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Canonical Signed Digit Study

Part II: FIR Digital Filter Simulation Results

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National Aeronautics and
Space Administration

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INTRODUCTION

Finite Impulse Response digital filter using Canonical Signed-Digit (CSD) number representation for the coefficients has been studied and its computer simulation results are presented here. Minimum Mean Square Error (MMSE) criterion is employed to optimize filter coefficients into the corresponding CSD numbers. To further improve coefficients optimization process, an extra non-zero bit is added for any filter coefficients exceeding $1/2$. This technique improves frequency response of filter without increasing filter complexity almost at all. The simulation results shows outstanding performance in bit-error-rate (BER) curve for all CSD implemented digital filters included in this presentation material.

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❑ **CSD applications for FIR filters**

■ **Issues and Concerns on high speed digital filter implementation**

☞ **Problems:** - **Digital Finite Impulse Response (FIR) filters are widely used in digital signal processing**

- **Multipliers in filter are the most complex hardware realization which increase the cost of filters**

- **Multiplication is the most time and power consuming process limits variety of high speed applications like high data rate digital modem implementation**

☞ **Solution:** - **Eliminate the multipliers from filter by employing CSD represented filter coefficients to increase the speed and reduce hardware complexity**

Question: How can CSD filter coefficients help high speed FIR digital filter design?

Answer: Less nonzero digits in a CSD form cuts number of addition stages and thereby also reduce both hardware complexity and power consumption

Note: Number of adder/subtractor required to realize a CSD coefficient is one less than the number of nonzero digits in the code

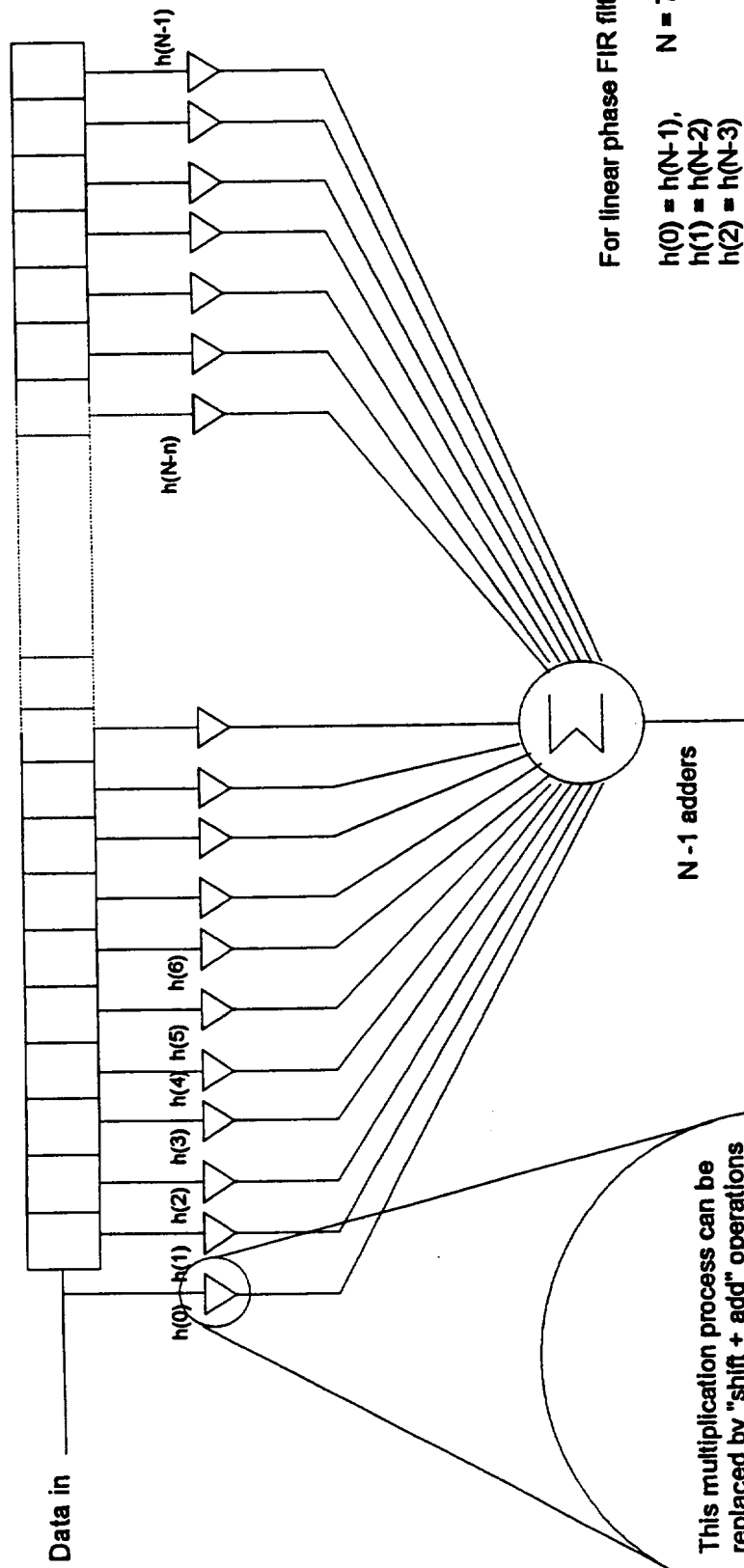
example

Consider a filter coefficients in 8-bit with value = 0.9921875

$$0.9921875_{10} = 0.1111111_2 = 1.000000\bar{1}_{\text{csd}} = 2^0 - 2^{-7}$$

Only one subtractor as opposed to seven adders

General Digital FIR Filter



For linear phase FIR filters,

- $h(0) = h(N-1)$, $N = 73$
- $h(1) = h(N-2)$
- $h(2) = h(N-3)$
- $h(3) = h(N-4)$
- ...
- ...
- ...

Total hardware:

Number of multipliers:	37
Number of adders:	72
Number of delays:	72

This multiplication process can be replaced by "shift + add" operations
 Load these filter taps with CSD filter coefficients which has minimal nonzero digits

□ CSD filter coefficients search guidelines:

① Select the number of nonzero digits and total code length in CSD format

- Code length (quantization bits) is not very critical for optimizing CSD coefficients

- 8 to 15 were found to satisfactory for most filter specifications

- The number of nonzero digits directly affect hardware complexity and the worst-case coefficient quantization error which results in undesirable frequency response of the filter and also in ISI

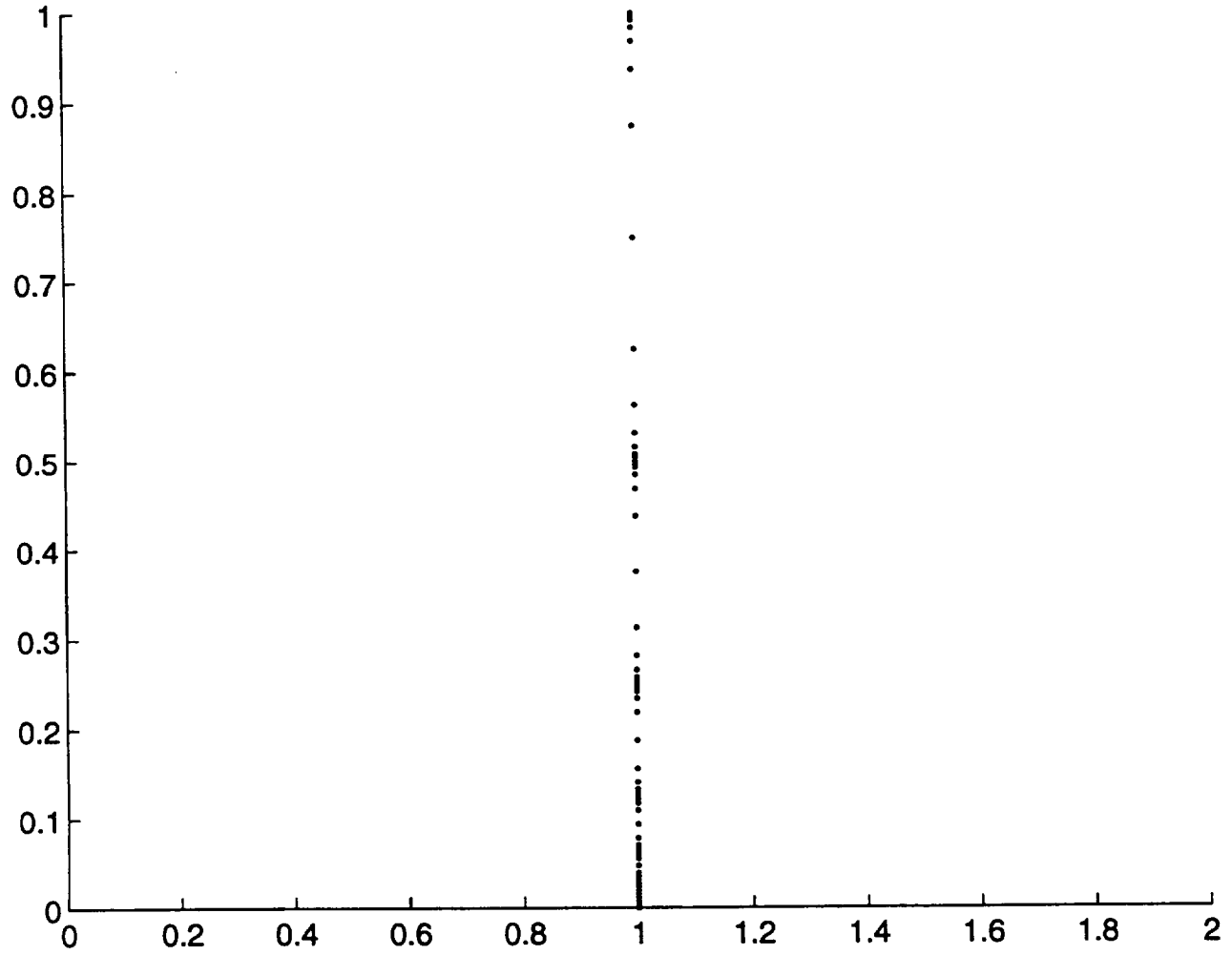
- 2 to 4 nonzero digits are satisfactory for most cases

- Rule of thumb: One nonzero digit required in the CSD code for each 20 dB of stopband attenuation

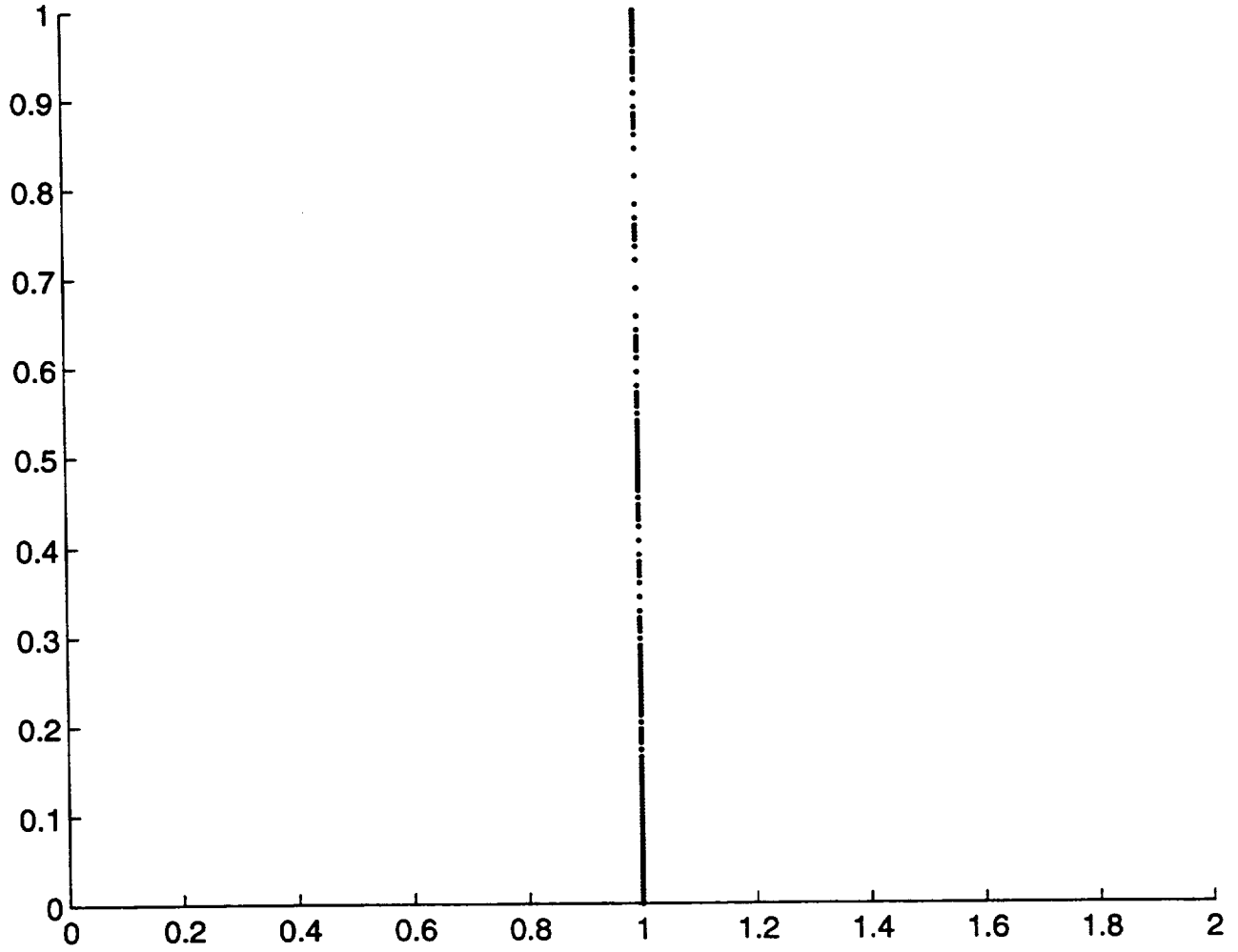
- Quantization errors caused by the nature of nonuniform distribution of CSD coefficients

② Optimize CSD filter coefficients through various search algorithms

CSD numbers distribution: $n = 8, nz = 2$



CSD numbers distribution: $n = 8$, $n_z = 3$



❑ Optimization of search algorithm for FIR filter coefficients

- Search for an optimal scale factor and bivariate local search to improve frequency response

Mixed Integer linear programming (MILP):

- Very effective but required a lot of computing powers and limited to about 40 filter taps due to exponentially growing CPU time with filter length

Local search:

- Performs nearly as well as the MILP and requires substantially less computing resources

◆ Optimization Criterion:

- ① FFT approach: scale the filter coeffs., perform an FFT and compare the frequency responses
- ② MMSE approach: select the scale factor that results in minimum value of coefficient quantization error
- ③ Hybrid approach: coarse time-domain search and then fine-tune with frequency domain search

Method of Choice

- ① Assign an extra nonzero digit in CSD form for filter coefficients greater than or equal to 0.5 (optional)
 - ✖ will result in substantial improvements in frequency response at the cost of a small increase in hardware complexity
- ② Prescale filter coefficients and then convert it into CSD represented filter coefficients
 - ✖ will improve ISI substantially as well as frequency response
 - ✖ the distribution of CSD coefficients is more dense for smaller valued coefficients
- ③ Change a prescale factor by one tenth (optional)
 - ✖ choose the step size, Δ , between scale factors to achieve a reasonable tradeoff between search time and frequency response
- ④ Optimize CSD coefficients in Minimum Mean Square Error (MMSE)

□ **Root-Raised-Cosine Filter equation,**

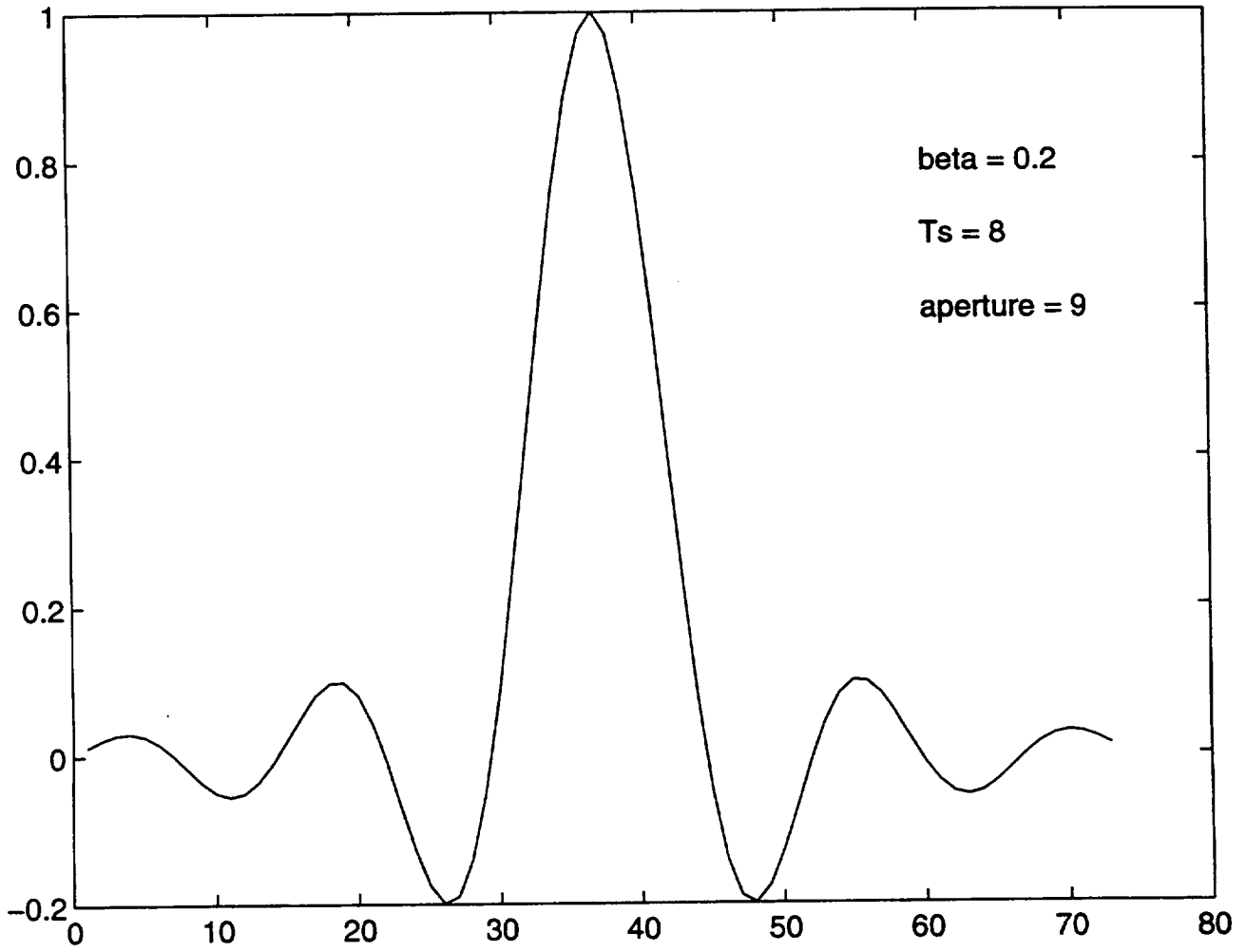
$$h(t) = \left(\frac{4\beta}{\pi(1-\beta) + 4\beta} \right) \left(\frac{T_s \sin\left(\frac{\pi}{T_s}(1-\beta)t\right)}{4\beta t} + \frac{\cos\left(\frac{\pi}{T_s}(1+\beta)t\right) + \frac{4\beta t}{T_s} \sin\left(\frac{\pi}{T_s}(1-\beta)t\right)}{1 - \left(\frac{4\beta t}{T_s}\right)^2} \right), \quad t \neq 0, \pm \frac{T_s}{4\beta}$$

$$h\left(\pm \frac{T_s}{4\beta}\right) = \left(\frac{4\beta}{\pi(1-\beta) + 4\beta} \right) \left(\frac{T_s \sin\left(\frac{\pi}{4\beta}(1-\beta)t\right)}{2} + \frac{\pi \cos\left(\frac{\pi}{4\beta}\right) + \pi \sin\left(\frac{\pi}{4\beta}\right)}{4\sqrt{2}} \right)$$

$h(t = 0) = 1$ and

where T_s is the symbol period in seconds, β is the rolloff constant

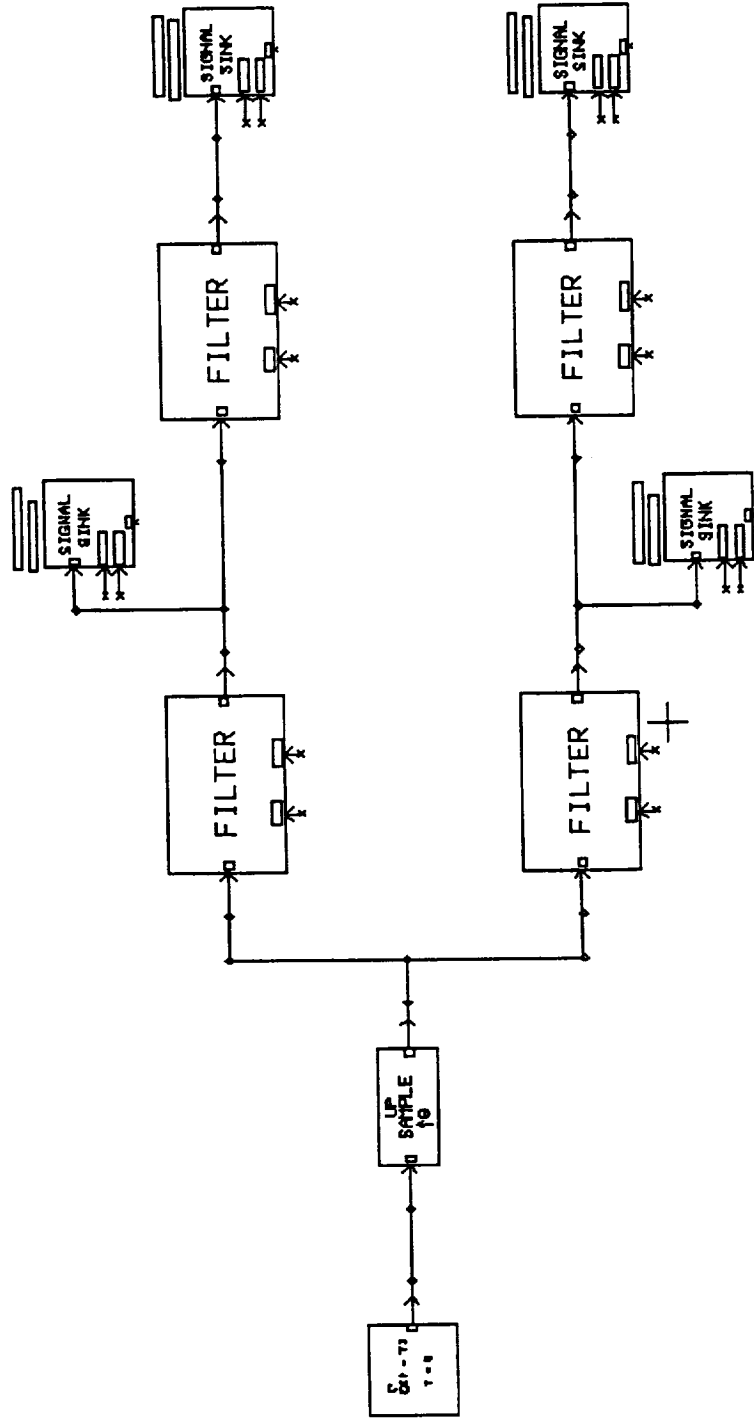
Impulse response of Root_Raised_Cosine filter



□ *SPW Simulation Results*

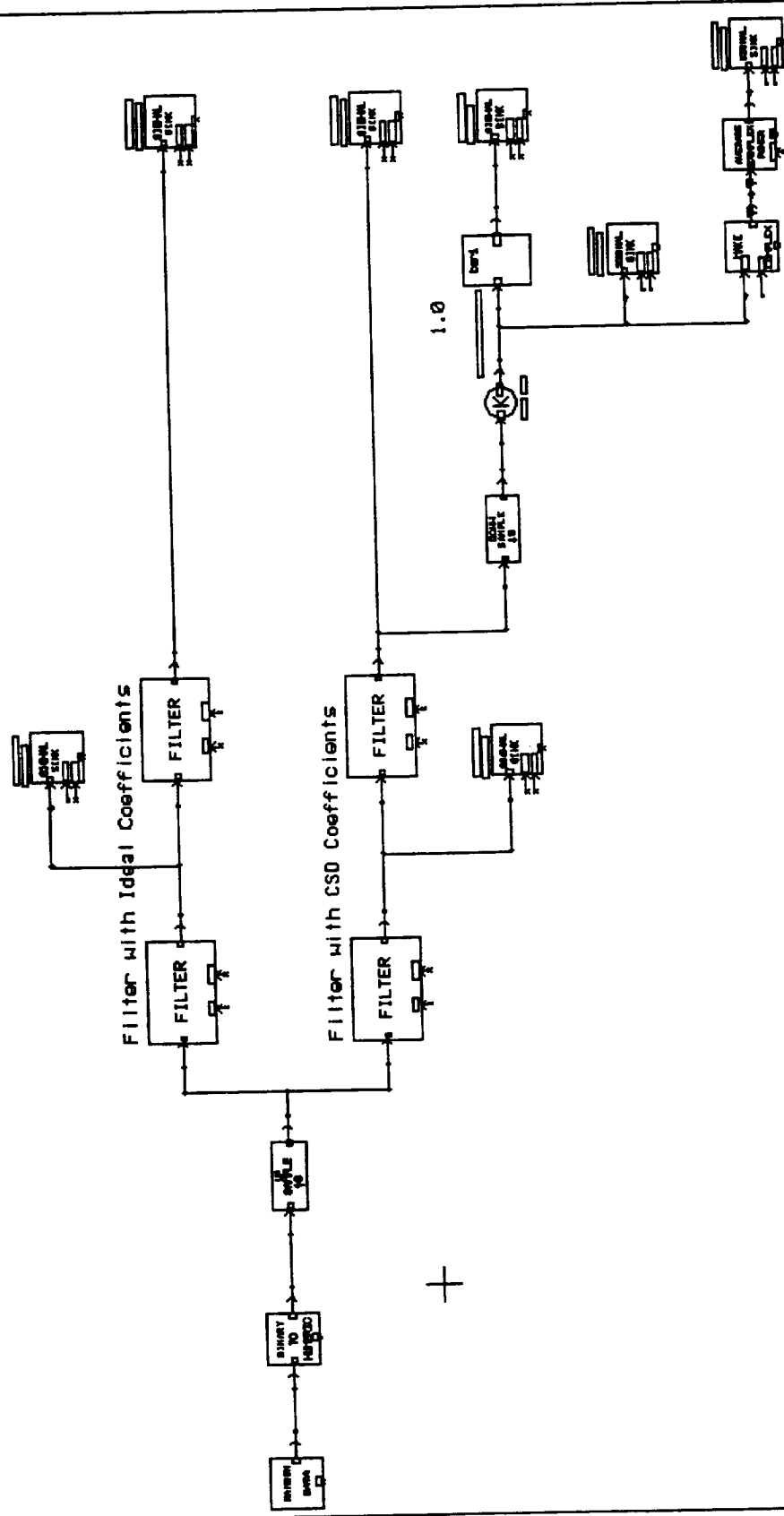
- **Simulation Models for Filter Systems**
- **CSD Filter Frequency Responses**
- **Eye Diagrams**
- **Comparison of CSD Filter Frequency Responses**
- **Bit-Error-Rate Performance Curves**

Impulse Response for CSD Filter



3-15-1998 by Heechul Kim

CSD FILTER BER MEASUREMENT SYSTEM

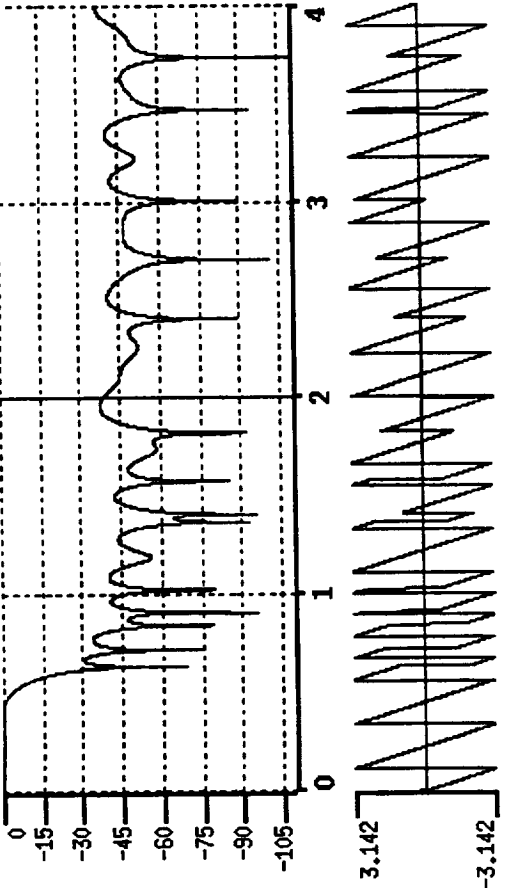


3-15-1996 by Heechul Kim

Impulse Response for CSD289829

A1

Frequency Response
 Point# = 6150
 Bin# = 2054
 # Pts = 8193
 Freq = 2.00586
 Mag. = -39.3914
 Phase = 2.97592



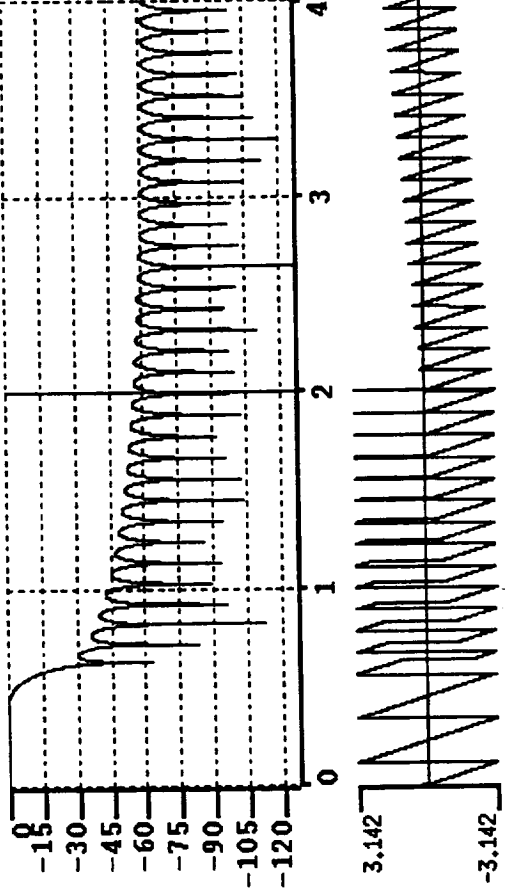
Mag (dB)

Phase (radians)

Impulse Response for hoft289

A2

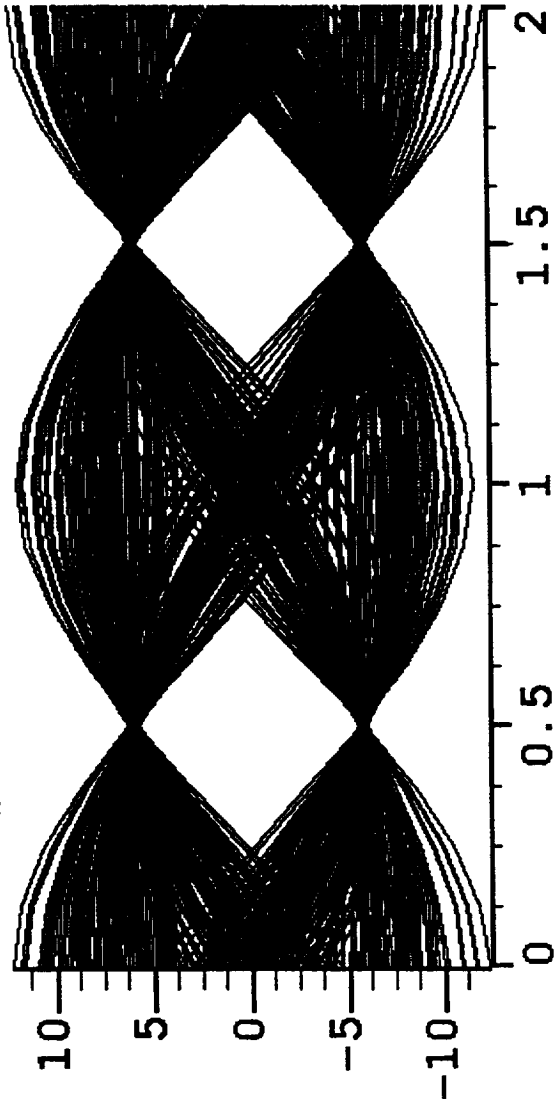
Frequency Response
 Point# = 6150
 Bin# = 2054
 # Pts = 8193
 Freq = 2.00586
 Mag. = -69.254
 Phase = -0.16567



Mag (dB)

Phase (radians)

Eye Diagram for CSD289829

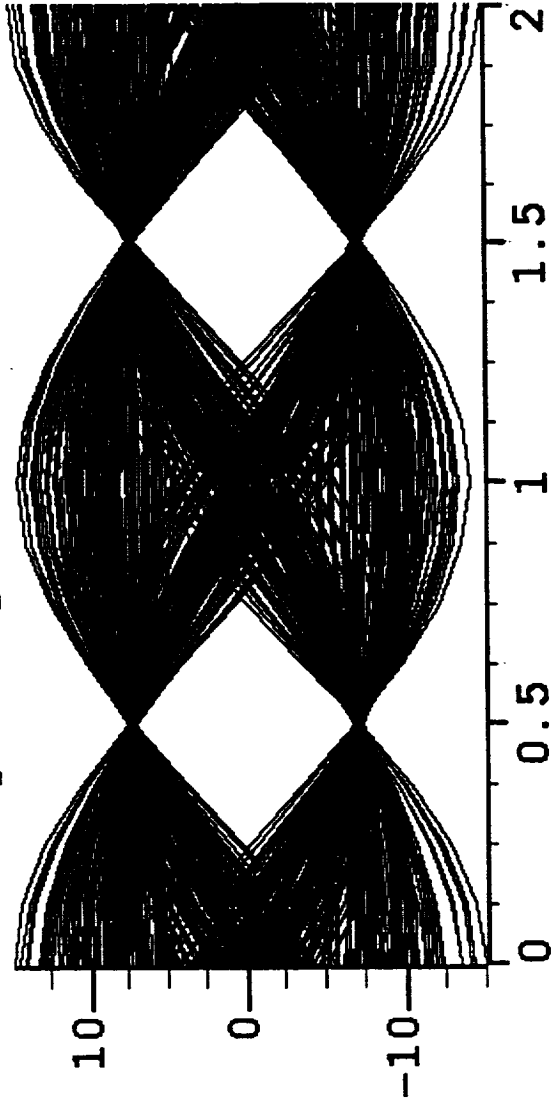


Eye Diagram

Points = 7900

A1

Eye Diagram for hoft289



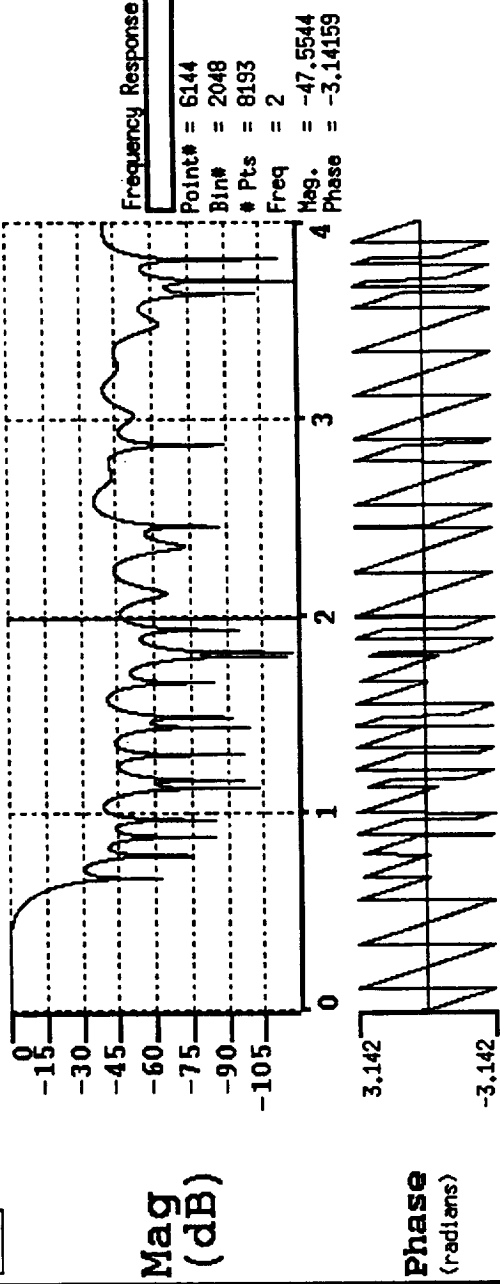
Eye Diagram

Points = 7900

A2

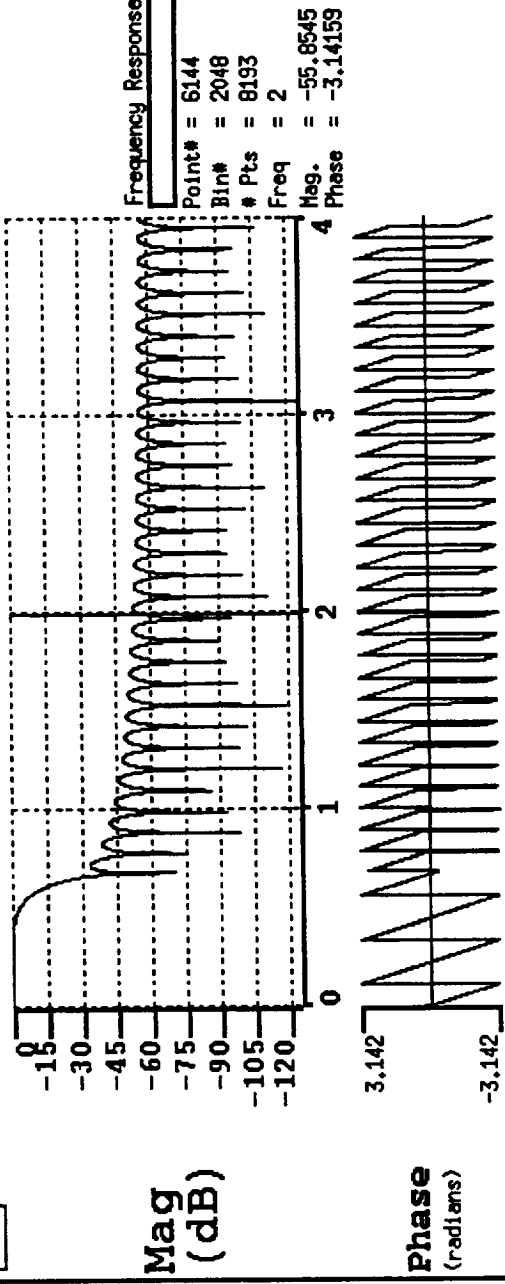
Impulse Response for CSD389821

A1

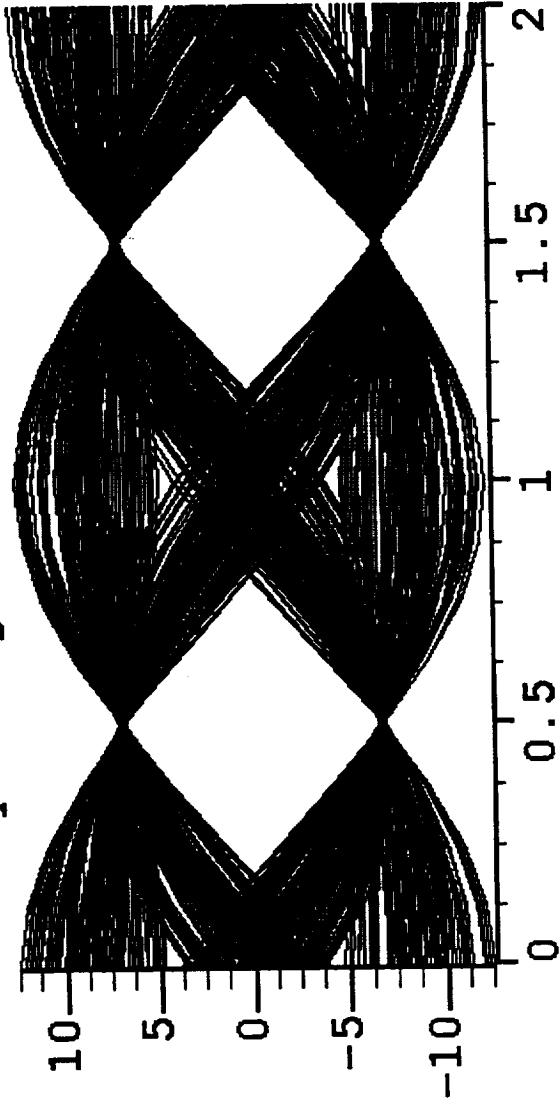


Impulse Response for hoft389

A2



Eye Diagram for CSD389821

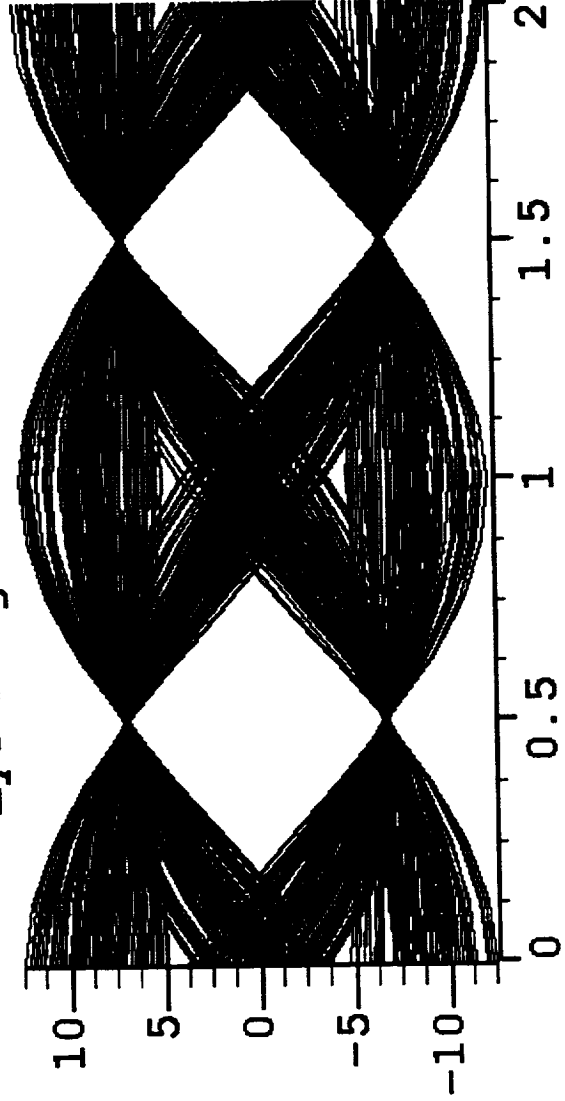


Eye Diagram

Points = 7900

A1

Eye Diagram for hoft389



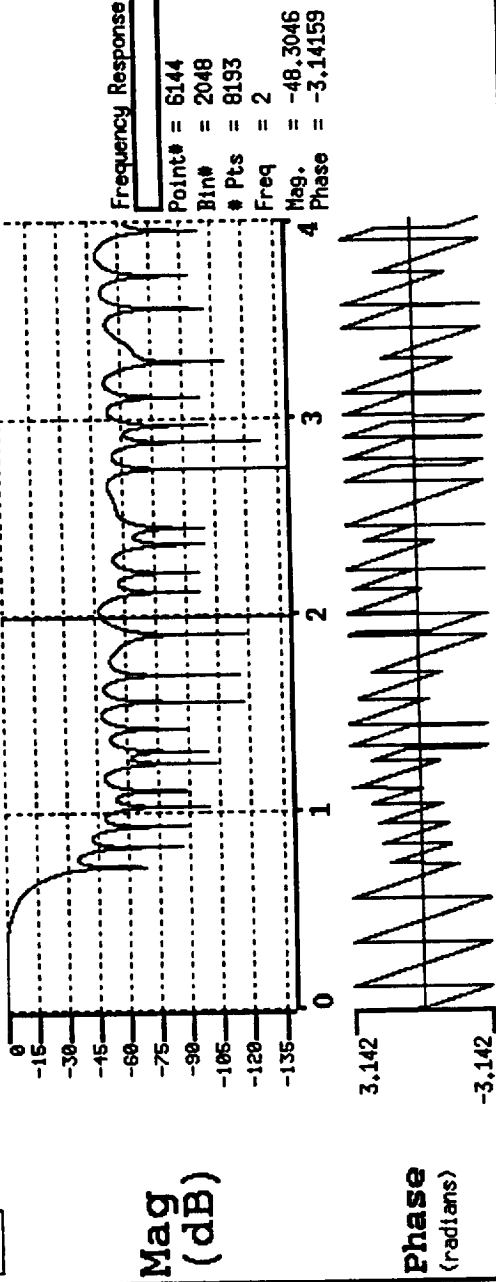
Eye Diagram

Points = 7900

A2

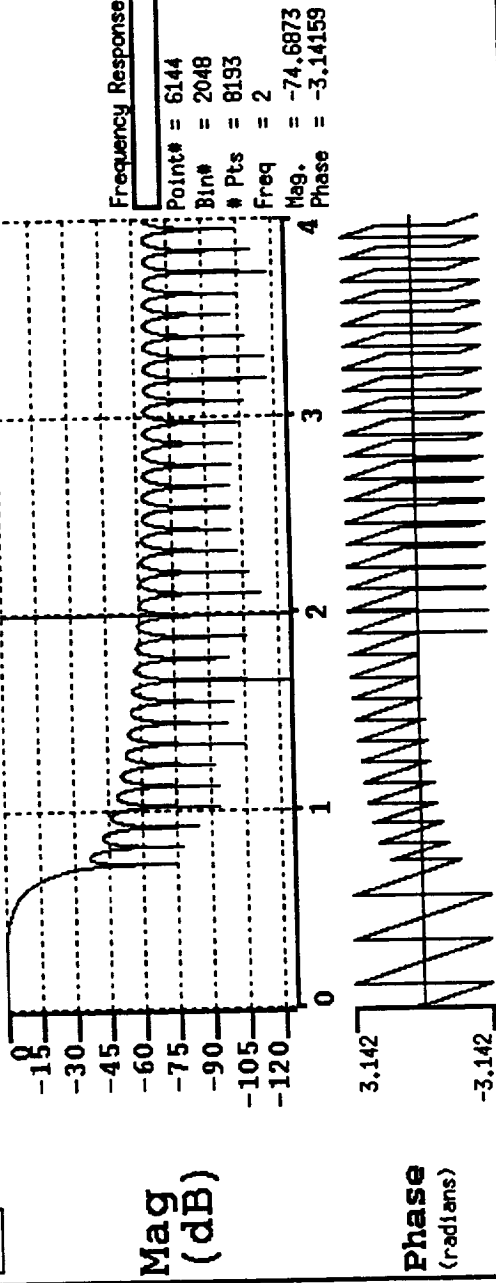
Impulse Response for CSD489847

A1

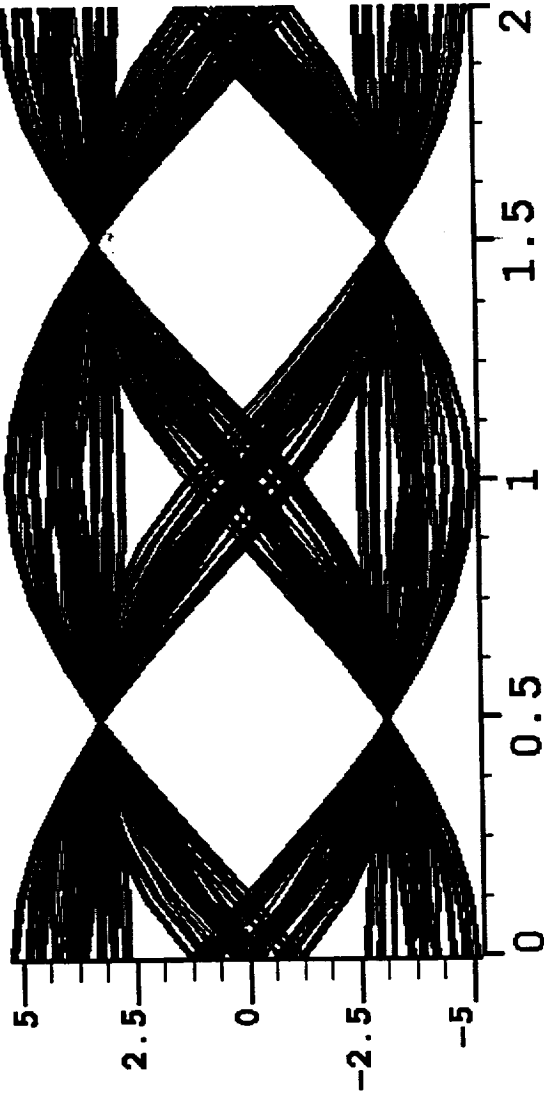


Impulse Response for hoft489

A2



Eye Diagram for CSD489847

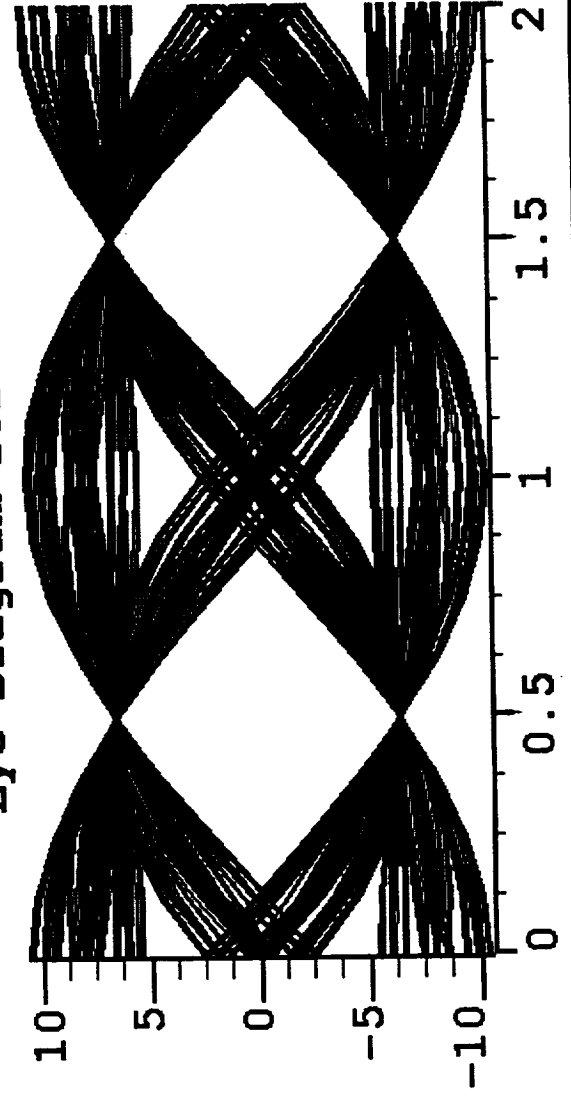


Eye Diagram

Points = 7900

A1

Eye Diagram for hoft489



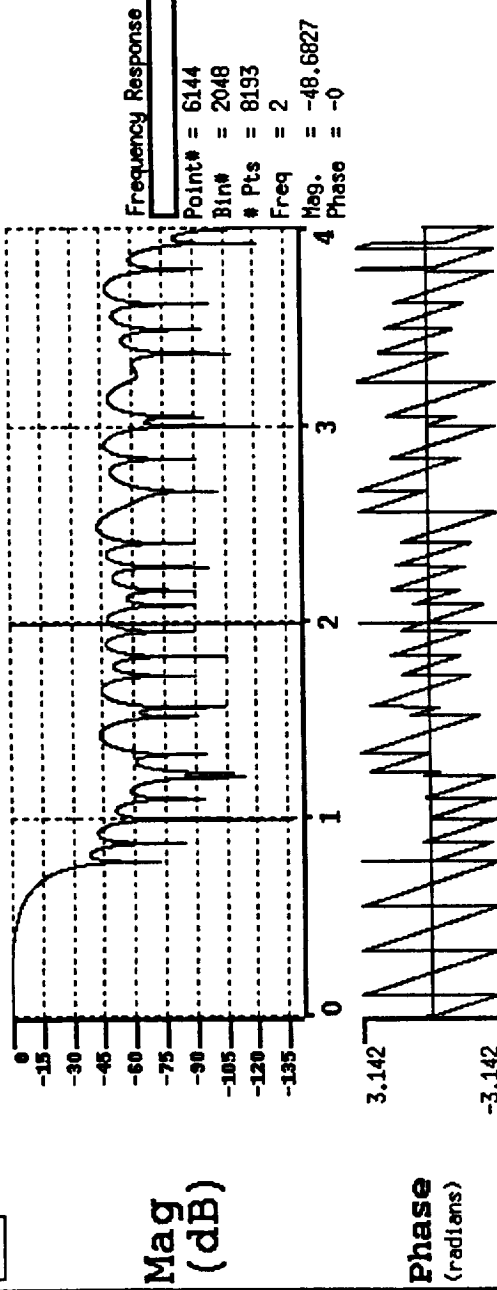
Eye Diagram

Points = 7900

A2

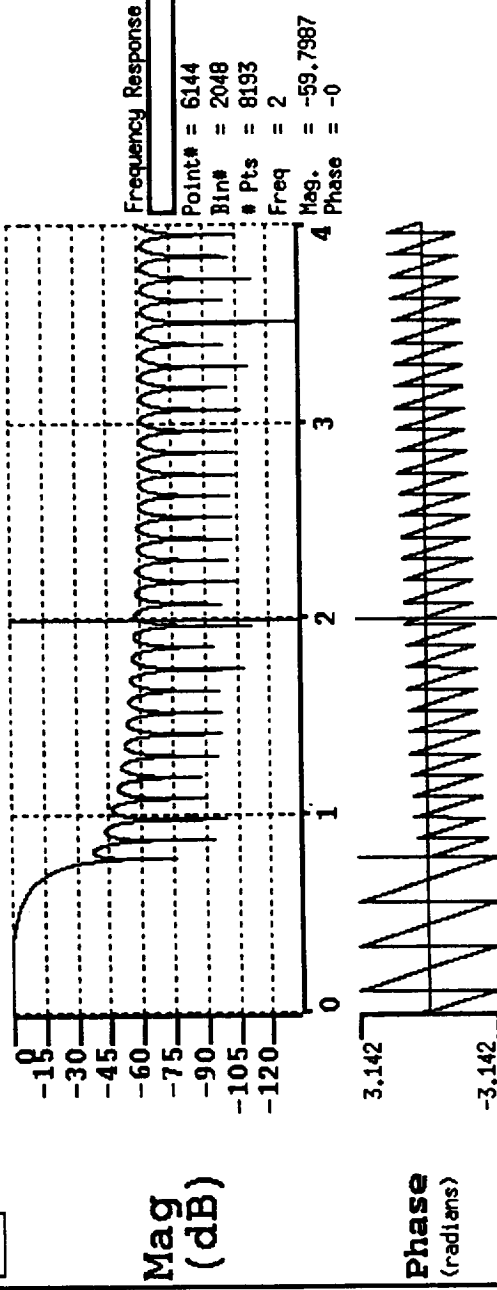
Impulse Response for CSD589836

A1

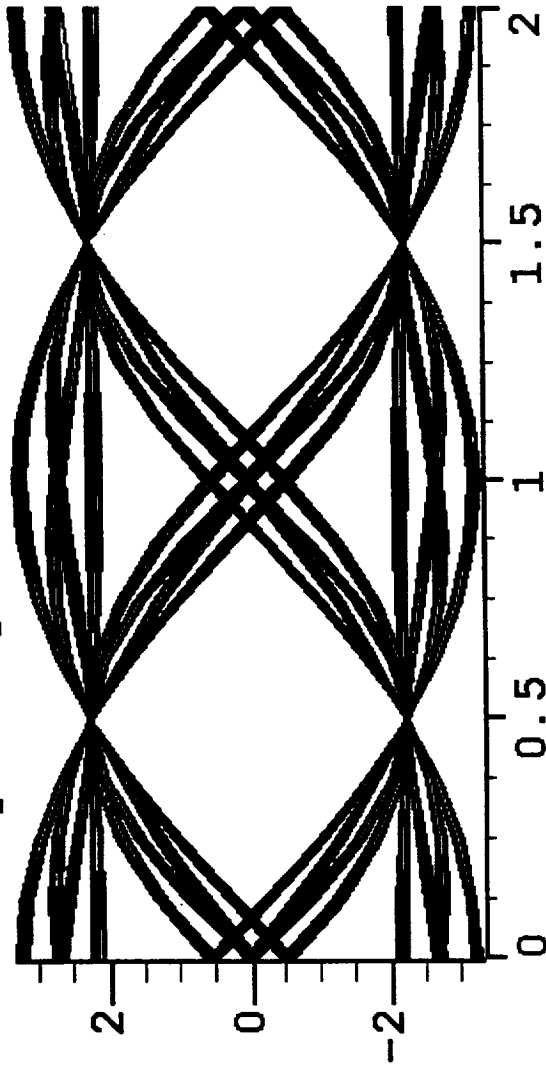


Impulse Response for hoft589

A2



Eye Diagram for CSD589836

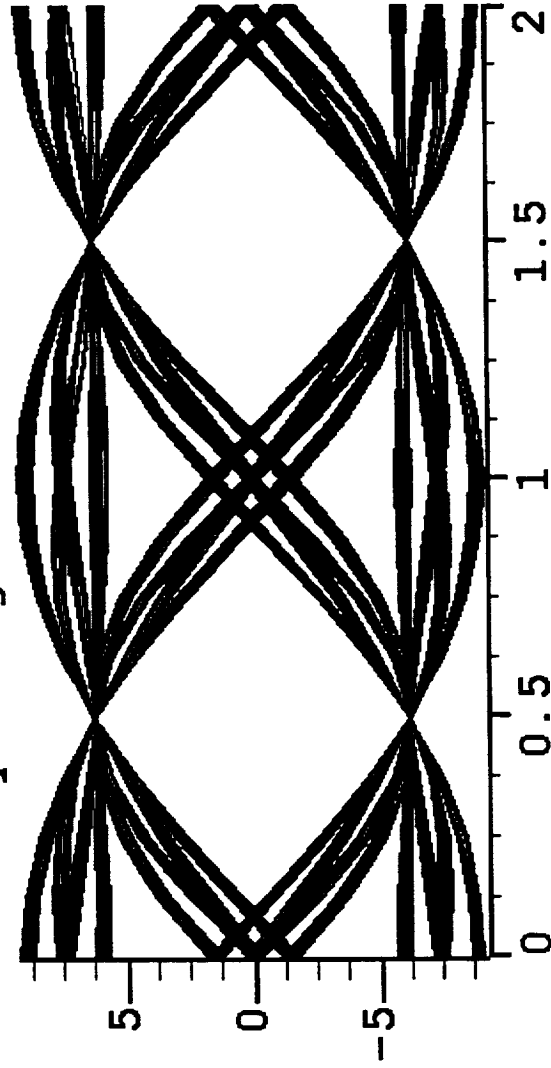


Eye Diagram

Points = 7900

A1

Eye Diagram for hoft589



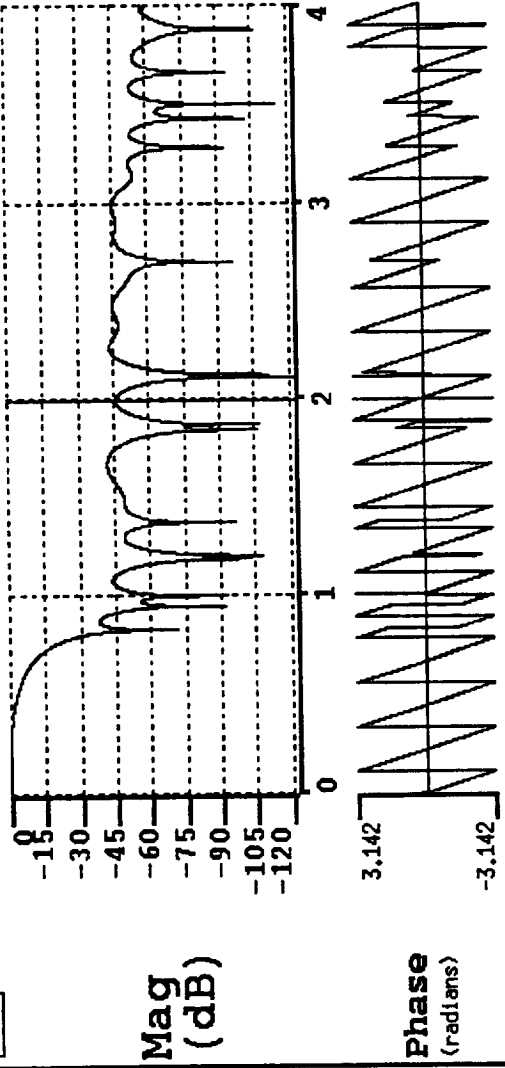
Eye Diagram

Points = 7900

A2

Impulse Response for CSD689829

A1

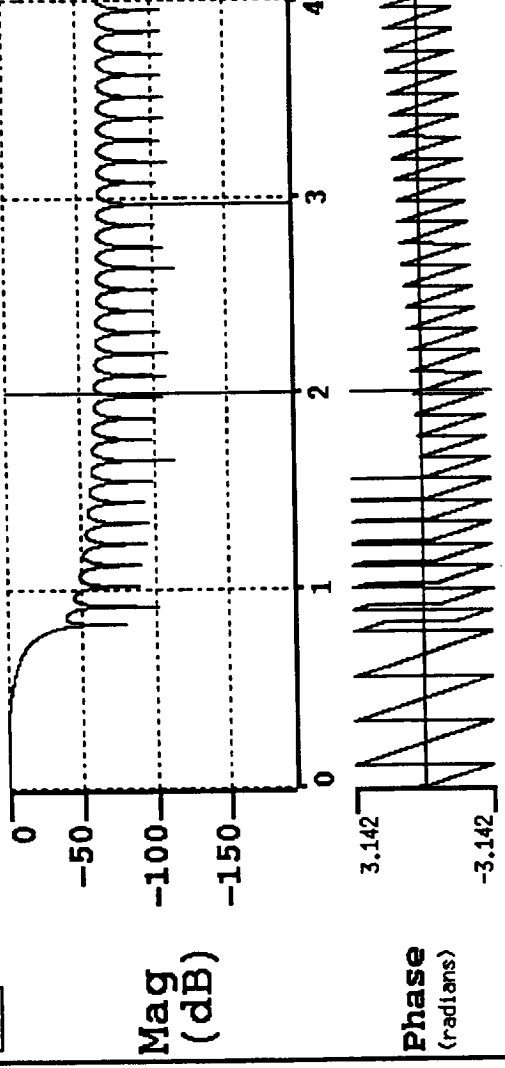


Frequency Response

Point# = 6144
 Bin# = 2048
 # Pts = 8193
 Freq = 2
 Mag. = -46.0103
 Phase = -0

Impulse Response for hoft689

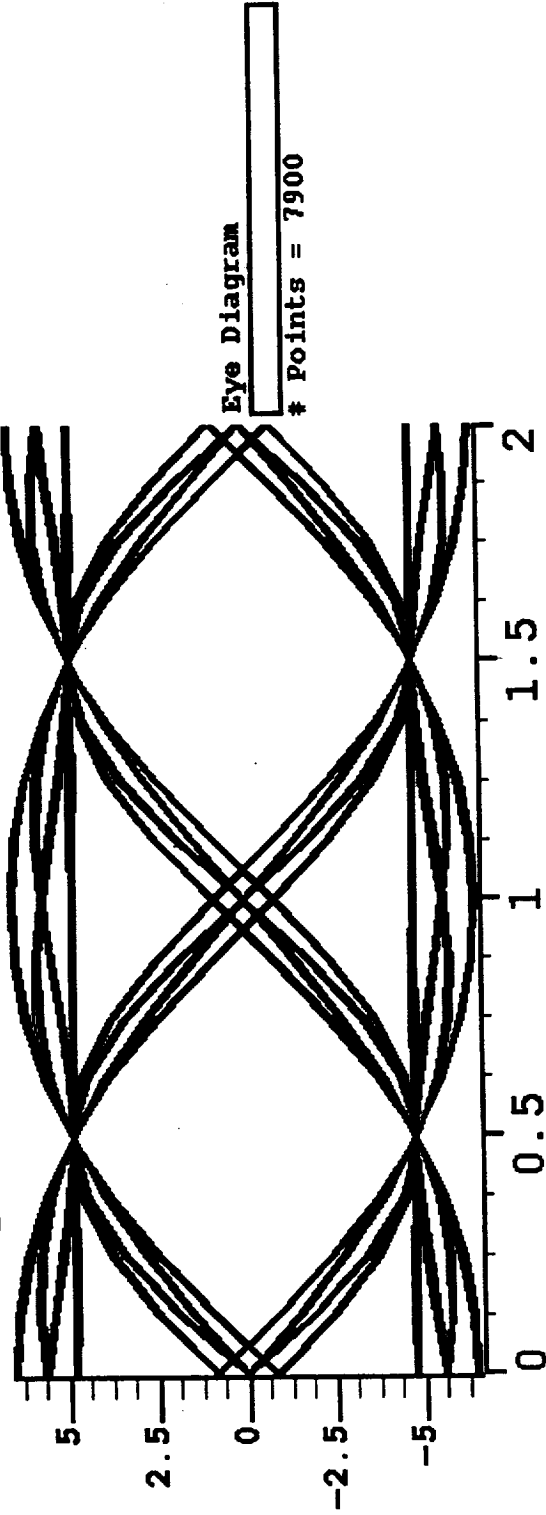
A2



Frequency Response

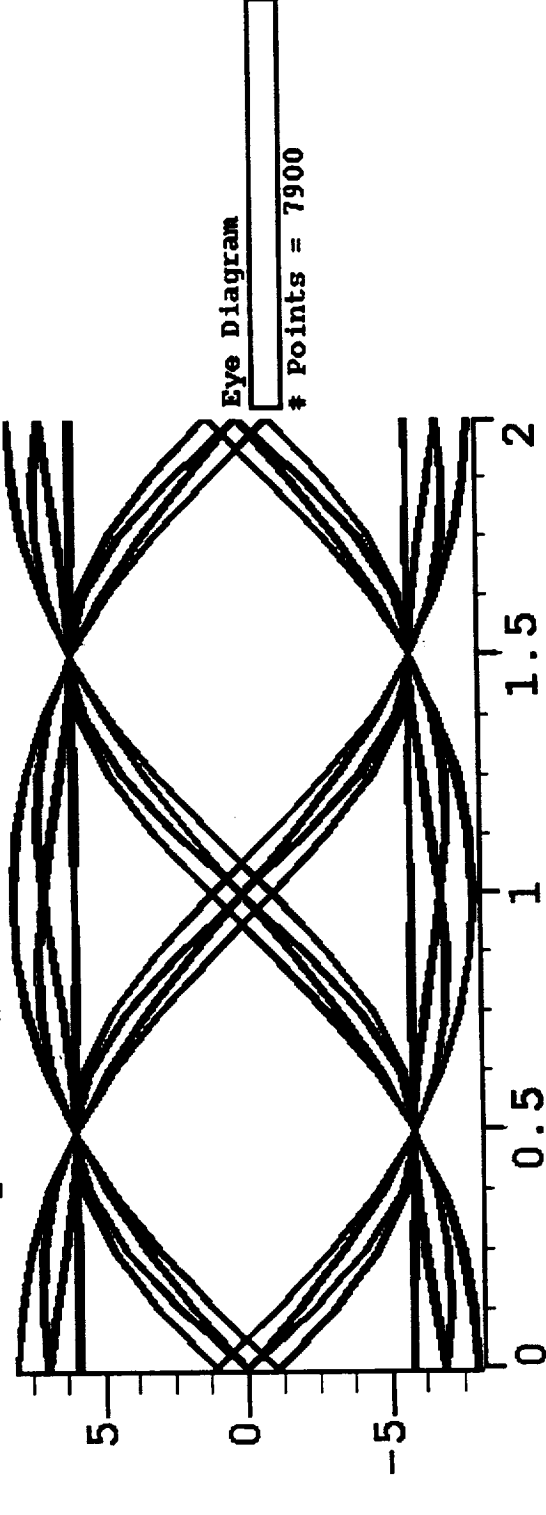
Point# = 6145
 Bin# = 2049
 # Pts = 8193
 Freq = 2.00098
 Mag. = -70.0278
 Phase = -0.0276117

Eye Diagram for CSD689829



A1

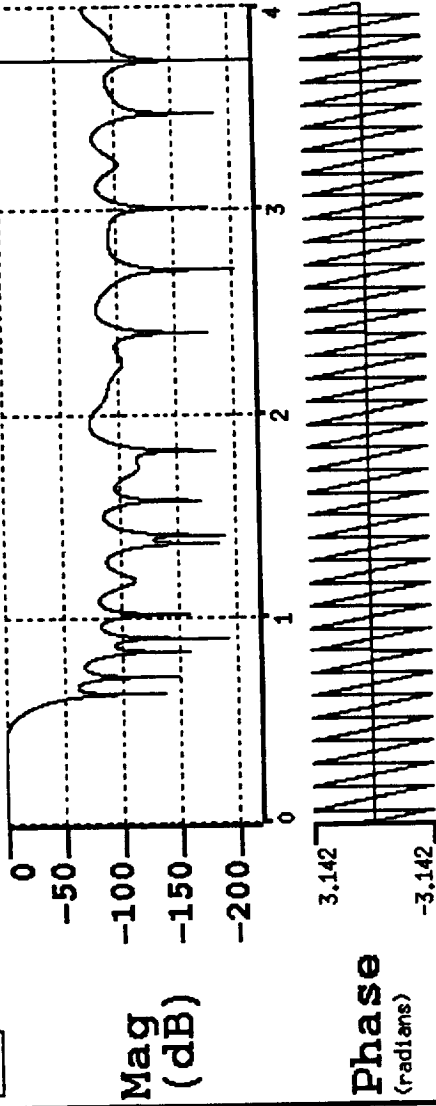
Eye Diagram for hoft689



A2

Frequency response for CSD289829

A1

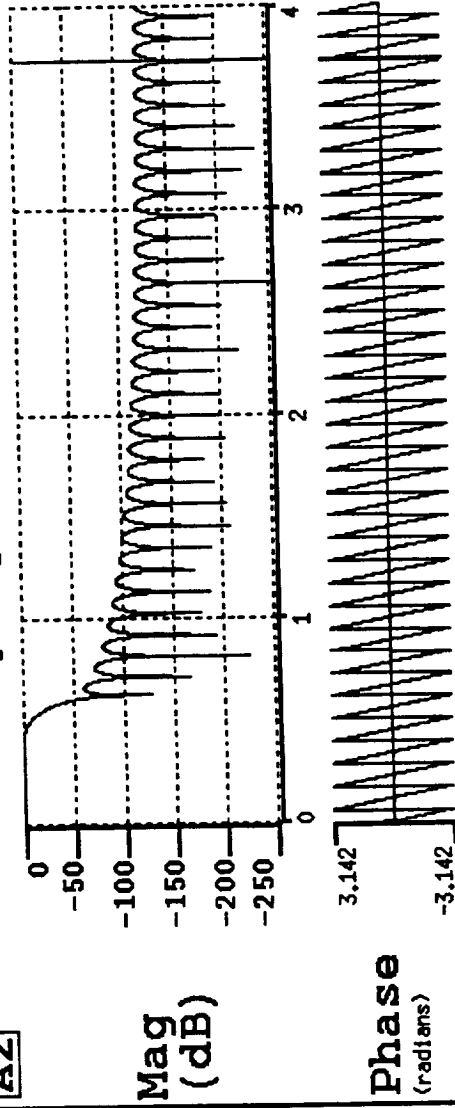


Frequency Response

Point# = 7924
 Bin# = 3828
 # Pts = 8193
 Freq = 3.73828
 Mag. = -139.345
 Phase = 2.23348

Frequency Response for hoft289

A2

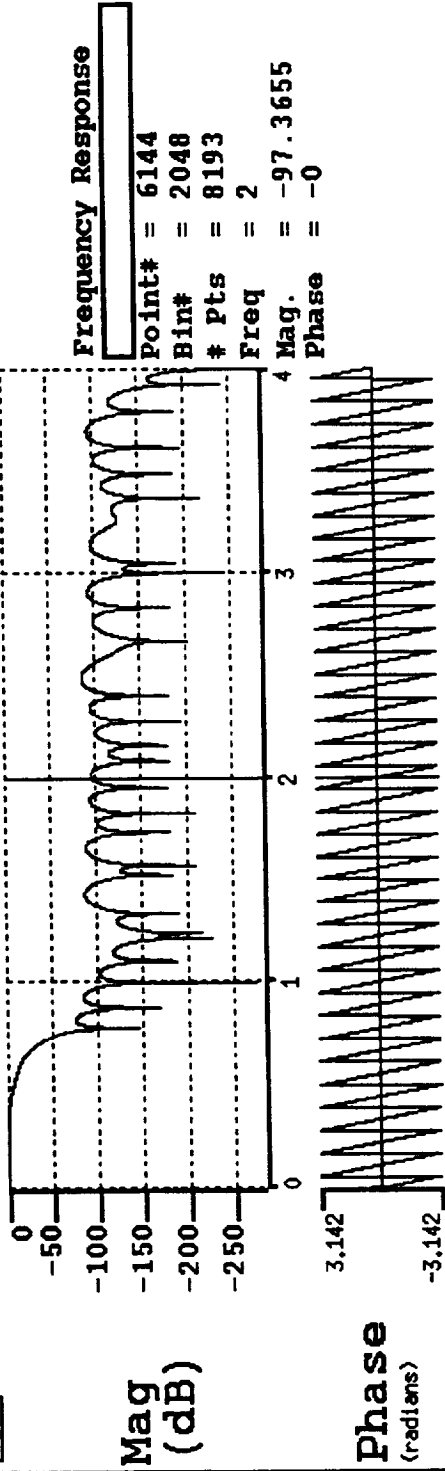


Frequency Response

Point# = 7924
 Bin# = 3828
 # Pts = 8193
 Freq = 3.73828
 Mag. = -145.86
 Phase = 2.23348

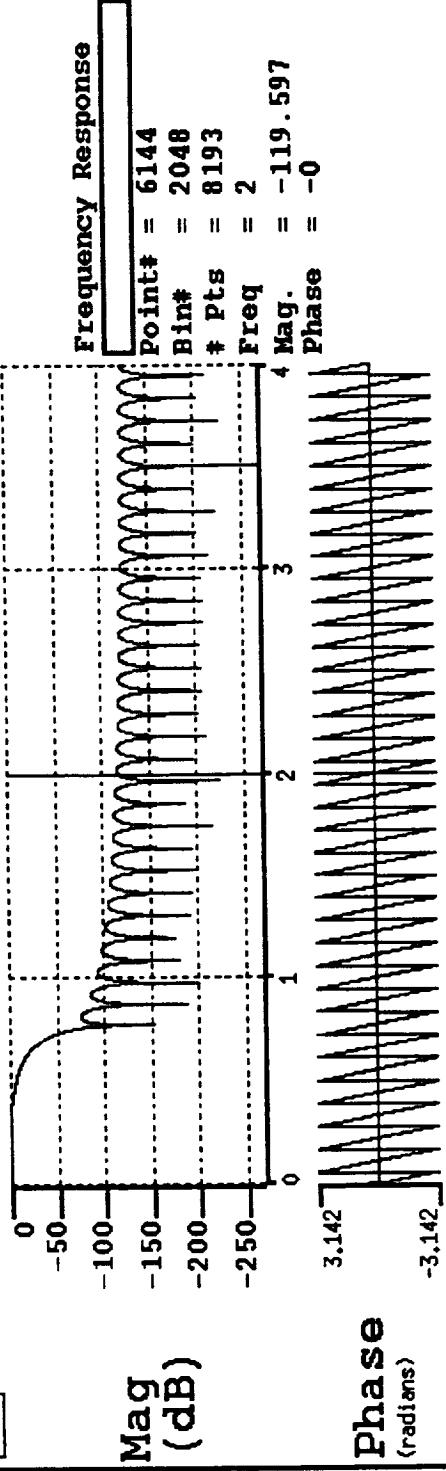
Frequency response for CSD589836

A1



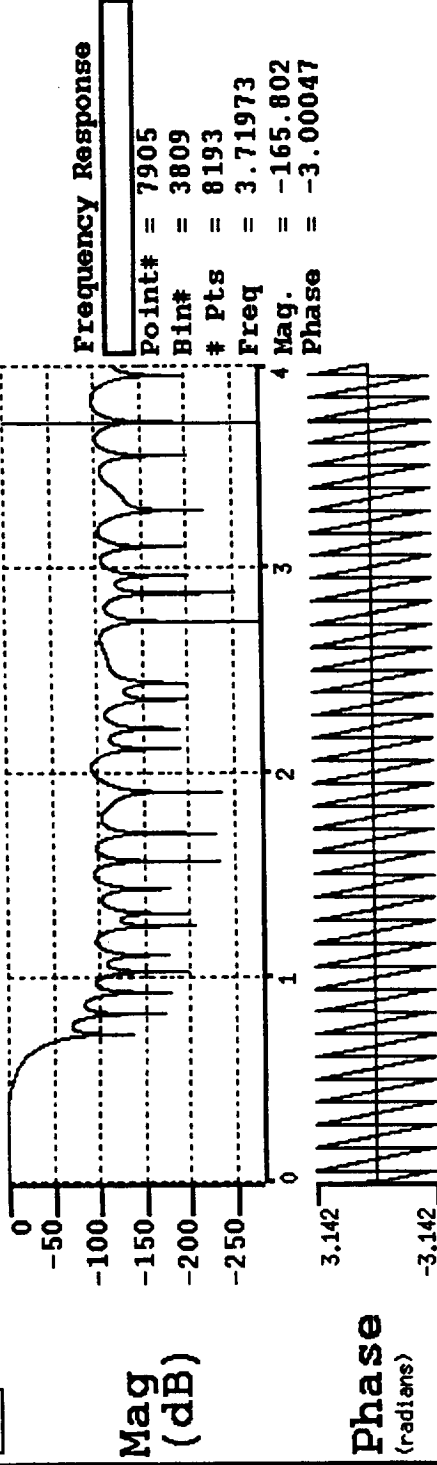
Frequency response for hoft589

A2



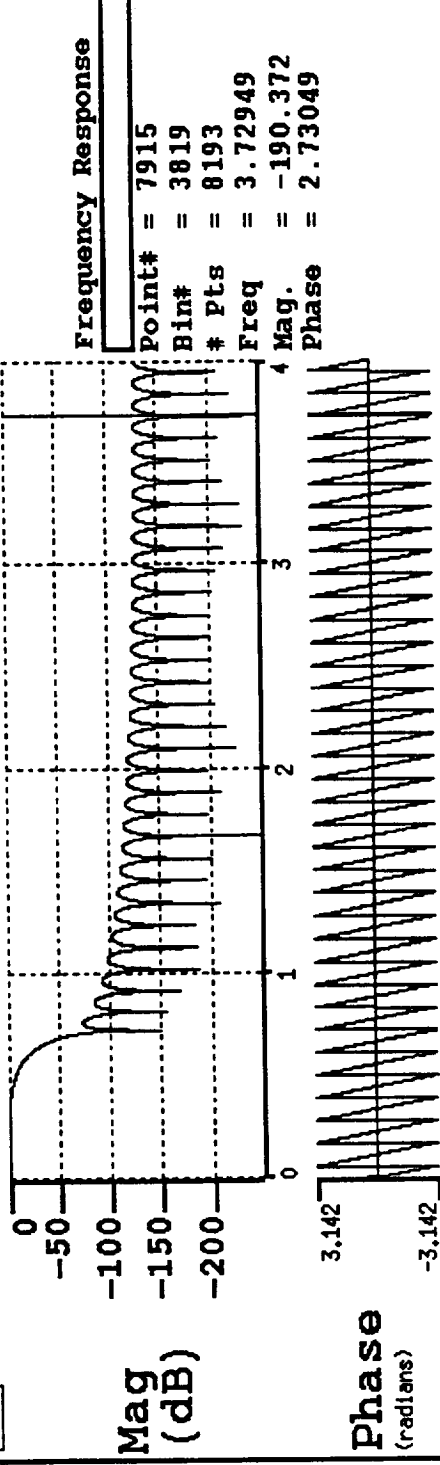
Frequency response for CSD489847

A1



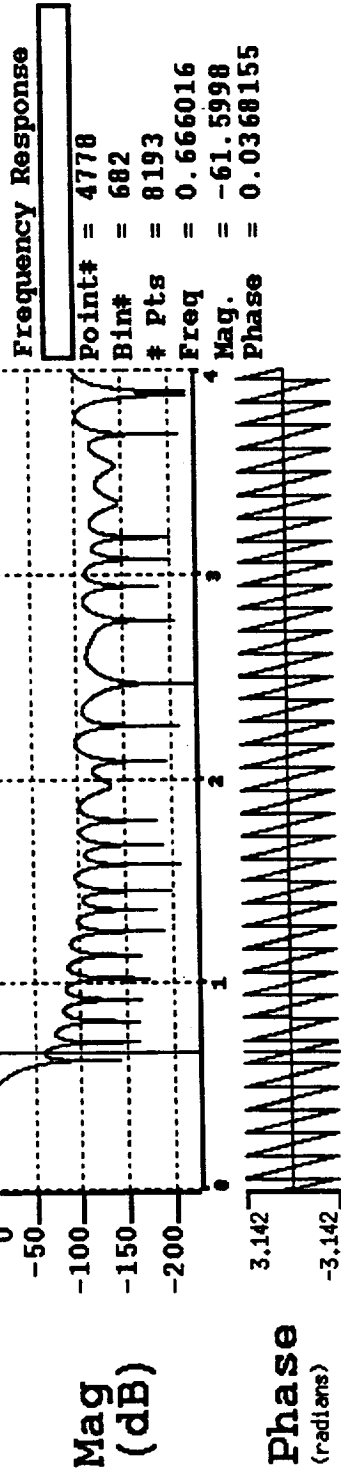
Frequency response for hoft489

A2



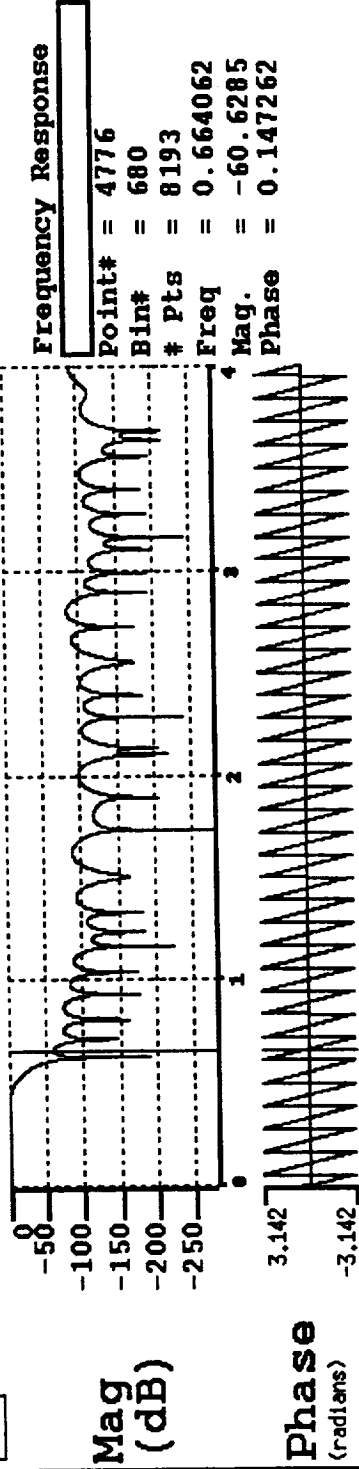
Frequency Response for CSD289848, nz = 4

A1



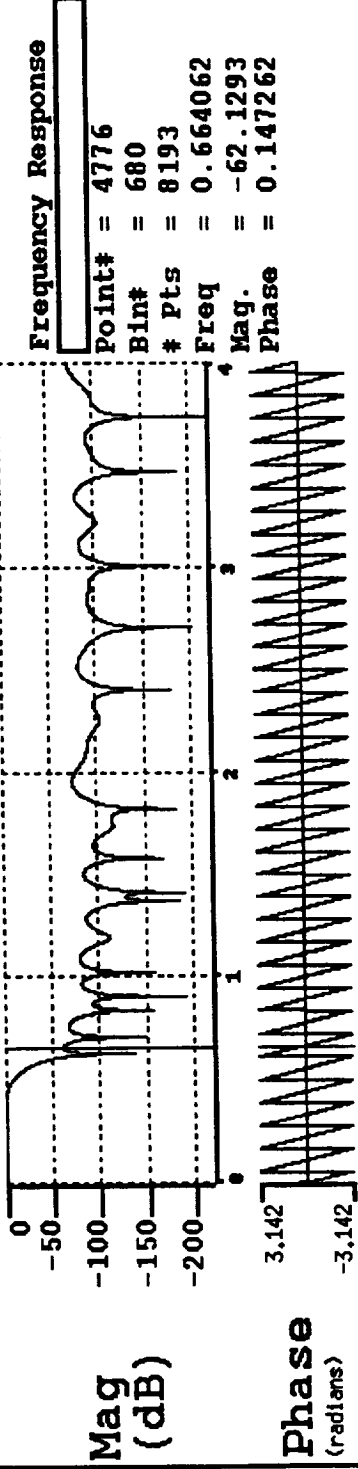
Frequency Response for CSD289836, nz = 3

A2



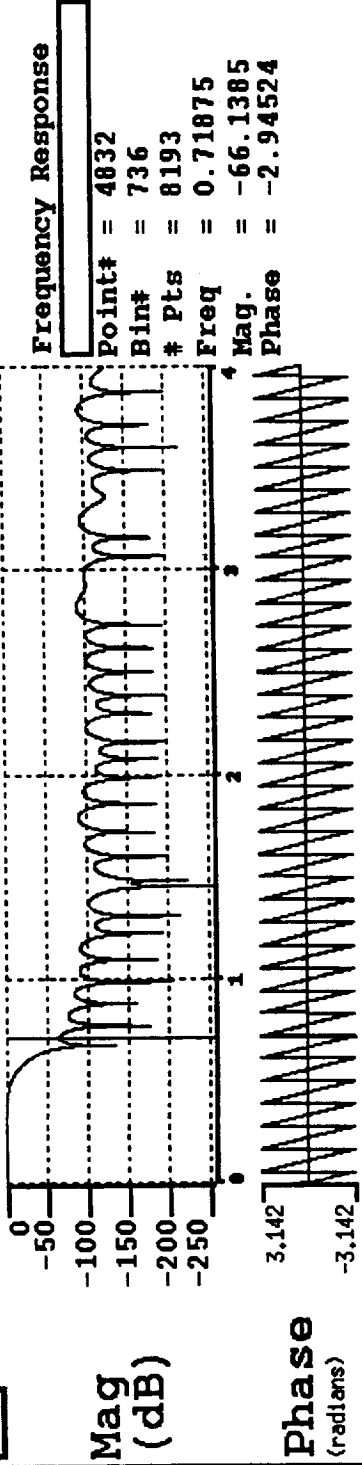
Frequency Response for CSD289829, nz = 2

A3



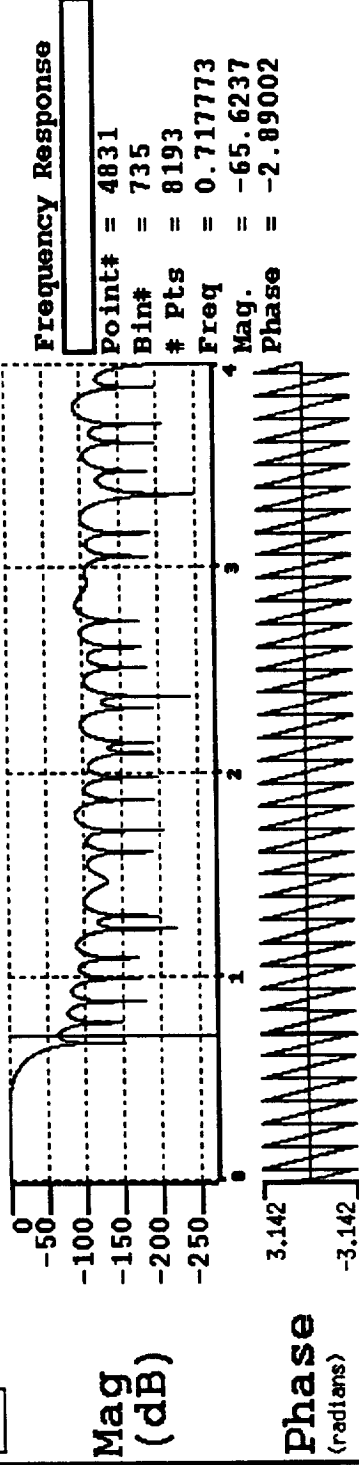
Frequency response for CSD389848, nz = 4

A1



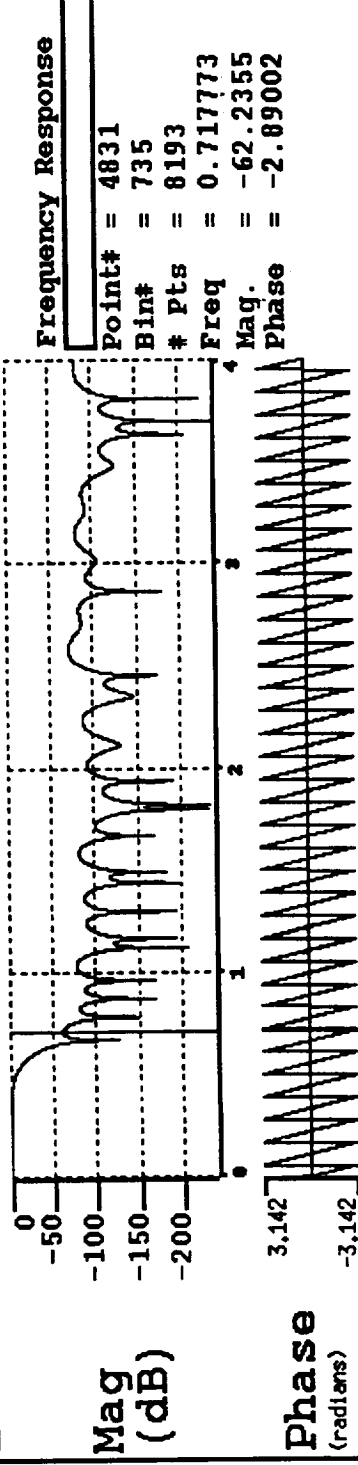
Frequency response for CSD389838, nz = 3

A2

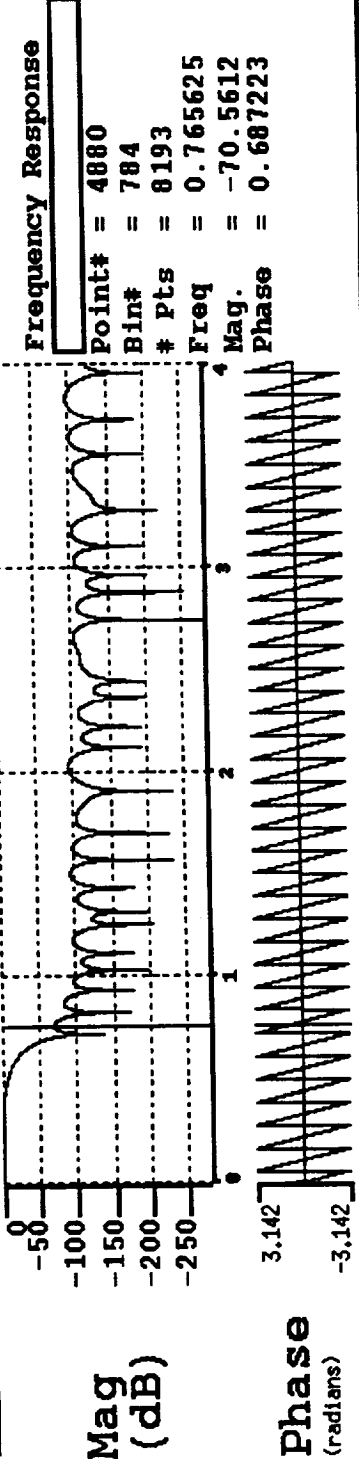


Frequency response for CSD389821, nz = 2

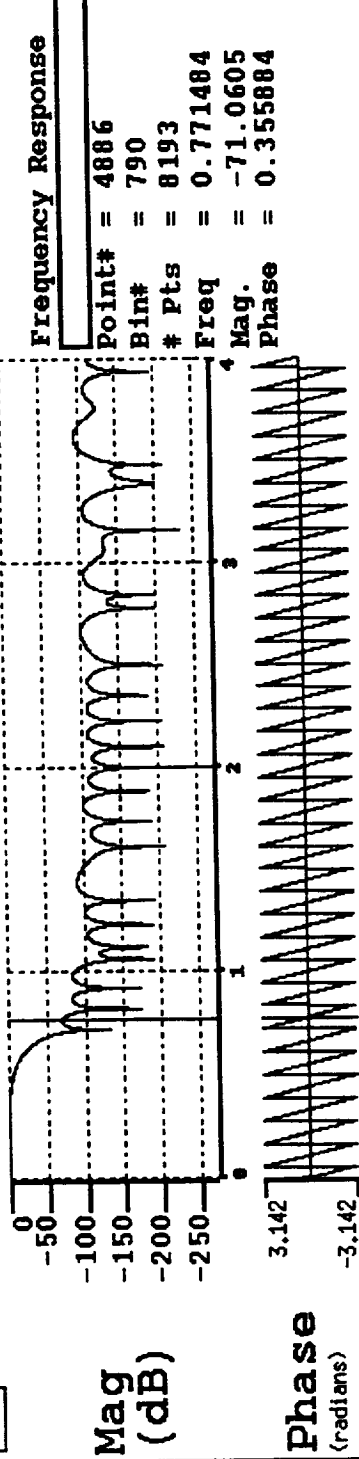
A3



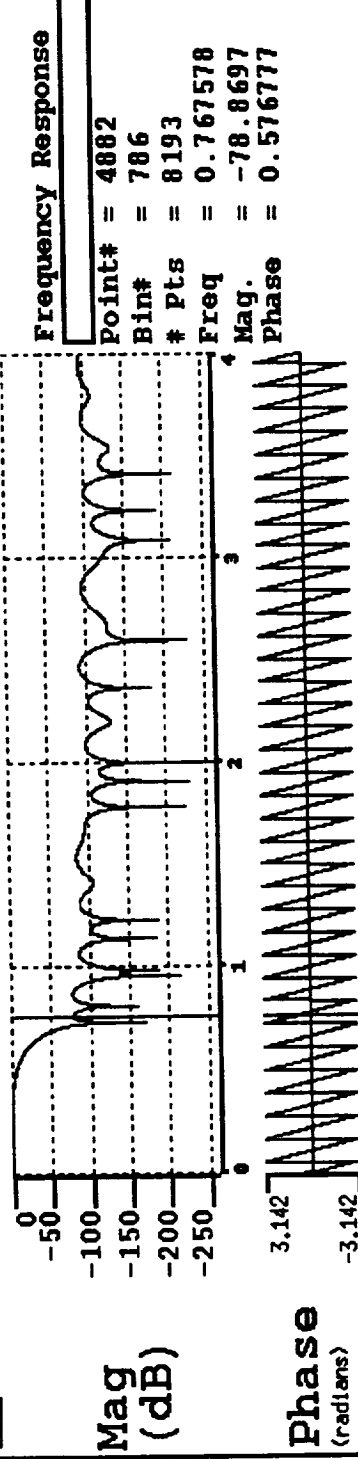
A1 Frequency Response for CSD489847, nz = 4



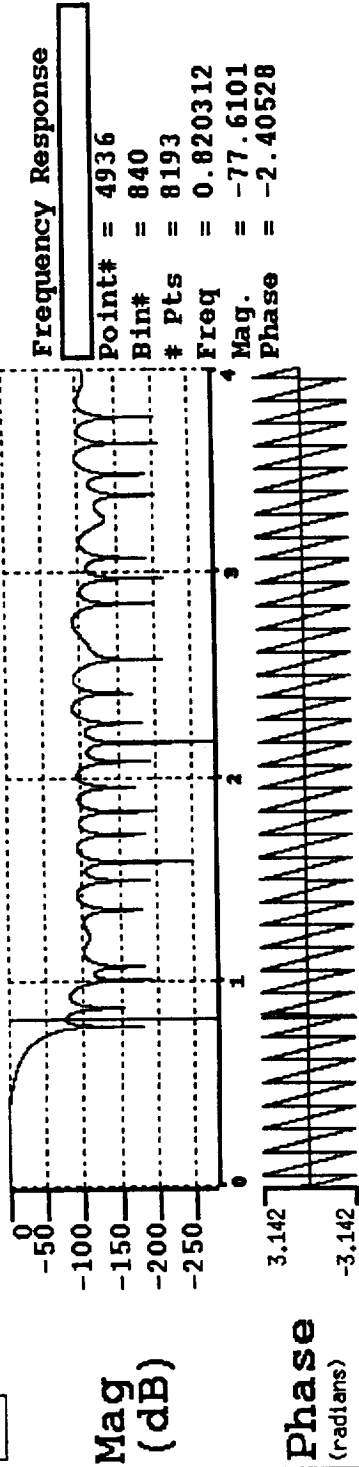
A2 Frequency Response for CSD489838, nz = 3



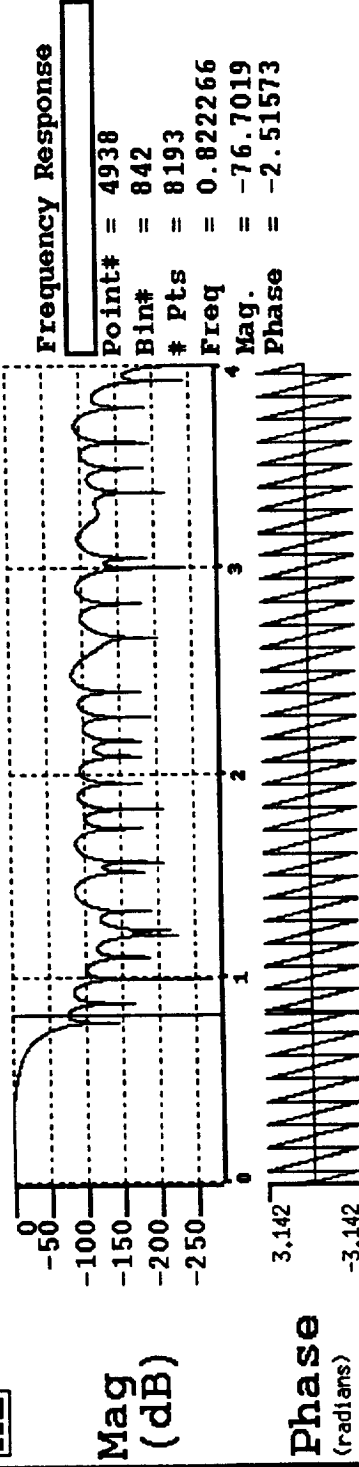
A3 Frequency Response for CSD489829, nz = 2



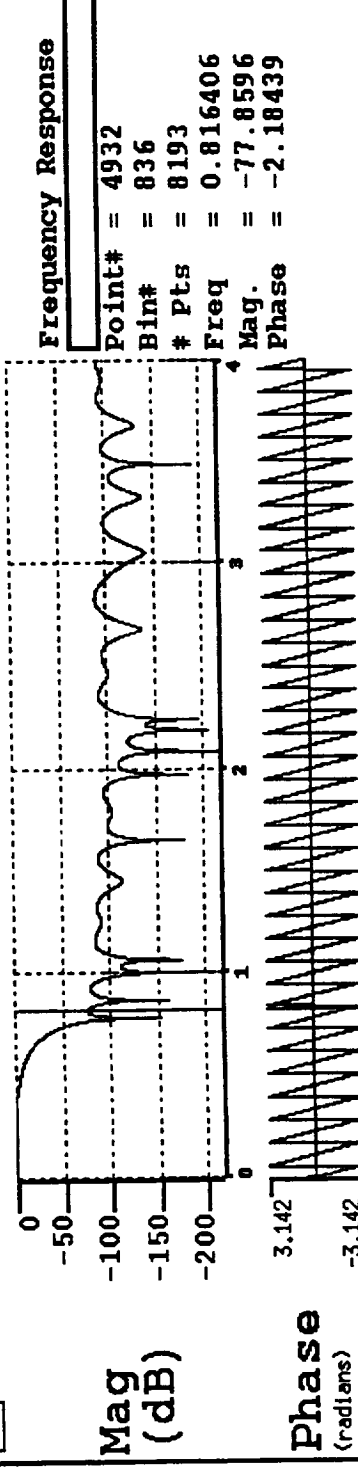
A1 Frequency Response for CSD589846, nz = 4



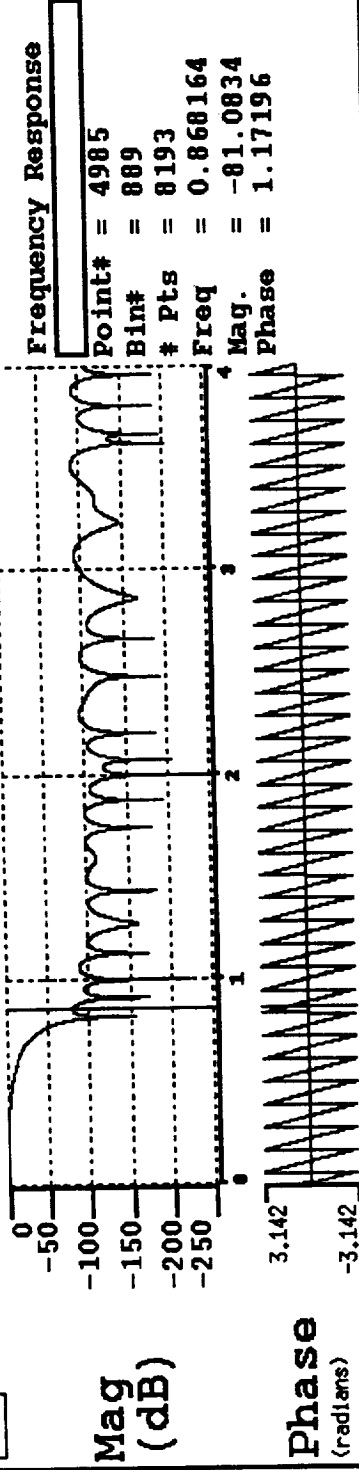
A2 Frequency Response for CSD589836, nz = 3



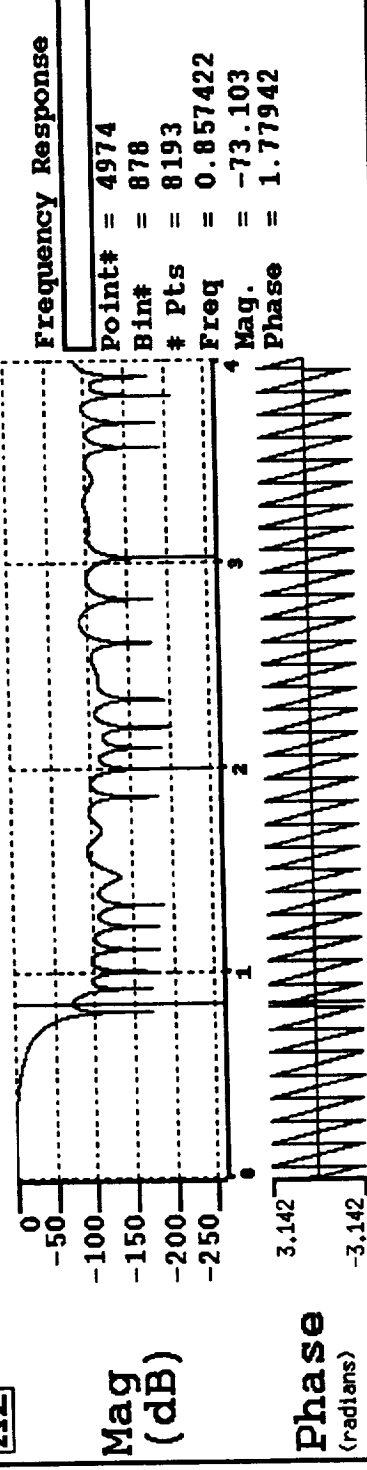
A3 Frequency Response for CSD589821, nz = 2



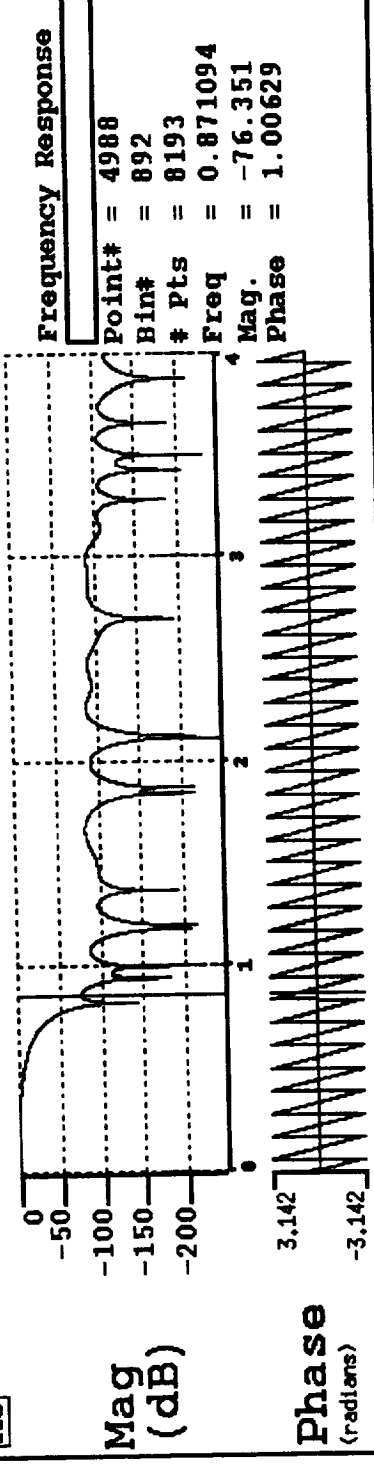
A1 Frequency Response for CSD689846, nz = 4



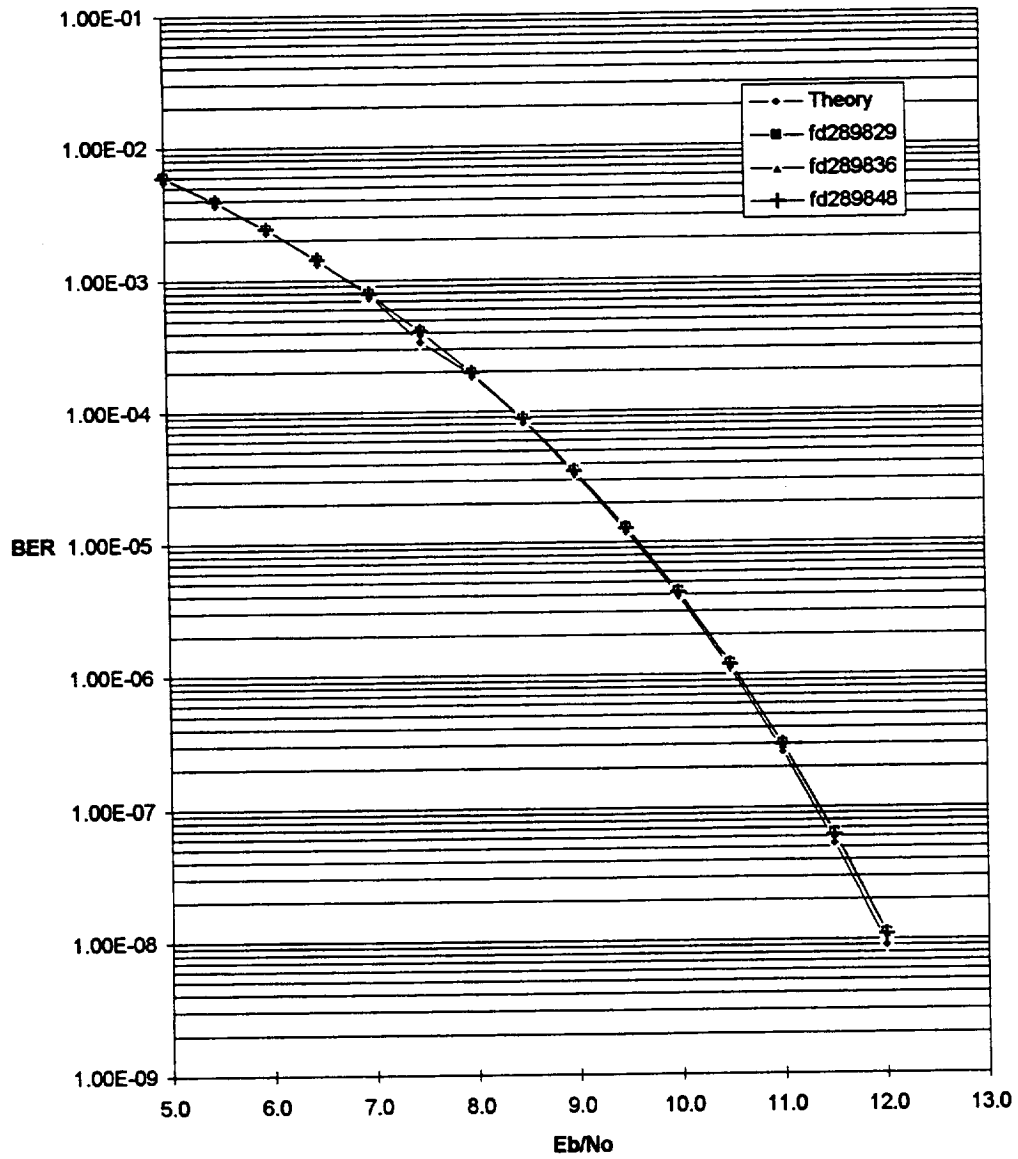
A2 Frequency Response for CSD689836, nz = 3



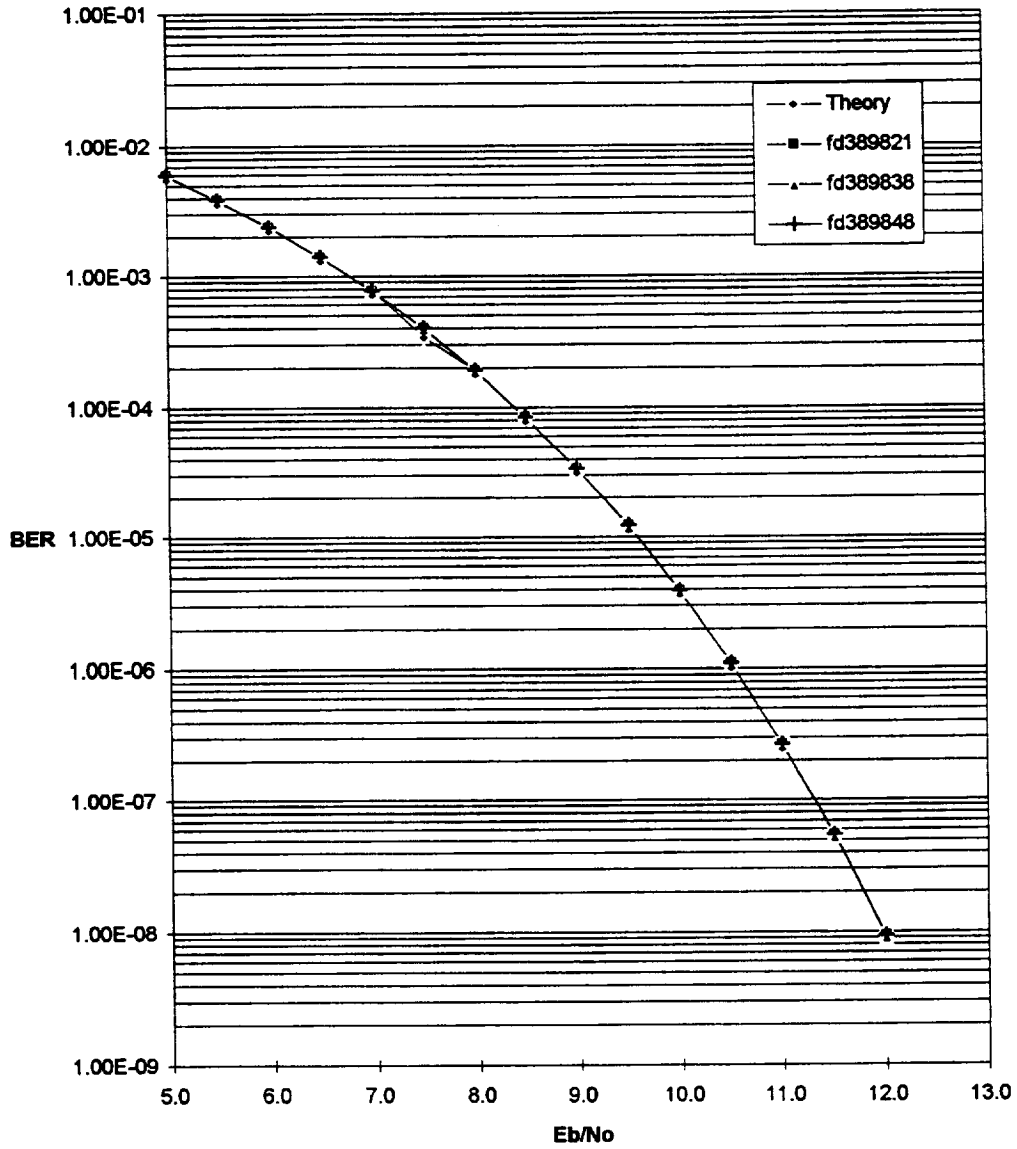
A3 Frequency response for CSD689829, nz = 2



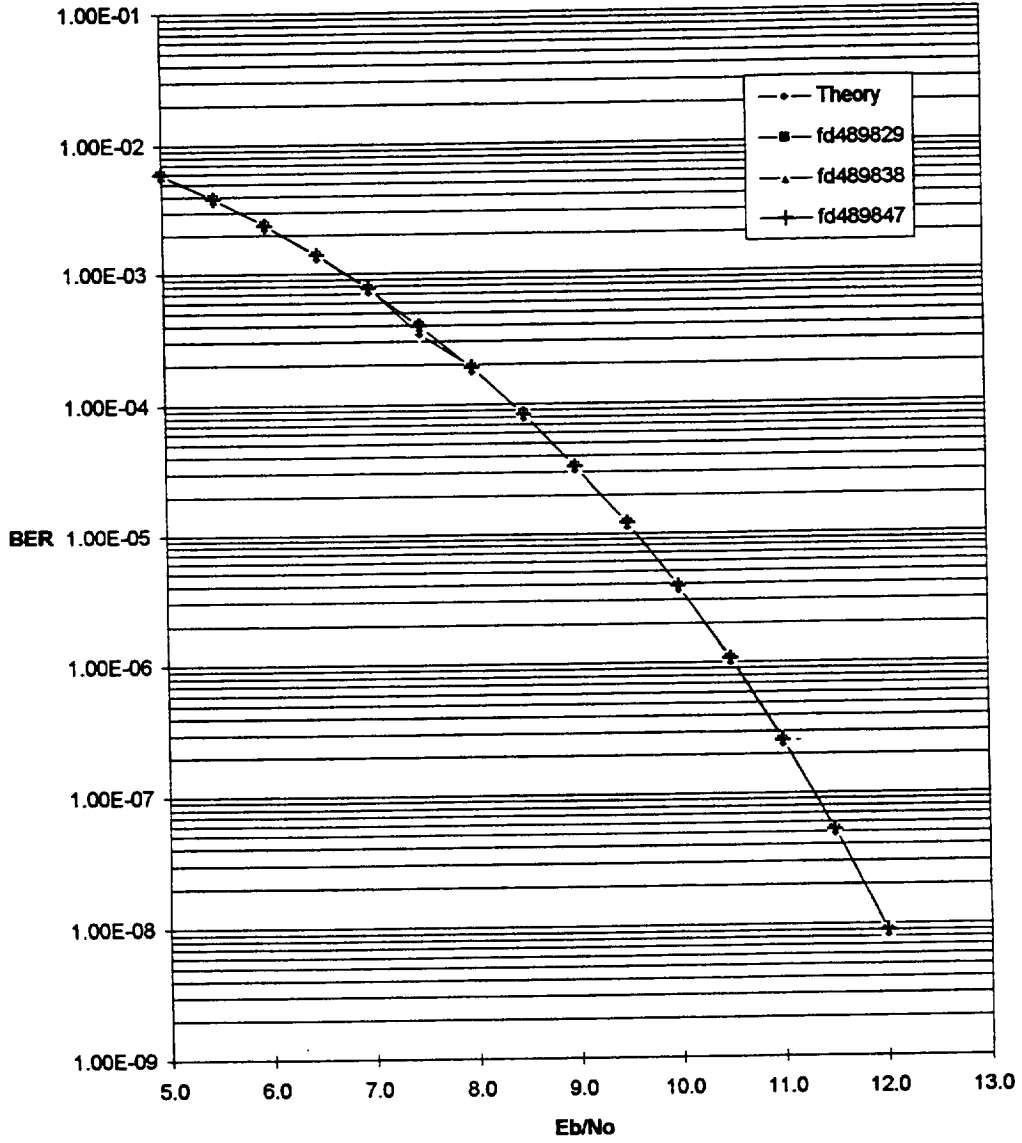
Simulated CSD Filter BER Curves



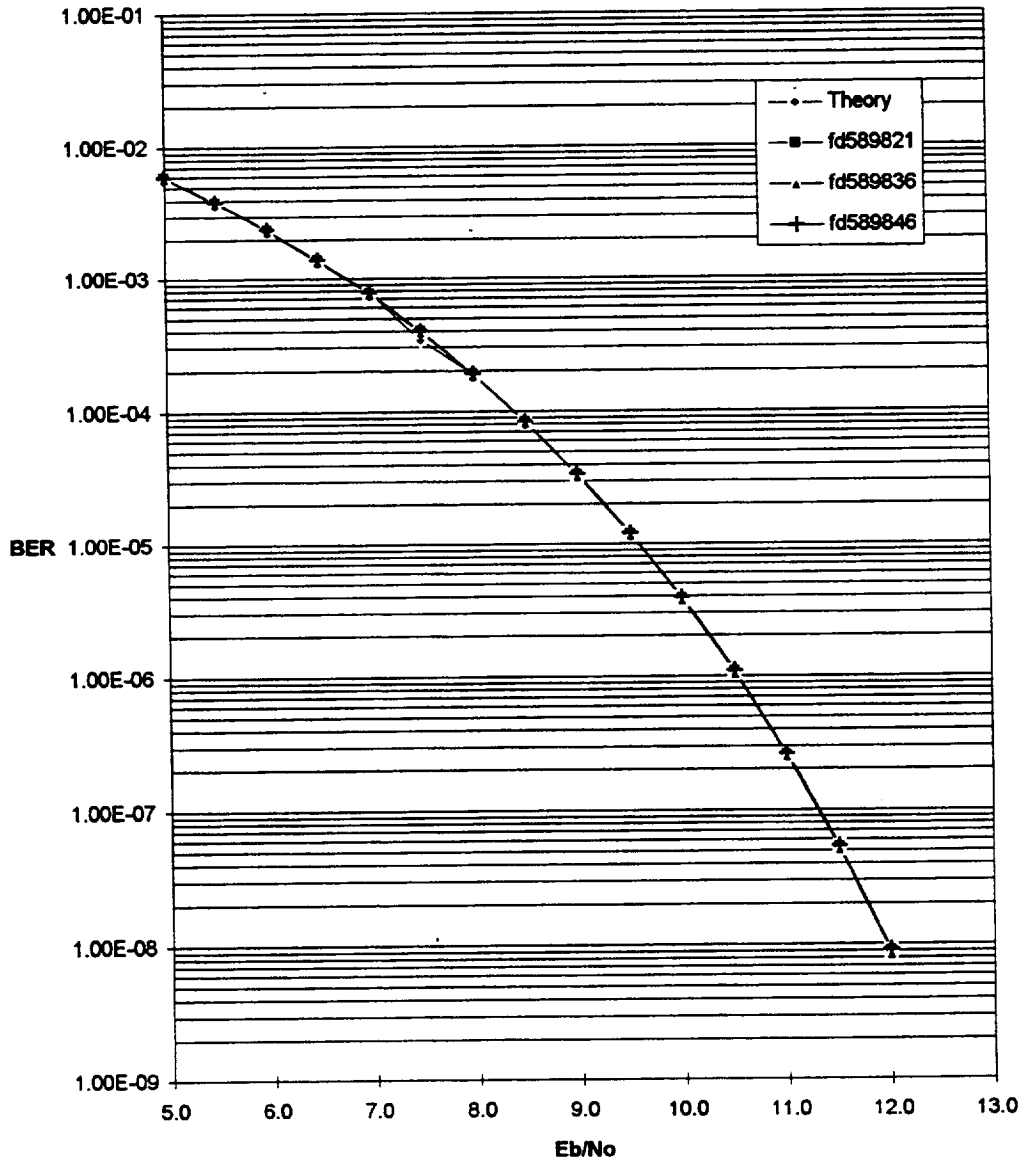
Simulated CSD Filter BER Curves



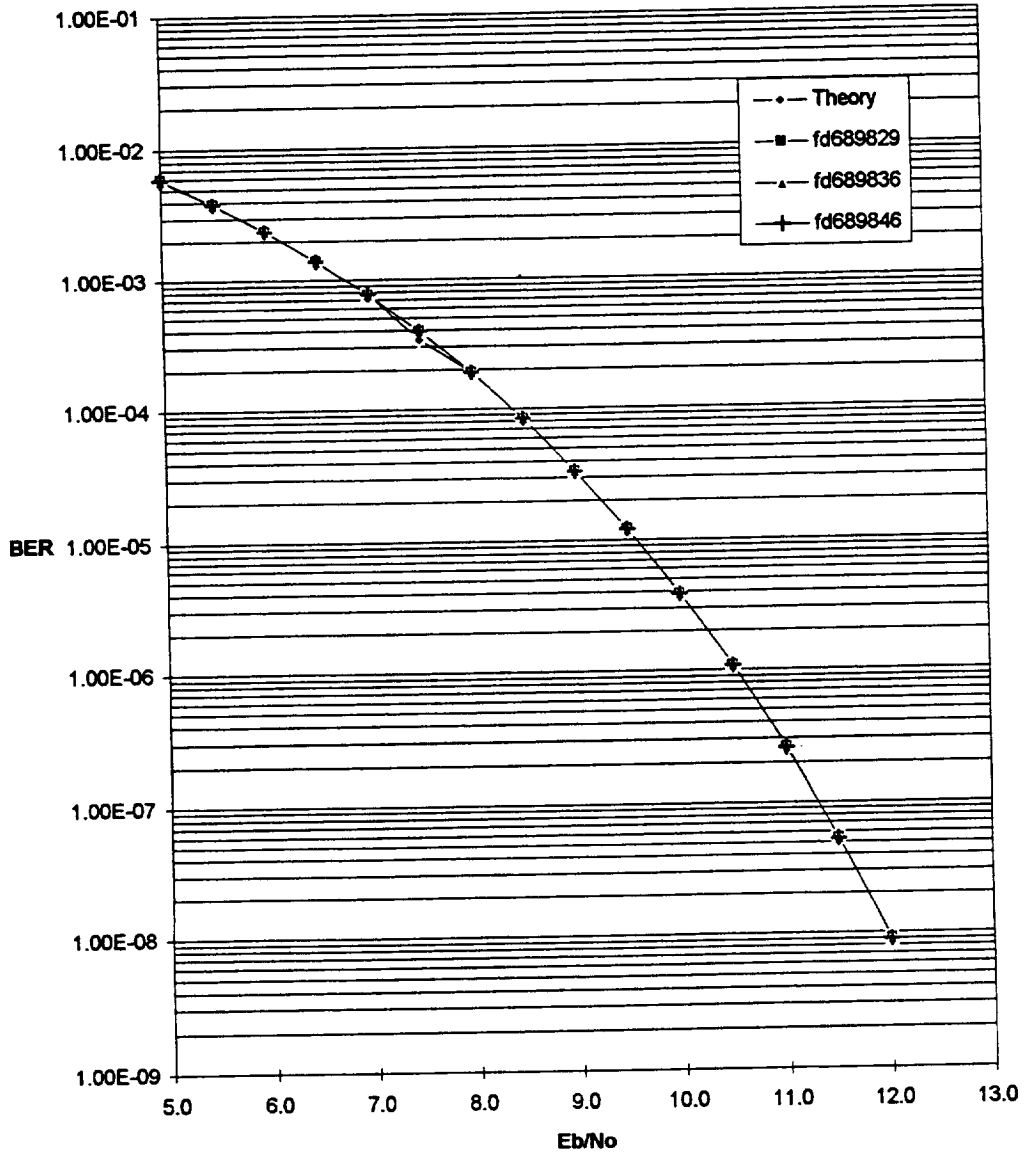
Simulated CSD Filter BER Curves



Simulated CSD Filter BER Curves



Simulated CSD Filter BER Curves



□ Hardware Complexity Comparison

example 1 CSD coefficients for 73-tap FIR 289829 filter

$$\begin{aligned} h(0) &= 2^6 - 2^8 \\ h(1) &= 2^6 + 2^8 \\ h(2) &= 2^5 - 2^8 \\ h(3) &= 2^5 - 2^8 \\ h(4) &= 2^5 - 2^7 \\ h(5) &= 2^6 \\ h(6) &= 0 \\ h(7) &= -2^6 \\ h(8) &= -2^6 \\ h(9) &= -2^4 + 2^6 \\ h(10) &= -2^4 + 2^6 \\ h(11) &= -2^4 + 2^6 \end{aligned}$$

$$\begin{aligned} h(12) &= -2^6 \\ h(13) &= -2^7 \\ h(14) &= 2^6 + 2^8 \\ h(15) &= 2^4 - 2^6 \\ h(16) &= 2^4 + 2^7 \\ h(17) &= 2^3 - 2^6 \\ h(18) &= 2^3 - 2^6 \\ h(19) &= 2^4 + 2^7 \\ h(20) &= 2^6 + 2^7 \\ h(21) &= -2^7 \\ h(22) &= -2^4 \\ h(23) &= -2^3 + 2^7 \end{aligned}$$

$$\begin{aligned} h(24) &= -2^3 - 2^6 \\ h(25) &= -2^2 + 2^4 \\ h(26) &= -2^3 - 2^6 \\ h(27) &= -2^3 \\ h(28) &= -2^4 + 2^6 \\ h(29) &= 2^4 + 2^7 \\ h(30) &= 2^2 - 2^6 \\ h(31) &= 2^1 - 2^3 \\ h(32) &= 2^1 + 2^6 + 2^8 \\ h(33) &= 2^0 - 2^3 - 2^6 \\ h(34) &= 2^0 - 2^3 + 2^6 \\ h(35) &= 2^0 - 2^4 \\ h(36) &= 2^0 - 2^4 + 2^6 \end{aligned}$$

Hardware: for one multiplier, 8 x 37 = 296 adders (On average) - 0.86 adders/tap

	Conventional	CSD Implemented	Difference	Ratio
Adders	72	32 + 72 = 104	264	1 : 3.54
Multipliers	37	0	**	**
Delays	72	72	0	1 : 1

example 2 CSD coefficients for 73-tap FIR 289836 filter

$h(0) = 2^7$	$h(12) = -2^6 - 2^8$	$h(24) = -2^3 + 2^6 + 2^8$
$h(1) = 2^6 - 2^8$	$h(13) = -2^7$	$h(25) = -2^3 + 2^8$
$h(2) = 2^6$	$h(14) = 2^6 - 2^8$	$h(26) = -2^3 + 2^6 - 2^8$
$h(3) = 2^6 + 2^8$	$h(15) = 2^6$	$h(27) = -2^3 + 2^6 + 2^7$
$h(4) = 2^6$	$h(16) = 2^4 - 2^6$	$h(28) = -2^6$
$h(5) = 2^7$	$h(17) = 2^4 - 2^8$	$h(29) = 2^4 - 2^6$
$h(6) = 0$	$h(18) = 2^4 - 2^8$	$h(30) = 2^3 + 2^4 + 2^8$
$h(7) = -2^6 + 2^8$	$h(19) = 2^4 - 2^6$	$h(31) = 2^2$
$h(8) = -2^6 + 2^7$	$h(20) = 2^5 - 2^8$	$h(32) = 2^1 - 2^3 - 2^6$
$h(9) = -2^6$	$h(21) = -2^8$	$h(33) = 2^1 - 2^4 + 2^6$
$h(10) = -2^6 - 2^8$	$h(22) = -2^4 + 2^6 + 2^8$	$h(34) = 2^1 + 2^6$
$h(11) = -2^6$	$h(23) = -2^4 - 2^6$	$h(35) = 2^1 + 2^4 + 2^6 + 2^8$
		$h(36) = 2^1 + 2^3 - 2^5 + 2^7$

Hardware: for one multiplier, 8 x 37 = 296 adders

	Conventional	CSD Implemented	Difference	Ratio
Adders	72	36 + 72 = 108	260	1 : 3.4
Multipliers	37	0	**	**
Delays	72	72	0	1 : 1

example 3 CSD coefficients for 73-tap FIR 289848 filter

$$\begin{aligned}
 h(0) &= 2^{-7} & h(12) &= -2^{-5} + 2^{-8} & h(24) &= -2^{-3} - 2^{-6} \\
 h(1) &= 2^{-6} & h(13) &= -2^{-7} & h(25) &= -2^{-3} - 2^{-5} - 2^{-8} \\
 h(2) &= 2^{-5} - 2^{-7} & h(14) &= 2^{-6} & h(26) &= -2^{-3} - 2^{-6} + 2^{-8} \\
 h(3) &= 2^{-5} - 2^{-7} & h(15) &= 2^{-4} - 2^{-6} - 2^{-8} & h(27) &= -2^{-3} + 2^{-6} - 2^{-8} \\
 h(4) &= 2^{-6} + 2^{-8} & h(16) &= 2^{-4} - 2^{-8} & h(28) &= -2^{-6} - 2^{-7} \\
 h(5) &= 2^{-6} - 2^{-8} & h(17) &= 2^{-4} + 2^{-6} & h(29) &= 2^{-4} \\
 h(6) &= 0 & h(18) &= 2^{-4} + 2^{-6} & h(30) &= 2^{-2} - 2^{-4} + 2^{-6} \\
 h(7) &= -2^{-6} & h(19) &= 2^{-4} - 2^{-8} & h(31) &= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} \\
 h(8) &= -2^{-6} + 2^{-8} & h(20) &= 2^{-5} + 2^{-8} & h(32) &= 2^{-1} - 2^{-6} + 2^{-7} \\
 h(9) &= -2^{-6} - 2^{-7} & h(21) &= -2^{-7} & h(33) &= 2^{-1} + 2^{-3} - 2^{-6} - 2^{-8} \\
 h(10) &= -2^{-4} + 2^{-6} + 2^{-8} & h(22) &= -2^{-4} + 2^{-7} & h(34) &= 2^0 - 2^{-3} - 2^{-6} - 2^{-8} \\
 h(11) &= -2^{-6} - 2^{-7} & h(23) &= -2^{-3} + 2^{-6} - 2^{-7} & h(35) &= 2^0 - 2^{-3} + 2^{-6} - 2^{-8} \\
 & & & & h(36) &= 2^0 - 2^{-3} + 2^{-6} - 2^{-7} + 2^{-8}
 \end{aligned}$$

Hardware: for one multiplier, 8 x 37 = 296 adders

	Conventional	CSD Implemented	Difference	Ratio
Adders	72	48 + 72 = 120	248	1 : 3.1
Multipliers	37	0	***	***
Delays	72	72	0	1 : 1

□ Conclusions

- For CSD implemented root-raised cosine filter with $\beta = 0.2, 0.3, 0.4$ and nonzero = 2, 3, 4 and 72 taps shows that all works nearly as well and the same as ideal one - aperture bounded
- Frequency response shows -30 dB or better stopband for all cases
- Sidelobes improve as the number of nonzero digits increases - also aperture bounded
- Eye diagrams look excellent and no timing jitters at the sampling instance
- Bit-Error-Rate test validates CSD implemented root-raised-cosine filter performs as well as one implemented conventionally
- Hardware saving from CSD implementation of filter becomes working model of Root-Raised-Cosine filter more dominant and powerful with less tap size filters

Future work

- **FPGA implementation of CSD filter**
- **Other optimization criteria, i.e. BER and ACI**
- **Other search methods, i.e. # of taps**

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