0	rad/sec	I_2=1.0361	Amp
a=0.5	m	b=0.5	m
Fuzzy Control Parameters:			
PL=0.01	Amp	PS=0.005	Amp
ZE=0.0002	Amp	NS=-0.005	Amp
NL=-0.01	Amp		

Figure 5 illustrates the numerical results of a control simulation along with four force impulses imposed on the four axes of bearing #1 and #2. The forces are of magnitude +/-10.0 N acting on axes x_1, x_2, y_1, and y_2 around 0.5, 0.6, 1.0, and 1.1 seconds, respectively. It can be seen in Figs. 5(a) - 5(d) that the four displacements respond synchronously to each other under the synchronization control. This implies the interactive dynamics reaction between the bearing axes due to the coupling control law. Above all, the peak magnitudes in Figs. 5(a) - 5(d) are considerably reduced within the range of 0.1 mm. Furthermore, the attendant oscillations are damped out within 0.4 seconds. Therefore, one disturbed axis can rapidly be recovered by other undisturbed axes in the presence of the fuzzy-based synchronization control. Figures 5(e) and 5(f) show the responses of the control currents. As can be seen, the amplitudes of the currents are restricted within the range of 0.015 Amp.
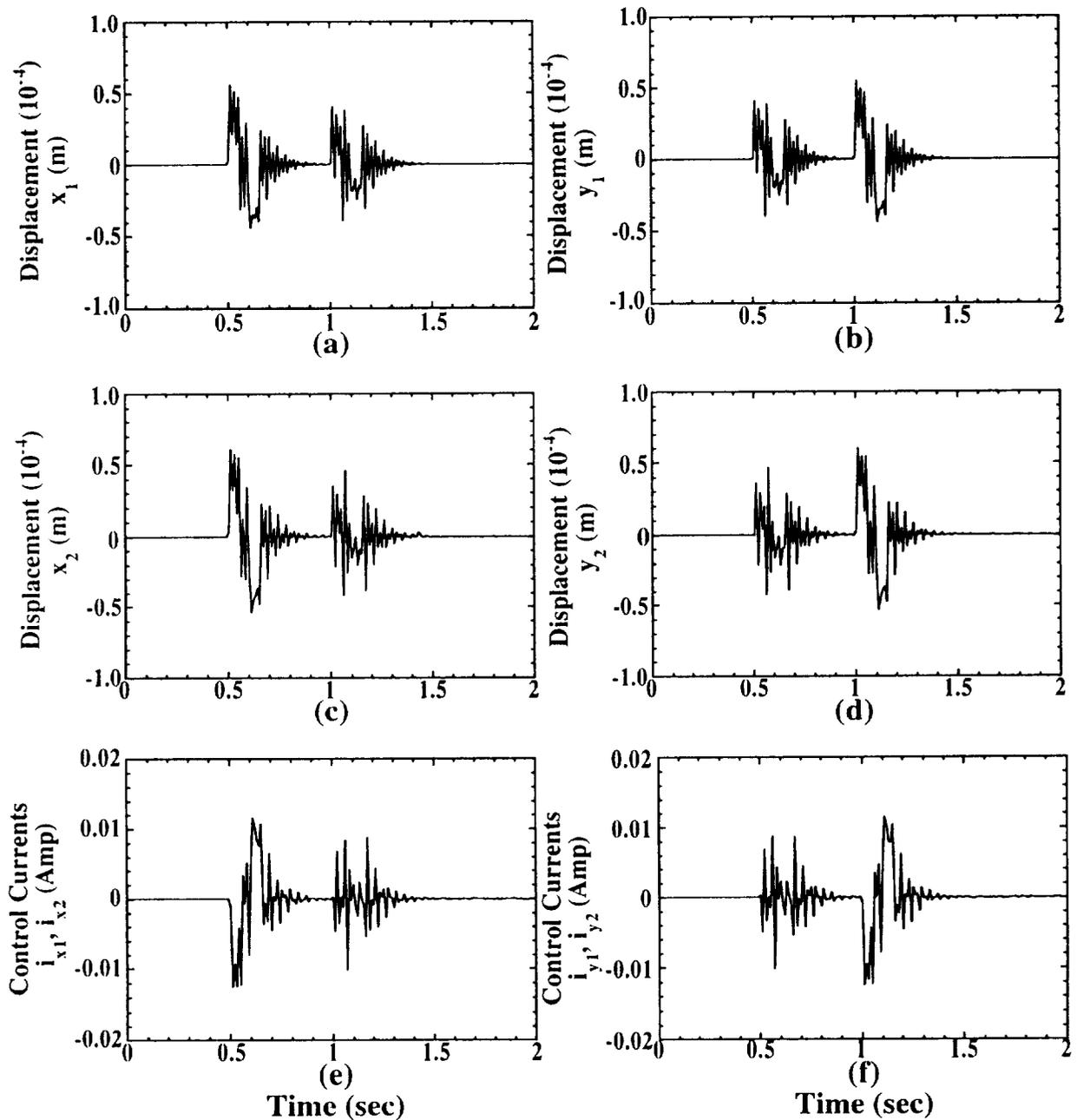
Figure 5: Simulation results subjected to impulsive disturbances

Another simulation demonstrates the capability of the proposed controller in handling the parametric change of the rotor-bearing system. Assume that the spinning speed of the rotor is suddenly changed from 1000.0 to 1010.0 rad/s around 0.4 seconds. The corresponding results are shown in Figure 6. Figures 6(a) - 6(d) show the transient responses of the displacements that are greatly eliminated with fast convergence. The corresponding time-histories of four control currents are illustrated in Figs. 6(e) and 6(f), respectively.
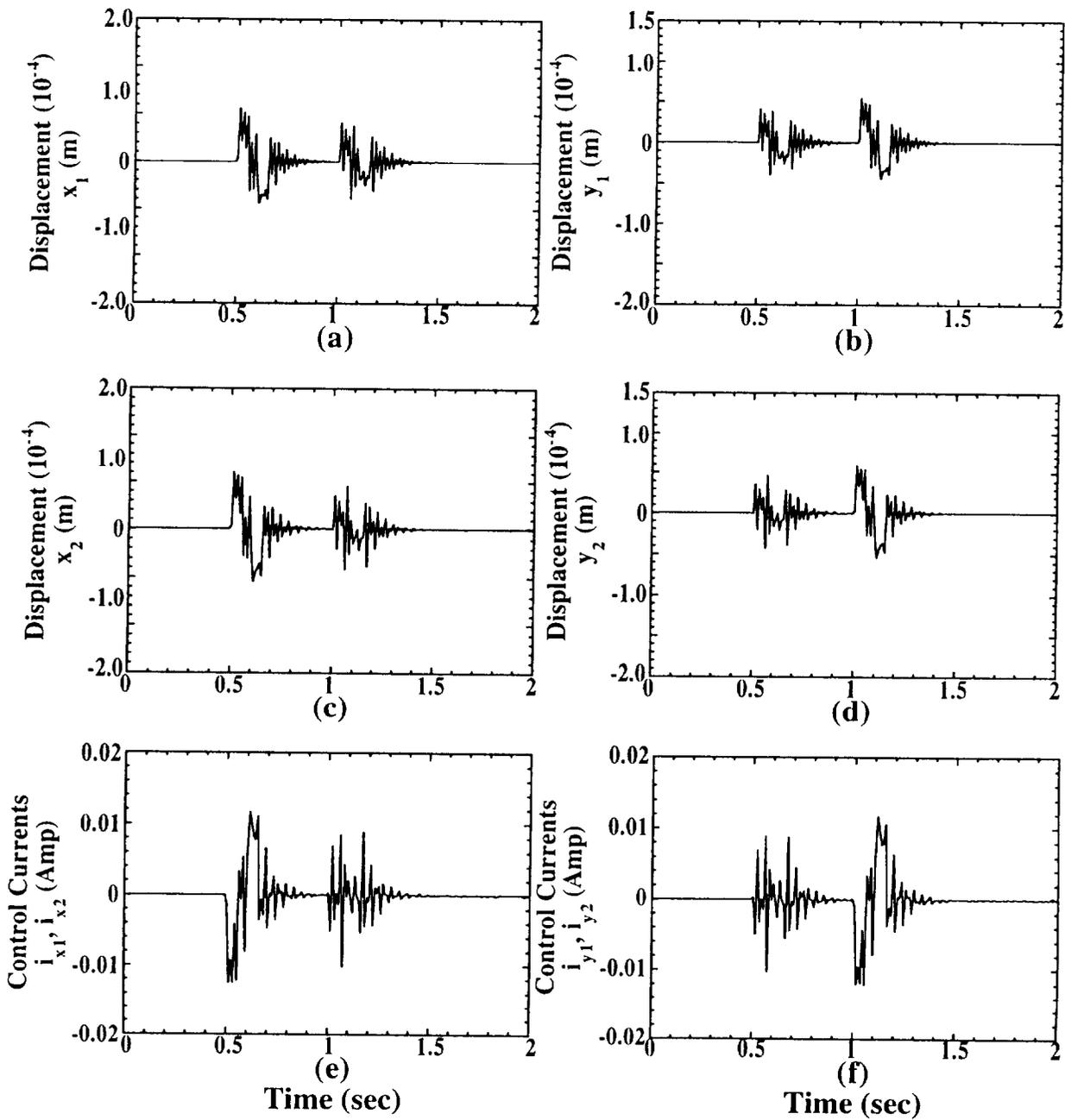
Figure 6: Simulation results subjected to the change of rotor speed

The motion of a whirling disk, under the presence of disturbances and plant parameter variations, has been shown to converge to the translatory whirl mode for fast recovery when the fuzzy-based synchronization control law is invoked. The errors appearing in synchronization have been effectively suppressed through the fuzzy control system. The simulation results in Figs. 5 and 6 thus validate the potential applicability of the proposed control design for the high-speed magnetic bearing system.

CONCLUSIONS

The problem of synchronizing the rotor motion in the magnetic bearings has been proposed and analyzed under fuzzy-logic control. An investigation of a fuzzy-based synchronization controller has been conducted for the magnetically suspended rotor system subject to disturbances and parametric variations. The fuzzy inference system was constructed by four local disturbance compensators cross-coupled to one another by using three coupling controllers in order to compromise the differential errors between the bearing axes. The stability of a rotor system can greatly be improved due to the elimination of unbalanced momenta and gyroscopic effects. The fuzzy-logic controller as designed through the synchronization process has shown to be suitable for the magnetic bearing device in rejecting the disturbances and compensating for the parametric changes. Simulation results demonstrated the effectiveness of the current approach in motion synchronization for the rotor system in cooperation with magnetic bearings. Much work needs to be done before one contemplates to evaluate the proposed strategy on a real-board prototype. They include the robustness issue, actual hardware cost-effectiveness, and power efficiency, among others. We intend to address these aspects in the future.

ACKNOWLEDGMENTS

REFERENCES

1. Dussaux, M.:   The Industrial Applications of Active Magnetic Bearings Technology. *Proceedings of the 2nd International Symposium on Magnetic Bearings*, 1990, pp. 33-38.

2. Youcef-Toumi, K. and Reddy, S.: Dynamic Analysis and Control of High Speed and High Precision Active Magnetic Bearings. *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 114, 1992, pp. 623-633.

3. Sinha, A.; Meese, K.; and Wang, K. W.: Sliding Mode Control of a Rigid Rotor Via Magnetic Bearings. *Proceedings of ASME Biennial Conference on Mechanical Vibration and Noise*, Miami, FL, 1991.

4. Dzielski, J.; Rowell, D.; and Wormley, D.: Approach to Control Moment Gyroscope Steering Using Feedback Linearization. *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 14, No.1, 1991, pp. 96-106.

5. Yang, L.-F.; Mikulas, M.M., Jr.; Park, K. C.; and Su, R.: Slewing Maneuvers and Vibration Control of Space Structures By Feedforward/Feedback Moment-Gyro Controls. Will appear in *ASME Journal of Dynamic Systems, Measurement, and Control*, 1995.

6. Youcef-Toumi, K. and Ito, O.: Controller Design for Systems With Unknown Dynamics. *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 112, No. 1, 1990, pp. 133-142.

7. Youcef-Toumi, K. and Reddy, S.: Stability Analysis of Time Delay Control with Application to High Speed Magnetic Bearings. *M.I.T. Laboratory for Manufacturing and Productivity*, Report No. LMP-90-004, 1990.

8. Hac, A. and Tomizuka, M.: Application of Learning Control to Active Damping of Forced Vibration for Periodically Time Variant Systems. *ASME Journal of Vibration and Acoustics*, Vol. 112, 1990, pp. 489-496.

9. Takagi, T. and Sugeno, M.: Fuzzy Identification of Systems and Its Applications to Modeling and Control. *IEEE Journal of Systems, Man, and Cybernetics*, Vol. SMC-15(1), 1985, pp. 116-132.

10. Welstead, S. T.: *Neural Network and Fuzzy Logic Applications in C/C++*, John Wiley and Sons, Inc., New York, NY, 1994.