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Asymptotic Waveform Evaluation (AWE) Technique for Frequency Domain Electromagnetic Analysis

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CONTENTS

Abstract

The Asymptotic Waveform Evaluation(AWE) technique is applied to a generalized frequency domain electromagnetic problem. Most of the frequency domain techniques in computational electromagnetics result in a matrix equation, which is solved at a single frequency. In the AWE technique, the Taylor series expansion around that frequency is applied to the matrix equation. The coefficients of the Taylor's series are obtained in terms of the frequency derivatives of the matrices evaluated at the expansion frequency. The coefficients hence obtained will be used to predict the frequency response of the system over a frequency range. The detailed derivation of the coefficients (called 'moments') is given along with an illustration for electric field integral equation (or Method of Moments) technique. The Radar Cross Section(RCS) frequency response of a square plate is presented using the AWE technique and is compared with the exact solution at various frequencies.

List of Symbols

∇	Del operator
δ_{qo}	Kronecker delta defined in equation (12)
η_o	Free space intrinsic impedance 377Ω
AWE	Asymptotic Waveform Evaluation
$A(k_o)$	Impedance/Admittance matrix of order NXN evaluated at k_o
$A(k)$	Impedance/Admittance matrix of order NXN evaluated at k
$A^{(n)}(k)$	nth derivative of $A(k)$ with respect to k; $\frac{d^n}{dk^n}A(k)$
$A^{-1}(.)$	Inverse of matrix A
$b(k_o)$	Excitation vector of order N evaluated at k_c
$b(k)$	Excitation vector of order N evaluated at k
$b^{(n)}(k)$	nth derivative of $b(k)$ with respect to k; $\frac{d^n}{dk^n}b(k)$
ds	Surface integration
EFIE	Electric Field Integral Equation
$\mathbf{E}_i$	Incident electric field
$I(k)$	The coefficient vector for current distribution in method of moments for EFIE implementation
$\mathbf{J}$	Vector basis function for the current distribution
j	$\sqrt{-1}$

k_o	Wavenumber at frequency f_o
k	Wavenumber at any frequency f
M_n	nth moment of AWE ($n=0,1,2,3,4$ ....)
$n!$	Factorial of number n
$P(q,p)$	Permutation Function
R	Distance between the source point and the observation point
$\mathbf{T}$	Vector testing function used in EFIE
$V(k)$	The excitation vector due to the incident electric field
$x(k_o)$	Solution vector of order N evaluated at k_o
$x(k)$	Solution vector of order N evaluated at k
$x^{(n)}(k)$	nth derivative of $x(k)$ with respect to k; $\frac{d^n}{dk^n}x(k)$
$Z(k)$	Impedance matrix of order N in method of moments for EFIE implementation
$Z^{-1}(.)$	Inverse of the matrix $Z(.)$

1. Introduction

Frequency domain numerical techniques such as Method of Moments(MoM), Finite Element Method(FEM) and hybrid FEM/MoM have become popular over the last few years due to their flexibility to handle arbitrarily shaped objects and complex materials[1,2]. One of the disadvantages of frequency domain techniques, however is the computational cost involved in obtaining the solutions over a frequency range. Computations have to be repeated for each frequency to obtain the complete frequency response over a frequency range. For frequency dependent systems such as resonant structures, the number of frequencies required to capture the resonance can be very large. If the problem size is large, total CPU time to compute all the frequencies can be highly prohibitive. To overcome this problem, a technique called Asymptotic Waveform Evaluation(AWE) is proposed. Initially, this technique was applied to timing analysis of VLSI circuits[3,4] and extended later to finite element analysis for microwave circuits[5].

The AWE technique, basically makes use of the Taylor series expansion of a matrix equation which is common in all frequency domain techniques. The coefficients of the Taylor Series (called 'moments', not to be confused with moments in Method of Moments) are evaluated using frequency derivatives of the original system matrix. In this work, we derive the expressions for evaluating the AWE moments and discuss the validity of AWE over a frequency band. Also, as an illustration, the Electric Field Integral Equation (EFIE) will be considered to compute the AWE moments.

The AWE technique used with a single expansion frequency may not always produce accurate results over a desired frequency range. Once the desired frequency range is fixed, techniques such as Complex Frequency Hopping(CFH)[6] can be used to accurately predict the frequency response over the entire frequency range. CFH involves considering multiple expansion frequency points for applying AWE and checking the accuracy of the response.

The organization of the rest of the paper is as follows. In section 2, the derivation of AWE moments for any system matrix (resulting from a frequency domain technique) is given. An application to the EFIE is also discussed. Section 3 discusses the accuracy of single frequency AWE and possible application of CFH for accurate prediction of frequency response over a desired frequency range. Numerical results of RCS frequency response of a square plate are presented. These results are compared with the computations done at each frequency point to validate the analysis presented in this paper. Section 4 concludes the paper with remarks on the advantages and limitations of the current technique.

2. Derivation of AWE moments

Any frequency domain technique such as MoM or FEM depends on the solution of the matrix equation

$$A(k_o)x(k_o) = b(k_o) \tag{1}$$

where $A(k_o)$ is a square matrix of the order N (number of unknowns in the frequency domain technique) calculated at the frequency corresponding to k_o, the wave number. Similarly $b(k_o)$ is the excitation vector and $x(k_o)$ is the solution vector at the same frequency. The AWE technique approximates the frequency response by expanding $x(k)$ (where k is the wave number corresponding to any frequency within the frequency range) in a Taylor series around k_o.

$$x(k) = x(k_o) + x^{(1)}(k_o)(k-k_o) + \frac{x^{(2)}(k_o)(k-k_o)^2}{2!} + \frac{x^{(3)}(k_o)(k-k_o)^3}{3!} + \ldots\ldots +$$

$$\frac{x^{(n)}(k_o)(k-k_o)^n}{n!} + \ldots\ldots\ldots\ldots\ldots\ldots \tag{2}$$

where $x^{(n)}(k_o)$ is the nth derivative of $x(k)$ evaluated at k_o.

Writing the moments

$$M_n = \frac{x^{(n)}(k_o)}{n!} \tag{3}$$

with $x^{(0)}(k_o) = x(k_o)$, equation (2) can be rewritten as

$$x(k) = \sum_{n=0}^{\infty} M_n (k-k_o)^n \tag{4}$$

Equation (1) can be rewritten for any frequency as

$$A(k)x(k) = b(k) \tag{5}$$

At $k = k_o$;

$$M_o = x(k_o) = A^{-1}(k_o)\,b(k_o) \tag{6}$$

Differentiating equation (5) with respect to k

$$x^{(1)}(k) = A^{-1}(k)\left[b^{(1)}(k) - A^{(1)}(k)\,x(k)\right] \tag{7}$$

Evaluating equation (7) at k_o, the moment M_1 is given by

$$M_1 = A^{-1}(k_o)\left[b^{(1)}(k_o) - A^{(1)}(k_o)\,M_o\right] \tag{8}$$

Differentiating equation (7) with respect to k again;

$$x^{(2)}(k) = A^{-1}(k)\left[b^{(2)}(k) - 2A^{(1)}(k)\,x^{(1)}(k) - A^{(2)}(k)\,x(k)\right] \tag{9}$$

Evaluating equation (9) at k_o, the moment M_2 is given by

$$M_2 = A^{-1}(k_o)\left[\frac{b^{(2)}(k_o)}{2!} - \frac{A^{(1)}(k_o)\,M_1}{1!} - \frac{A^{(2)}(k_o)\,M_o}{2!}\right] \tag{10}$$

From equations (8) and (10), a recursive relationship can be written for evaluating the moments as

$$M_n = A^{-1}(k_o)\left[\frac{b^{(n)}(k_o)}{n!} - \sum_{q=0}^{n}\frac{(1-\delta_{qo})\,A^{(q)}(k_o)\,M_{n-q}}{q!}\right] \tag{11}$$

where the Kronecker delta δ_{qo} is defined as

$$\delta_{qo} = \begin{cases} 1 & q = 0 \\ 0 & q \neq 0 \end{cases} \tag{12}$$

$A^{(n)}(k_o)$ and $b^{(n)}(k_o)$ are the nth derivatives of $A(k)$ and $b(k)$ at the frequency corresponding to k_o.

Once the moments are evaluated, the solution vector at any frequency (within the frequency range of accuracy) can be found by equation (4). It can be noticed from the equations that if the inverse of matrix $A(k_o)$ is calculated once, it can be repeatedly used to compute the moments. In practice, instead of finding the inverse of matrix $A(k_o)$ a LU factorization of the matrix is done once and all the moments are evaluated by computationally less intensive forward/ backward substitution.

Application of AWE to EFIE:

The Electric Field Integral Equation is widely used in MoM, for radar cross section analysis of complex perfect Electric Conductor(PEC) bodies. The analysis involves solving the following matrix equation:

$$Z(k)I(k) = V(k) \tag{13}$$

where

$$Z(k) = \frac{jk\eta_o}{4\pi}\iint \mathbf{T} \bullet \iint \mathbf{J}\frac{\exp(-jkR)}{R}ds'ds - \frac{j\eta_o}{4\pi k}\iint (\nabla\bullet\mathbf{T}) \iint (\nabla\bullet\mathbf{J})\frac{\exp(-jkR)}{R}ds'ds \tag{14}$$

and

$$V(k) = \iint \mathbf{E}_i(k) \bullet \mathbf{T}ds \tag{15}$$

$\mathbf{J}$ is the current distribution over the surface and $\mathbf{T}$ is the testing function. $\mathbf{E}_i$ is the incident plane wave. For a subdomain approach, the surface integrals are evaluated over the surfaces of the subdomains. For more details on subdomain MoM using EFIE, the reader is referred to [7].

Applying AWE to the equation (13), $I(k)$ is expanded in Taylor series as

$$I(k) = \sum_{n=0}^{\infty} M_n(k-k_o)^n \tag{16}$$

with the moments given by

$$M_n = Z^{-1}(k_o)\left[\frac{V^{(n)}(k_o)}{n!} - \sum_{q=0}^{n}\frac{(1-\delta_{qo})Z^{(q)}(k_o)M_{n-q}}{q!}\right] \tag{17}$$

$Z^{(q)}(k_o)$ is the qth derivative with respect to k, of $Z(k)$ given in equation (14) and evaluated at k_o. Similarly $V^{(n)}(k_o)$ is the nth derivative with respect to k, of $V(k)$ given in equation (15). After performing a number of differentiations, one can show that the explicit representation of $Z^{(q)}(k_o)$ is given by

$$Z^{(q)}(k_o) = \frac{jk_o\eta_o}{4\pi}\iint \mathbf{T} \bullet \iint \mathbf{J}(-jR)^q\left(1-\frac{q}{jk_oR}\right)\frac{\exp(-jk_oR)}{R}ds'ds$$

$$- \frac{j\eta_o}{4\pi k_o}\iint(\nabla\bullet\mathbf{T})\iint(\nabla\bullet\mathbf{J})\,(-jR)^q\left(\sum_{p=0}^{q}\frac{P(q,p)}{(jk_oR)^p}\right)\frac{\exp(-jk_oR)}{R}ds'ds \tag{18}$$

where the permutation function $P(q,p)$ is defined as[8]

$$P(q,p) = \frac{q!}{(q-p)!} \tag{19}$$

Once the moments are obtained, the current distribution can be obtained for different frequencies within the frequency range of accuracy using the equation (16). The Radar Cross Section is obtained using the current distribution on the PEC surface.

3. Numerical Results

The AWE technique described above is implemented in a method of moments code to obtain the RCS frequency response of a square plate. Figures 1 and 2 show the frequency response of 1cmX1cm square plate over two different frequency bands for H-polarization with normal incidence. In figure 1, the frequency response at the center frequency 30GHz is shown with a frequency range of $\pm 10\,\text{GHz}$. Even with two or three moments, we can see a very good agreement over the complete frequency band. In figure 2, the frequency response at the center frequency 12GHz is shown with a frequency range of $\pm 6\,\text{GHz}$. The first, third and sixth order AWE solutions are plotted. As it can be seen from figure 2, third order AWE showed a reasonable agreement in the frequency range of $12 \pm 3\,\text{GHz}$, the sixth order gave a very good agreement. These results validate the application of AWE for electromagnetic analysis. Many more examples along with storage and timing requirements are presented in [9] to show the flexibility of AWE technique in different EM environments.

As it can be seen from the two examples presented, the frequency range and accuracy depends on the number of moments used and also the electromagnetic phenomena occurring in a particular problem. By numerical experimentation, it was also noted that after certain number of moments, the frequency range of AWE has not increased further. Methods such as Complex Frequency Hopping(CFH) can be implemented to estimate the error in AWE predictions and hence improve the reliability of the calculations and also increase the frequency range[6].

4. Concluding Remarks

Application of AWE for numerical electromagnetic analysis is considered. The moments required in the AWE analysis are derived and explicit expressions are presented. Application of AWE for an electric field integral equation for RCS frequency response is demonstrated. A numerical example is considered to validate the analysis presented. AWE seems to be a viable approach to obtain the frequency response of an electromagnetic system through a frequency domain analysis. By expanding at many frequency points, a RCS over a wide frequency range can be obtained. Computationally, AWE increases the storage and CPU time requirements, compared to single point calculations. But considering the number of frequency points required to compute the frequency response of a system, AWE provides much better performance. The accuracy and frequency range of AWE can be further improved by implementing techniques such as Complex Frequency Hopping(CFH)[6].

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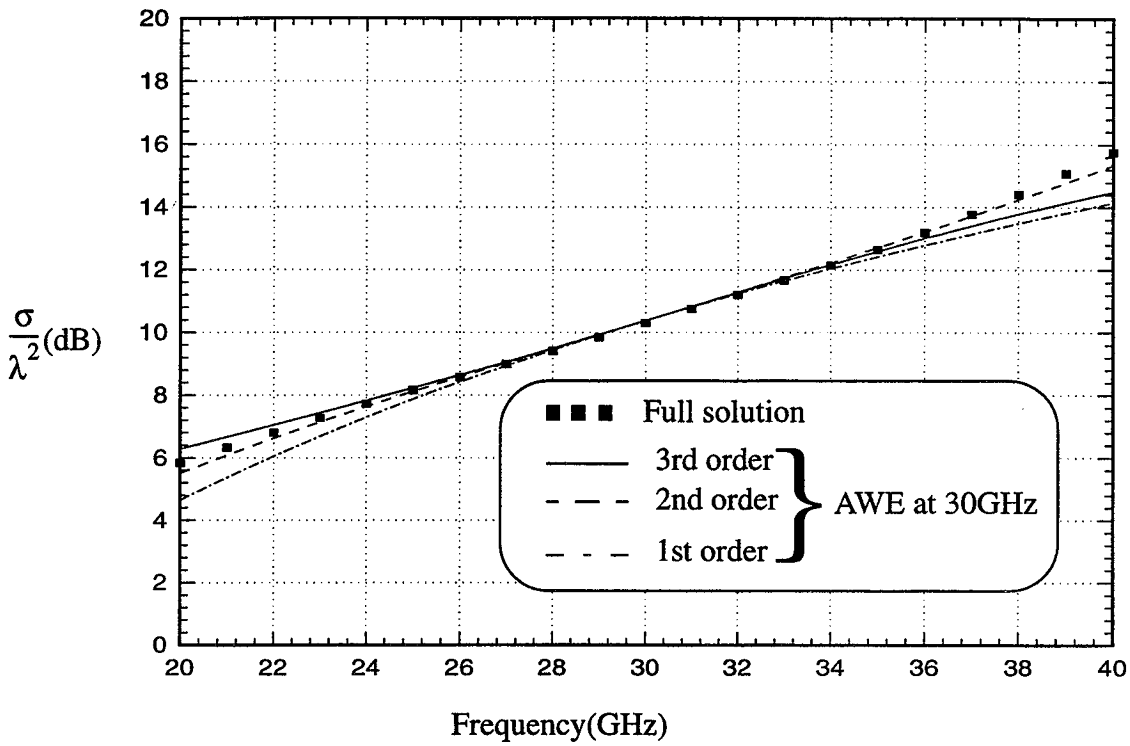

Figure 1 Frequency response of a square plate (1cmX1cm). Center frequency 30GHz (H-Polarized, normal incidence)

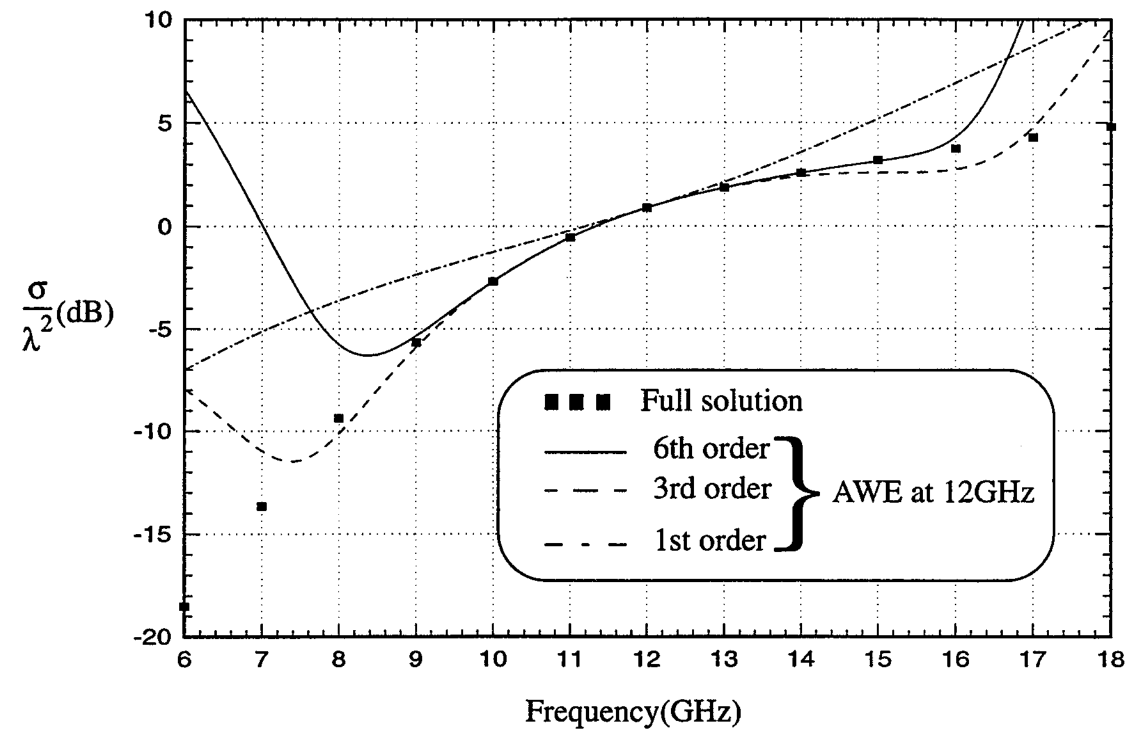

Figure 2 Frequency response of a square plate (1cmX1cm). Center frequency 12GHz (H-Polarized normal incidence).

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