

On using Taylor's hypothesis for three-dimensional mixing layers

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In the present study, errors in using Taylor's hypothesis to transform measurements obtained in a temporal (or phase) frame onto a spatial one were evaluated. For the first time, phase-averaged ("real") spanwise and streamwise vorticity data measured on a three-dimensional grid were compared directly to those obtained using Taylor's hypothesis. The results show that even the qualitative features of the spanwise and streamwise vorticity distributions given by the two techniques can be very different. This is particularly true in the region of the spanwise roller pairing. The phase-averaged spanwise and streamwise peak vorticity levels given by Taylor's hypothesis are typically lower (by up to 40%) compared to the real measurements. © 1995 American Institute of Physics.

Taylor¹ hypothesized that the time history of the flow obtained from a stationary probe can be regarded as that due to convection of a spatial pattern. The underlying basic assumption of this hypothesis is that the flow structure remains unchanged or "frozen" as it passes by the measurement location. This Taylor's hypothesis (hereinafter referred to as T-H) has been extensively used on 2-D planes to educe organized structures in turbulent shear flows.^{2,3} The limitations of T-H (in two dimensions) have been widely studied and are fairly well understood. The four main causes for the breakdown of T-H have been identified as: temporal evolution of the flow field, spatial nonuniformity of convection velocity, anisotropy produced by the primary shear, and aliasing due to unsteadiness in convection velocity.⁴ Knowing that the mixing layer is dominated by large-scale structures which are continuously evolving and interacting with each other, it would appear from the onset that T-H will not work very well for the study of mixing layer structure. However, the use of T-H in mixing layers has been widespread and, furthermore, the errors incurred by using T-H are hardly ever discussed. The first attempt at optimizing the application of T-H to mixing layers, and at quantifying the resulting errors, was that of Zaman and Hussain.⁵ They compared actual 2-D spanwise vorticity (spatial) distributions with those given by T-H along a 2-D longitudinal cut through a jet mixing layer. In the regions of interaction, T-H gave unacceptably large errors regardless of the choice of convection velocity. Overall, the least objectionable results with T-H were obtained using the structure passage velocity.

Despite the findings of Zaman and Hussain,⁵ T-H has been routinely applied to the study of mixing layer structure. Furthermore, since transitioning mixing layers have been shown to contain a relatively strong secondary structure in the form of streamwise vortices,⁶ T-H has even been applied to the study of the three-dimensional (3-D) structure.⁷⁻⁹ The main objective of the present work was to establish quantitatively the errors incurred when employing T-H in 3-D mixing layers. The "real" phase-averaged spanwise and stream-

wise vorticity measurements¹⁰ (using the same database) in the present investigation. Comparisons of only the streamwise vorticity in the region upstream of and during pairing are presented in this Brief Communication.

The experiments were conducted in a mixing layer wind tunnel consisting of two separate legs which were set to 12 m/s and 7.2 m/s. The boundary layers on the splitter plate were laminar and nominally two dimensional at these operating conditions. Velocity measurements were made in two-coordinate planes (uv and uw) using a single cross-wire probe. A digital sine wave generator was used to provide the forcing signal consisting of the fundamental roll-up frequency and its subharmonic. This signal was supplied (via an audio amplifier) to a spanwise array of speakers located directly across from the splitter plate trailing edge. Averages over 768 ensembles of 16 samples per cycle were used for this study. The measurement grid resolution was 0.5 cm in the X direction, 0.25 cm in the Z direction, and ranged from 0.1 to 0.26 cm in the Y direction.

A single convection velocity ($U_c = 9.6$ m/s) equal to the average of the streamwise velocities on the two sides of the layer (suggested by Zaman and Hussain)⁵ was used to transform the sampling time (t) to $X = X_{ref} - U_c t$, where X_{ref} is the reference location at which the time series were measured. Then the resulting streamwise T-H grid spacing for the sampling rate used turned out to be $\Delta X = 0.24$ cm (i.e., less than half the true grid spacing). To facilitate quantitative comparisons and to avoid potential bias due to differences between the T-H grid, and the true measurement grid, the T-H velocity field was linearly interpolated to the measurement grid. Further reduction of both the real and approximated measurements to vorticity was achieved using a central difference scheme with forward and backward differences at the grid boundaries.

The streamwise evolution of spanwise vorticity along the mixing layer centerline for the reference phase is depicted in Fig. 1. The evolution and pairing of primary rollers is easily tracked in this figure. Clearly, signs of subharmonic

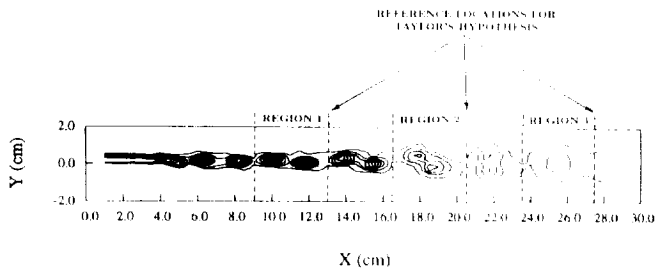


FIG. 1. Centerplane phase-averaged spanwise vorticity ($\langle \Omega_z \rangle / U_0$, cm^{-1}) contours. Lowest level = -0.25 , increment = -0.5 .

forcing are present early in the mixing layer development since distinguishable *pairs* of primary rollers are discernible from the onset of their development. The peak phase-averaged spanwise vorticity levels drop by an order of magnitude during the pairing process.

The three regions covered in the comparisons are shown in Fig. 1, as are the reference locations used for T-H. All the comparisons are made at a fixed phase (phase 1 of LeBoeuf and Mehta);¹⁰ changing the phase will only affect the locations and relative orientations of the structures, and not the conclusions drawn from these comparisons.

The region upstream of pairing (region 1) was defined to cover *two* spanwise rollers since, as noted above, pairs of vortices are discernible from the onset. A Y - Z plane cut at $X = 10$ cm showing phase-averaged streamwise vorticity in this domain is presented in Fig. 2. A vertically oriented "three-tier" distribution consisting of the upstream and downstream rib vortices of the same sign with opposite-signed vorticity (due to kinking of the roller) in between is exhibited in this cut through a spanwise roller. Some interesting differences are noted at $Z \approx 3$ cm. The T-H results show signs of the three-tier structure with positive core vorticity which one would expect to see in between the two neighboring structures of opposite sign. However, this structure is not at all visible in the true measurements. This implies that amplification of the positive ribs, and the resulting primary roller kinking, occur while the flow moves from $X = 10$ cm to the reference location at $X = 13$ cm. Another noteworthy feature is that some of the T-H structures, such as the positive one at $Z \approx 2$ cm for example, have a *higher* peak level. Again this is due to the local amplification occurring as the structure moves from $X = 10$ cm to the reference location. These observations imply that the vortical structures (both streamwise and spanwise) are undergoing a rapid change, even in this upstream, supposedly "dormant" region. Examination of the streamwise vorticity in the Y - Z plane at $X = 12$ cm (not shown here) revealed a qualitatively good comparison between the T-H and the true measurements. However, peak levels in the T-H results were lower by between 15% and 30% even though the plane is very close to the reference location. Clearly, T-H should not be used over more than one primary roller in this pre-pairing region.

The streamwise extent of region 2 is chosen to completely encompass the two merging rollers as shown in the

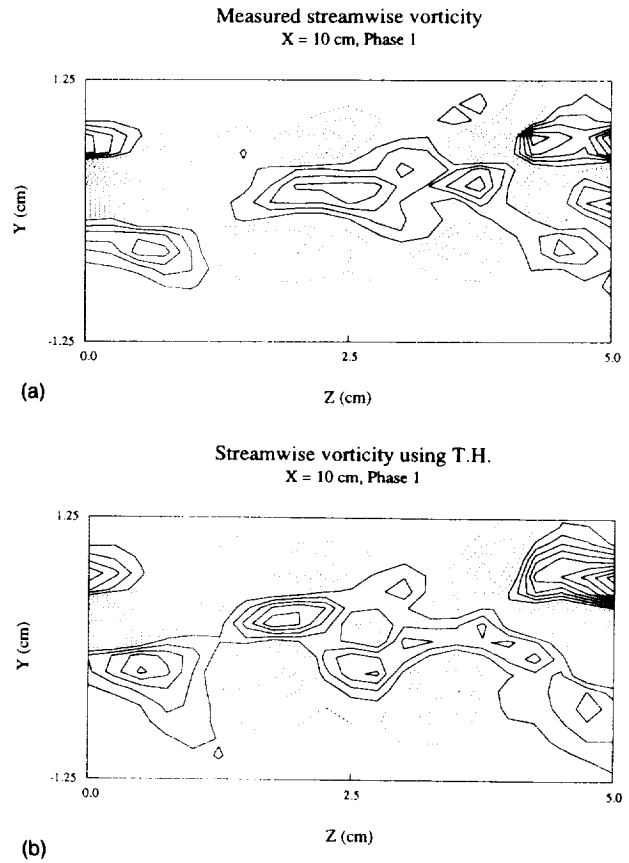
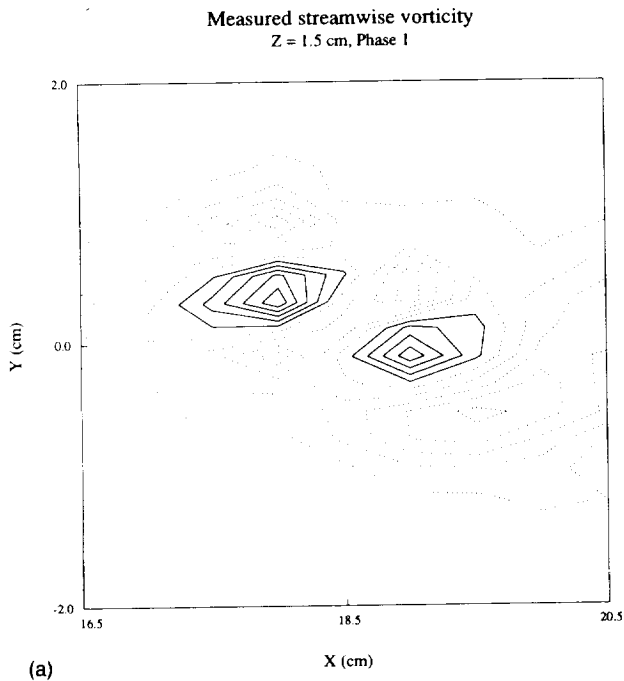


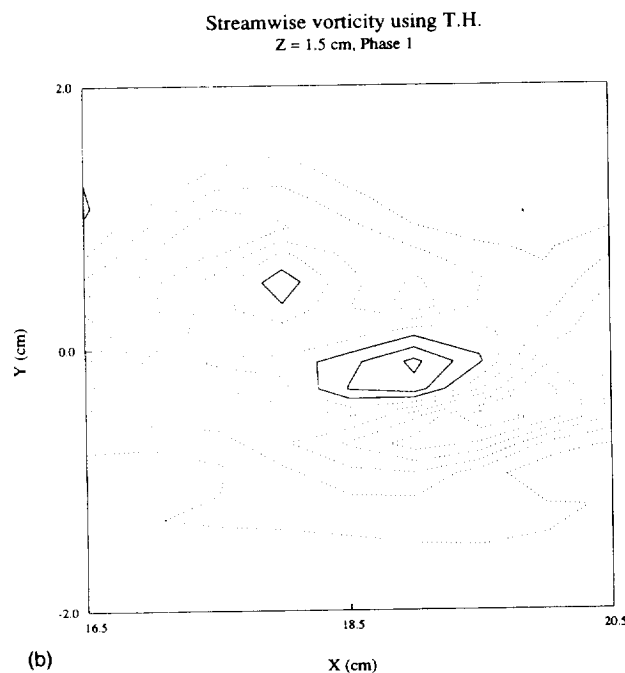
FIG. 2. Phase-averaged streamwise vorticity ($\langle \Omega_x \rangle / U_0$, cm^{-1}) contours at $X = 10$ cm. Negative \cdots , positive $—$, lowest level = ± 0.075 , increment = ± 0.15 . (a) Actual spatial distribution. (b) T-H approximation using a reference at $X = 13$ cm.

X - Y plane cut at $Z = 1.5$ cm (Fig. 3). It is apparent in the real measurements how the two rollers, each consisting of a three-tier distribution, begin to rotate around each other. Apart from the peak levels being lower in the T-H results, the distribution of streamwise vorticity peaks characterizing streamwise oriented ribs is not represented at all for the upstream pairing roller. Additionally, peaks due to the primary roller kinking have moved farther away from the centerline in the T-H results because the corotation progresses before the reference location is reached. The substantially lower peaks in the T-H streamwise vorticity results at this station were anticipated since the peak vorticity is generally decreasing with downstream distance in this region.¹⁰ So, as the flow moves from its T-H inferred location to the reference location, the vorticity experiences a significant decrease.

The results show that even the qualitative features of the phase-averaged spanwise and streamwise vorticity distributions given by the two techniques can be very different in *all* regions of the mixing layer development. This can occur in the form of different shapes of the vortical structures or the appearance or absence of some structures. This is particularly true in regions of the spanwise vortex pairing, when the spanwise and streamwise structures are undergoing rapid changes in time, thus invalidating the "frozen flow" approximation of Taylor.¹ In terms of the dynamics, the vorticity



(a)



(b)

FIG. 3. Phase-averaged streamwise vorticity $\langle\langle\Omega_x\rangle\rangle/U_0$, cm^{-1} contours at $Z = 1.5$ cm. Negative \cdots , positive — , lowest level $= \pm 0.075$, increment $= \pm 0.15$. (a) Actual spatial distribution. (b) T-H approximation using a reference at $X = 20.5$ cm.

distributions given by T-H show an earlier completion of a given interaction (e.g., pairing) than is actually the case.

Furthermore, the peak levels of spanwise and streamwise vorticity are generally underestimated in all regions when

using T-H. The underestimation typically ranges between 20% and 40% which is higher than the estimated vorticity measurement accuracy of 15%.¹⁰ This is a direct consequence of both the peak spanwise and streamwise phase-averaged vorticity decaying with streamwise distance, as shown in the recent measurements of LeBoeuf and Mehta.¹⁰ So obviously, when using T-H, since the peak vorticity is decaying continuously within a given structure, a lower peak level is measured as it passes through the reference location. Exceptions to this trend were observed in the region preceding pairing, where *locally* some ribs were still amplifying. This resulted in a local overestimation of some peaks in the T-H inferred streamwise vorticity. Not surprisingly, differences or errors increase as the distance from the T-H reference location is increased.

The present comparisons clearly show that if details of the vortical structures (peak vorticity levels and morphology) are important in an investigation, then Taylor's hypothesis should not be used to transform time (or phase) onto a spatial domain. Instead, phase-averaged data must be obtained on 3-D grids.

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