

Temperature Distributions in Semitransparent
Coatings - A Special Two-Flux Solution

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Radiative transfer is analyzed in a semitransparent coating on an opaque substrate and in a semitransparent layer for evaluating thermal protection behavior and ceramic component performance in high temperature applications. Some ceramics are partially transparent for radiative transfer, and at high temperatures internal emission and reflections affect their thermal performance. The behavior is examined for a ceramic component for which interior cooling is not provided. Two conditions are considered: 1) the layer is heated by penetration of radiation from hot surroundings while its external surface is simultaneously film cooled by convection, and 2) the surface is heated by convection while the semitransparent material cools from within by radiant emission leaving through the surface. By using the two-flux method, which has been found to yield good accuracy in previous studies, a special solution is obtained for these conditions. The analytical result includes isotropic scattering and requires only an integration to obtain the temperature distribution within the semitransparent material. Illustrative results are given to demonstrate the nature of the thermal behavior.

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Nomenclature

| | |
|----------------|--|
| a | absorption coefficient of semitransparent material, m^{-1} |
| C | integration constant in energy equation, W/m ; $\tilde{C} = C/D\sigma T_g^4$ |
| D | thickness of coating or one-half of layer (Fig. 1), m |
| G | flux quantity defined in Eq. (2b), W/m^2 ; $\tilde{G} = G/\sigma T_g^4$ |
| H | dimensionless convection-radiation parameter, $h/\sigma T_g^3$ |
| h | heat transfer coefficient at boundary, W/m^2K |
| k | thermal conductivity, W/mK |
| K | extinction coefficient, $a+\sigma_s$, m^{-1} |
| N | conduction-radiation parameter, $k/\sigma T_g^3 D$ |
| n | refractive index of layer |
| q | heat flux, W/m^2 ; $\tilde{q} = q/\sigma T_g^4$ |
| q_r | radiative heat flux, W/m^2 ; $\tilde{q}_r = q_r/\sigma T_g^4$ |
| q_r^+, q_r^- | radiative fluxes in + and - x directions, W/m^2 |
| q_r^o | externally incident radiation flux, W/m^2 ; $\tilde{q}_r^o = q_r^o/\sigma T_g^4$ |
| T | absolute temperature, K |
| t | dimensionless temperature, T/T_g |
| T_g | gas temperature at coating surface, K |
| x | coordinate in layer (Fig. 1), m ; $X = x/D$ |
| ϵ_D | emissivity of substrate measured in air or vacuum |
| κ_D | optical thickness, KD |
| ρ | diffuse reflectivity of interface |
| σ | Stefan-Boltzmann constant, W/m^2K^4 |
| σ_s | scattering coefficient in layer, m^{-1} |
| Ω | scattering albedo in layer, $\sigma_s/(a+\sigma_s) = \sigma_s/K$ |

Subscripts

tot total heat flux by combined conduction and radiation

Superscripts

- i inside of layer
- o outside of layer

Introduction

The development of ceramic coatings to protect materials for high temperature use is critical for advanced aircraft engines where high thermal efficiency is required. Some coatings partially transmit radiant energy in certain wavelength regions. In high temperature surroundings such as in a combustion chamber, infrared and visible radiation transmitted within the coating provides internal heating that affects temperatures of the coating and its substrate.

At elevated temperatures radiant emission within a material can be large. For a material with a high refractive index this is especially true since internal emission depends on the refractive index squared. Since radiation leaving through an interface cannot exceed that from a blackbody, internal reflections occur when radiation passes into a material with a lower refractive index. In addition to internal emission, absorption and reflection, radiant scattering and heat conduction take part in the energy transfer. The interaction of internal radiation and conduction must be understood for semitransparent components and protective coatings subjected to radiative and convective environments. Internal scattering must be examined as it can influence the temperature distribution for some conditions.

An important area involving radiation within hot materials with refractive indices larger than one and with significant

internal emission, is the heat treating and cooling of glass plates.¹ The related literature has been briefly reviewed in previous papers.^{2,3} In these papers, temperature distributions and heat flows in partially transmitting materials with high refractive indices are predicted using the radiative transfer equations coupled with heat conduction. The resulting integral equations, including the scattering source function for some of the work, are solved numerically. Each exterior boundary is heated by radiation and convection, and diffuse interface reflections are included. For comparisons during the development of approximate solutions in Ref. 4, the numerical solutions were extended to a three-layer composite with three spectral bands in each layer and with isotropic scattering. This simulates a ceramic layer with a reinforcing layer, or with coatings of other ceramic materials for protection from corrosive atmospheres such as combustion gases.

Since the formulation and solution of the exact radiative transfer equations including scattering is rather complex, it is desirable to develop more convenient approximate methods such as the two-flux method if these can provide accurate results. The two-flux equations are given in Refs. 5 and 6. The two-flux method was shown to give accurate results for a gray layer with a refractive index of one between boundaries with specified temperatures.^{7,8} Two-flux and diffusion solutions, and combinations of the two for layers with optically thin and thick spectral bands, were derived in Ref. 4 for materials with refractive indices larger than one. This included heating

conditions where the boundary temperatures are not specified and must be determined during the solution. The two-flux formulation yielded very accurate results by comparisons with exact solutions. In Ref. 9 two-flux solutions were obtained with two spectral bands for a packed bed with two layers of particles.

In the present work it is shown that for some types of heating and cooling conditions the two-flux solution can be reduced to a single integration to generate the material temperature distribution. Results are given that illustrate the behavior of a semitransparent protective coating on an opaque substrate when the substrate is not internally cooled. This models a protected component that is either heated on all sides by hot gas while it is cooled by radiating to cold surroundings, or is being film cooled on all sides while being subjected to radiation from hot surroundings.

Analysis

Energy and two-flux equations

Figure 1 shows the geometries and conditions for which the present analysis was developed. In Fig. 1a a semitransparent layer such as a ceramic component is in a high temperature environment that provides symmetric radiative and convective conditions at both boundaries. In Fig. 1b there is a semitransparent protective coating of thickness D on each side of a high temperature opaque component. Both sides of the component have the same coating and are subjected to the same external conditions; from this symmetry there is no net heat flow through the entire coated composite. The results also apply for a coating

on one side of a substrate where the uncoated side is insulated or has negligible heat loss. The layer in Fig. 1a and the coatings in Fig. 1b are semitransparent and are gray emitting, absorbing, scattering, and heat conducting materials with refractive indices larger than one. The substrate in Fig. 1b is opaque. The convective and radiative conditions are uniform on the coating boundaries. The temperature distribution is to be found in the semitransparent layer of Fig. 1a and in the coating material in Fig. 1b, and how the distribution depends on the parameters will be illustrated.

From symmetry the energy equation combining the energy fluxes by conduction and radiation within the layers in Fig. 1 is

$$-k \left. \frac{dT}{dx} \right|_x + q_r(x) = q_{tot} = 0 \quad (1)$$

Thermal properties are assumed independent of temperature. Since the substrate in Fig. 1b is opaque it does not have an internal radiative energy source. Then from Eq. (1) with $q_r(x) = 0$, $dT/dx = 0$ within the substrate, and with symmetric boundary conditions or an insulated back side, the substrate is at a uniform temperature equal to $T(D)$.

The solution method developed here to determine the heat transfer behavior of the semitransparent material is based on the two-flux method^{4,5} for the radiative fluxes. To validate this approximate method some results are compared with numerical solutions of the exact radiative transfer equations using the computer program from Ref. 3. In the two-flux method the radiative fluxes $q_r^+(x)$ and $q_r^-(x)$ are in the positive and negative

directions (Fig. 1), and each flux is assumed isotropic. The radiative flux $q_r(x)$ in the x direction and a quantity $G(x)$ are related to $q_r^+(x)$ and $q_r^-(x)$ by,⁵

$$q_r(x) = q_r^+(x) - q_r^-(x); \quad G(x) = 2[q_r^+(x) + q_r^-(x)] \quad (2a,b)$$

Solving Eqs. (2a) and (2b) for $q_r^+(x)$ and $q_r^-(x)$ gives the useful relations,

$$q_r^+(x) = \frac{1}{2} \left[\frac{G(x)}{2} + q_r(x) \right]; \quad q_r^-(x) = \frac{1}{2} \left[\frac{G(x)}{2} - q_r(x) \right] \quad (3a,b)$$

The two-flux equations including isotropic scattering are,⁵

$$\frac{dq_r(x)}{dx} = K(1 - \Omega) [4n^2 \sigma T^4(x) - G(x)] \quad (4)$$

$$\frac{dG(x)}{dx} = -3Kq_r(x) \quad (5)$$

Integration of energy and two-flux equations

The $q_r(x)$ in Eq. (5) is substituted into Eq. (1) which is then integrated to give,

$$kT(x) + \frac{1}{3K} G(x) = C \quad (6)$$

where C is a constant. The two-flux relation Eq. (4) is now used. The energy equation Eq. (1) is differentiated and used to eliminate $dq_r(x)/dx$ on the left side of Eq. (4). The $G(x)$ on the right side of Eq. (4) is eliminated by using Eq. (6). This yields the following equation in terms of $T(x)$,

$$k \frac{d^2 T}{dx^2} = K(1 - \Omega) [4n^2 \sigma T^4(x) + 3KkT(x) - 3KC] \quad (7)$$

The order of Eq. (7) is reduced by multiplying the entire

equation by dT/dx and integrating each term. The constant of integration is eliminated by subtracting the integrated equation evaluated at $x = 0$ to give,

$$\frac{k}{2} \left[\left(\frac{dT}{dx} \right)_x^2 - \left(\frac{dT}{dx} \right)_{x=0}^2 \right] = K(1-\Omega) \left\{ \frac{4}{5} n^2 \sigma [T^5(x) - T^5(0)] + \frac{3}{2} K k [T^2(x) - T^2(0)] - 3 K C [T(x) - T(0)] \right\} \quad (8)$$

Boundary conditions

At the boundary $x = 0$ external convection is equal to conduction within the layer; radiation does not enter this balance since radiation is a volume process and hence, for a semitransparent material, there is no absorption or emission at the surface itself. Then

$$-k \frac{dT}{dx} \Big|_{x=0} = h[T_g - T(0)] \quad (9)$$

To derive the boundary condition at $x = D$ in Fig. 1b, the radiative flux leaving the opaque gray substrate at $x = D$ is expressed in terms of emitted and reflected radiation,

$$q_r^-(D) = \epsilon_D n^2 \sigma T^4(D) + (1 - \epsilon_D) q_r^+(D) \quad (10)$$

where n is the refractive index of the coating and ϵ_D is the substrate emissivity as measured in air or vacuum. Some substitutions into Eq. (10) are now made. The $q_r^+(D)$ and $q_r^-(D)$ are eliminated by using Eqs. (3a,b). The $G(D)$ and $q_r(D)$ that this introduces are eliminated by using Eqs. (6) and (1). The result is a relation for $dT/dx|_{x=D}$ in terms of $T(D)$ and the constant C ,

$$k \frac{dT}{dx} \Big|_{x=D} = \frac{2\epsilon_D}{2-\epsilon_D} \left[\frac{3}{4} C K - \frac{3}{4} k K T(D) - n^2 \sigma T^4(D) \right] \quad (11)$$

For the symmetric single layer in Fig. 1a, $dT/dx|_{x=D} = 0$ which is also equivalent to the limit in Eq. (11) when $\epsilon_D \rightarrow 0$.

To obtain an expression for the constant C, Eq. (6) is used at $x = 0$ to give $C = kT(0) + (1/3K)G(0)$. The $G(0)$ depends on the externally incident radiation and on the flux internally reflected at the boundary; an expression for $G(0)$ is derived in Ref. 4 as,

$$G(0) = 4 \frac{1-\rho^o}{1-\rho^i} q_r^o - 2 \frac{1+\rho^i}{1-\rho^i} q_r(0) \quad (12)$$

Using Eq. (12) the C is given by,

$$C = kT(0) + \frac{1}{3K} \left[4 \frac{1-\rho^o}{1-\rho^i} q_r^o - 2 \frac{1+\rho^i}{1-\rho^i} q_r(0) \right] \quad (13)$$

The $q_r(0)$ is eliminated from Eq. (13) by using Eq. (1) at $x = 0$ and replacing the conduction term with the convection term in Eq. (9); this yields,

$$C = kT(0) + \frac{1}{3K} \left\{ 4 \frac{1-\rho^o}{1-\rho^i} q_r^o + 2 \frac{1+\rho^i}{1-\rho^i} h [T_g - T(0)] \right\} \quad (14)$$

Form of differential equation for integration

The boundary condition, Eq. (9), is substituted into the right side of Eq. (8) and the result is solved for dT/dx to yield,

$$\frac{dT}{dx} = \pm \left[\frac{2K}{k} (1-\Omega) \left\{ \frac{4}{5} n^2 \sigma [T^5(x) - T^5(0)] + \frac{3}{2} K k [T^2(x) - T^2(0)] - 3KC [T(x) - T(0)] \right\} + \left\{ \frac{h}{k} [T_g - T(0)] \right\}^2 \right]^{\frac{1}{2}} \quad (15)$$

The sign in front of the square root is selected according to the initial slope of the temperature profile expected from the heating and cooling imposed by external radiation and convection.

Before discussing the numerical procedure, the differential equation, constant C , and boundary condition at $x = D$ for Figs. 1a and 1b are placed in dimensionless form:

$$\frac{dt}{dX} = \pm \left[\frac{2\kappa_D}{N} (1-\Omega) \left\{ \frac{4}{5} n^2 [t^5(X) - t^5(0)] + \frac{3}{2} \kappa_D N [t^2(X) - t^2(0)] - 3\kappa_D \tilde{C} [t(X) - t(0)] \right\} + \left\{ \frac{H}{N} [1-t(0)] \right\}^2 \right]^{\frac{1}{2}} \quad (16a)$$

$$\tilde{C} = N t(0) + \frac{1}{3\kappa_D} \left\{ 4 \frac{1-\rho^0}{1-\rho^1} q_r^0 + 2 \frac{1+\rho^1}{1-\rho^1} H[1-t(0)] \right\} \quad (16b)$$

$$\left. \frac{dt}{dX} \right|_{x=1} = 0 \quad ; \quad \left. \frac{dt}{dX} \right|_{x=1} = \frac{1}{N} \frac{2\epsilon_D}{2-\epsilon_D} \left[\frac{3}{4} \tilde{C} \kappa_D - \frac{3}{4} N \kappa_D t(1) - n^2 t^4(1) \right] \quad (17a,b)$$

Numerical solution

To obtain the temperature distribution, the \tilde{C} in Eq. (16b) is substituted into Eq. (16a); dt/dX is then a function of $t(0)$ and the specified parameters. A value of $t(0)$ is guessed and forward integration is performed from $X = 0$ to 1 using a fourth-order Runge-Kutta method. At the end of the integration the boundary condition Eq. (17a) or (17b) is checked to see if it is satisfied; if not, $t(0)$ is adjusted until it is satisfied. For some values of the parameters the integration is very sensitive to $t(0)$; double precision was used for these cases. If the optical thickness is large there is radiative penetration for only small X and the temperature distribution is uniform except in a small region near $X = 0$. Then the forward integration is performed only over the X distance where the temperature is changing significantly, and the particular $t(0)$ is found for which

$dt/dx = 0$ as the integration proceeds. For these conditions the temperature distribution is independent of the condition in Eq. (17b), and $dt/dx|_{x=1} = 0$ which also corresponds to the symmetric single layer in Fig. 1a.

Within the substrate, Fig. 1b, the temperature is uniform and equal to the temperature $t(1)$ at the coating-substrate interface. This follows from the energy equation in the absence of heat sources within the opaque substrate, and the temperatures being equal at both sides of the substrate for the symmetric conditions considered here.

Special Case for a Coating at Uniform Temperature

In some applications a protective coating has a high thermal conductivity or is thin enough that its temperature does not vary much throughout its thickness. The following analysis is for the special case of a layer at uniform temperature.

Equation (4) is differentiated with T constant and is substituted into the left side of Eq. (5) to eliminate $dG(x)/dx$,

$$\frac{d^2 q_r(x)}{dx^2} = 3K^2(1-\Omega)q_r(x) \quad (18)$$

The general solution of Eq. (18) is

$$q_r(x) = C_1 e^{\alpha x} + C_2 e^{-\alpha x} \quad ; \quad \alpha = K[3(1-\Omega)]^{1/2} \quad (19)$$

Substituting $q_r(x)$ into Eq. (4) gives for $G(x)$,

$$G(x) = 4n^2\sigma T^4 - \left(\frac{3}{1-\Omega} \right)^{1/2} (C_1 e^{\alpha x} - C_2 e^{-\alpha x}) \quad (20)$$

An overall energy balance states that incident radiation and convection to the coating must equal reflected radiation and the

radiation leaving the coating, $q_r^\circ + h[T_g - T(0)] = q_r^\circ \rho^\circ + q_r^-(0)(1 - \rho^i)$. By applying the radiative conditions across $x = 0$, $q_r(0) = q_r^+(0) - q_r^-(0) = q_r^\circ(1 - \rho^\circ) + \rho^i q_r^-(0) - q_r^-(0)$, the energy balance reduces to

$$T(0) = \frac{q_r(0)}{h} + T_g \quad (21)$$

where $q_r(0)$ is inside the coating. Thus the uniform coating temperature $T = T(0)$ can be obtained by finding $q_r(0)$ from Eq. (19) by determining the integration constants C_1 and C_2 . These are evaluated by applying the boundary conditions to Eqs. (19) and (20). At $x = 0$ and D Eqs. (12) and (10) apply. Using Eqs. (3a,b), Eq. (10) becomes

$$G(D) = 2 \frac{2 - \epsilon_D}{\epsilon_D} q_r(D) + 4n^2 \sigma T^4 \quad (22)$$

Equations (19) and (20) evaluated at $x = 0$ and D are substituted into the boundary conditions (12) and (22). This yields two simultaneous equations for C_1 and C_2 that are solved algebraically. From Eq. (19) $q_r(0) = C_1 + C_2$, and this is inserted into Eq. (21); the result is placed into dimensionless form to obtain the following equation for t that can be evaluated with a root solver,

$$t = 1 + \frac{1}{H} \left\{ \frac{[(\beta_1 - \beta_3) e^{-2\beta_2 \kappa_D} + \beta_1 + \beta_3] 4(\beta_5 q_r^\circ - n^2 t^4)}{(-\beta_1 + 2\beta_4)(\beta_1 - \beta_3) e^{-2\beta_2 \kappa_D} + (\beta_1 + 2\beta_4)(\beta_1 + \beta_3)} \right\} \quad (23)$$

where

$$\beta_1 = \left(\frac{3}{1 - \Omega} \right)^{\frac{1}{2}} ; \quad \beta_2 = [3(1 - \Omega)]^{1/2} ; \quad \beta_3 = \frac{4 - 2\epsilon_D}{\epsilon_D} ; \quad \beta_4 = \frac{1 + \rho^i}{1 - \rho^i} ; \quad \beta_5 = \frac{1 - \rho^\circ}{1 - \rho^i}$$

Limit for a Transparent Coating on an Opaque Substrate

For $\kappa_D \rightarrow 0$ radiation is absorbed and emitted only by the substrate, but this absorption and emission is affected by multiple reflections within the coating. The net absorption by the substrate at $X = 1$ is conducted out through the coating and is equal to the convection from the coating surface. The temperature distribution is linear, and $t(1)$ and $t(0)$ are given by,

$$\epsilon_D \left[\frac{\bar{q}_r^o (1 - \rho^o) + \epsilon_D \rho^i n^2 t^4(1)}{1 - \rho^i (1 - \epsilon_D)} - n^2 t^4(1) \right] = H [t(1) - 1] \left[\frac{H}{N} + 1 \right]^{-1} \quad (24a)$$

$$t(0) = \left[t(1) + \frac{H}{N} \right] \left[\frac{H}{N} + 1 \right]^{-1} \quad (24b)$$

Apparent Surface Temperature from Emitted Radiation

For temperature measurements using pyrometry it is of interest to examine the coating surface temperature calculated from the radiation leaving the coating as compared with the $t(0)$ obtained from the solution of Eq. (16a). For a gray material the transmittance or absorptance of the coating surface is $1 - \rho^o$. From an overall energy balance the energy leaving the coating must equal the incident radiation transmitted across the surface and the energy transferred to the surface by convection. Then the apparent temperature T_{app} required to emit this energy, relative to the actual surface temperature $T(0)$, is given by,

$$\frac{T_{app}}{T(0)} = \frac{1}{t(0)} \frac{1}{(1 - \rho^o)^{1/4}} \left\{ \bar{q}_{r1}^o (1 - \rho^o) + H [1 - t(0)] \right\}^{1/4} \quad (25)$$

Results and Discussion

To illustrate the characteristics of the solution, temperature distributions are given for a symmetric semitransparent layer as in Fig. 1a, and for a protective coating on an insulated or symmetric substrate as in Fig. 1b. Results are given for two situations: 1) radiative heating with convective cooling to simulate a film cooled material in hot surroundings, and 2) convective heating with radiative cooling to simulate a material in hot gas with cooled surroundings. The results illustrate how the temperature of a substrate, which is to be thermally protected or protected from a corrosive atmosphere, is influenced by the physical parameters of its coating.

Symmetric Heating Conditions on a Semitransparent Layer

Results are given first to validate the two-flux method by comparison with numerical solutions of the radiative transfer equations. By using the computer program from Ref. 3 with symmetric boundary conditions, temperature distributions are obtained for the layer in Fig. 1a, or for the coating in Fig. 1b in the limit of a perfectly reflecting substrate or when $t(X)$ is independent of ϵ_D as will be discussed. For these conditions $dt/dX|_{X=1} = 0$. In Fig. 2 the distributions are for the semitransparent material exposed to a hot environment that provides an incident radiative flux $q_r^0 = \sigma(1.5 T_g)^4$. The surface is being cooled rather strongly by convection with a convection-radiation parameter $H = 10$. Radiation penetrates into the layer and the net absorbed energy is conducted to the surface at $X = 0$ and removed by convection.

The results from the numerical solution of the exact transfer equations show that the two-flux analysis provides excellent agreement even in regions with large temperature gradients. The two-flux solution will then be used to illustrate some aspects of the thermal performance of a coating on an opaque substrate for the boundary conditions considered here. The results in Fig. 2 are valid for Fig. 1b when $t(X)$ does not depend on the substrate emissivity. Figure 3 will show the effect of ϵ_D , which is found to be important only for a coating with a small optical thickness.

In Fig. 2 the largest effect of radiant absorption in the coating is for $\kappa_D \approx 0.1$ where the substrate temperature, $t(1)$, is only a little below $t = 1.5$ which is the effective blackbody temperature of the surroundings. For larger or smaller κ_D the substrate temperature decreases. The limit is shown when the coating is opaque so that radiative exchange is only at $X = 0$. For very small κ_D the low substrate temperature for $\kappa_D = 0.001$ is not realistic for actual coating behavior as it requires the substrate to be perfectly reflecting, $\epsilon_D \rightarrow 0$; even a small ϵ_D provides absorption at the substrate surface and can raise its temperature considerably as shown in Fig. 3. For a large κ_D and with large convective cooling there are large temperature changes near $X = 0$ that can produce thermal stresses; the temperature gradients are reduced if N is larger (see Fig. 6).

Effect of Substrate Emissivity

The previous results apply for a symmetric single layer and for a coating with the limiting condition $\epsilon_D \rightarrow 0$. It is now shown (for the present parameters) that these results also apply for

coatings with $\kappa_D > 0.03$ because for these κ_D the temperature distributions are independent of ϵ_D .

Figure 3a shows the effect of having a coating on a substrate that is either black or has $\epsilon_D = 0.005$; the results are compared with $t(X)$ from Fig. 2 for $\epsilon_D = 0$. For $\epsilon_D = 1$ and when κ_D is very small, radiation penetrates through the coating and is absorbed by the substrate. The substrate is almost at the temperature of the radiating surroundings, $t = 1.5$, and the internal temperature distribution is linear as provided by heat conduction. There is a substantial effect of ϵ_D which affects energy absorption at the substrate surface, and the results are sensitive to small ϵ_D as shown by $t(X)$ for $\epsilon_D = 0.005$. The effect of ϵ_D is substantially decreased when $\kappa_D = 0.01$, and a further increase to $\kappa_D = 0.03$ eliminates the effect of ϵ_D . Hence, for the parameters considered here, when $\kappa_D > 0.03$ there is a negligible effect of ϵ_D and the $t(X)$ from Fig. 2 apply for all ϵ_D . The sensitivity to ϵ_D for small κ_D is reduced when heat conduction is increased (larger N). A larger N also makes $t(0)$ change more with ϵ_D . These characteristics can be quantified by examining the behavior of the limiting solution for $\kappa_D \rightarrow 0$ in Eq. (24).

Figure 3b has results for a coating on a substrate when the effective blackbody temperature of the radiating surroundings is small so that $q_r^\circ = (0.25T_g)^4$; there is then radiative cooling acting along with convective heating. For small absorption in the coating (small κ_D) and with $\epsilon_D \rightarrow 0$ the temperature in the coating is close to T_g ($t \approx 1.0$) since there are very little radiative heat losses from either the coating or substrate surface for these

conditions. If, however $\epsilon_D = 1.0$, the temperature at $X = 1$ is substantially reduced for small κ_D by energy radiated from the substrate surface to the surroundings, and heat conduction provides a linear $t(X)$. Since the convective heating is at the coating surface the highest temperature for each κ_D is at $X = 0$. As κ_D increases, the coating radiates away energy that has been conducted into its interior. For $\epsilon_D = 0$ the lowest temperature at the substrate interface $X = 1$ is for $\kappa_D \approx 0.1$. For $\epsilon_D = 1$ the substrate temperature is further reduced by radiation loss from the substrate boundary. For $\kappa_D > 0.1$ the effect of ϵ_D is not significant, and for larger κ_D the $t(X)$ increases toward the opaque limit. A small ϵ_D has a significant effect if $\kappa_D < 0.1$, but this sensitivity to ϵ_D decreases if N is increased. The results demonstrate that for certain ranges of κ_D and ϵ_D , radiation from the coating and substrate can be effective in reducing the substrate temperature.

Refractive Index Effects for Radiative Heating with Convective Cooling

The previous results are for a refractive index $n = 1.5$. Results for $n = 1, 1.5$, and 2 in Fig. 4 illustrate how $t(X)$ is influenced by n for κ_D up to the opaque limit. For $\kappa_D = 0.001$ and 0.1 , $t(X)$ results are given for $\epsilon_D = 0$ and 1 ; for larger κ_D the $t(X)$ are independent of ϵ_D . For small κ_D the temperatures are increased for larger n , but for $\epsilon_D = 1$ the increase is small. For $\kappa_D \geq 1$ the trend is reversed and the maximum temperature is reduced with increasing n . This results from increased reflection at the external surface that prevents part of the incident

radiation from entering the layer. For small κ_D the effect of reflecting away part of q_r^e is over compensated by decreased internal emission, and by increased internal reflections providing a longer radiation path that augments internal radiative absorption and raises the internal temperatures. For $\kappa_D = 0.1$ the effect of ϵ_D is rather small for $n = 1.5$ as noted previously; the effect of ϵ_D is even smaller for $n = 2$. For $n = 1$ internal reflections do not occur and there is a more significant effect of ϵ_D .

Scattering Effects for Convective Heating with Radiative Cooling

The $t(X)$ in Fig. 5 illustrate the effect of scattering; for the previous figures $\Omega = 0$. The $t(X)$ are almost independent of ϵ_D for the κ_D values shown where all of the absorption optical thicknesses aD are ≥ 0.1 . For $aD = 0.1$ there is a small effect of ϵ_D . The results are for a coating with convective heating and radiative cooling. Two sets of distributions are given where aD is held constant at either 0.1 or 1 while scattering is added thereby increasing Ω . For the heating conditions considered, increased scattering reduced the ability of the layer to radiate away energy from the coating region near the substrate. This produced higher internal temperatures and an increased substrate temperature.

Conduction and Convection Effects

The influence of the conduction and convection parameters are illustrated in Figs. 6 and 7. The upper set of curves in Fig. 6 are for radiative heating with convective cooling. For a very small N the energy absorbed from incident radiation is not readily

The substrate and most of the coating are then near the surroundings temperature $t = 1.5$ with convective cooling effects only in the region close to $X = 0$. Increasing N to 10 makes $t(X)$ rather uniform in the coating thereby decreasing the substrate temperature except near $X = 0$. For large N there is an effect of ϵ_D but it is very small. Since the profiles all have dt/dX very close to zero at $X = 1$ they also apply for the single symmetric layer in Fig. 1a. The curves in the lower part of Fig. 6 show how radiative cooling can reduce the substrate temperature if N is small. For $N = 10$ the $t(X)$ is close to the uniform temperature limit from Eq. (23).

An increase in the external convection coefficient produces increased temperature gradients near the outer boundary; these can become large as in Fig. 7 for $N = 0.1$. For $\kappa_D = 1$ the temperatures are uniform over much of the coating interior, and the results are valid for any ϵ_D . For $\kappa_D = 0.003$ two sets of curves show the effect of ϵ_D ; for a coating on a substrate (Fig. 1b) there is a substantial difference between a black and perfectly reflecting substrate. The $t(X)$ for $\kappa_D = 0.003$ with $\epsilon_D = 0$, and for $\kappa_D = 1$, are valid for a symmetric single layer (Fig. 1a).

Apparent Surface Temperature

To measure the surface temperature of a coating with a radiation detector, it is of interest to compare the temperature calculated from the radiative flux leaving from within the coating with the actual surface temperature. The results for $H = 10$ in Fig. 8 are for large convection, and since conduction is small, there are large variations in $t(X)$ within the coating. Since the

interior temperatures are larger than $t(0)$ the T_{app} calculated from the exiting radiation is larger than $T(0)$. The optical thickness must be quite large for $T_{app}/T(0)$ to be near 1. For $H = 1$ the gradients of $t(X)$ are reduced and T_{app} is closer to $T(0)$; this would also be true for a larger N . The effects of H , N , and ϵ_D are readily examined using the present analytical relations. For radiative cooling with convective heating the $T_{app}/T(0) < 1$.

Conclusions

A convenient analytical expression has been derived to predict thermal behavior of a semitransparent coating on an opaque substrate when internal cooling is not provided. The substrate is either insulated on its side away from the coating, or the geometry and boundary conditions are symmetric, so there is no net heat flow through the coated material. The temperature distribution in the coating is a result of the local balance between internal radiation and conduction. When the coating is heated by hot gases in cooled surroundings, there can be a reduction in substrate temperature as a result of radiative cooling from within the coating. For the illustrative results given here a coating optical thickness larger than about $\kappa_D = 0.3$ gives a thermal behavior almost independent of the substrate emissivity. For these conditions the results also apply for a single semitransparent layer with symmetric boundary conditions. For very optically thin coatings the temperature distributions are sensitive to the substrate emittance for the illustrative parameters used here. Results for other parameters are easily examined by using the convenient analytical expressions that were developed.

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Figure Captions

Fig. 1 Geometry and coordinate system for two-flux analysis with symmetric thermal boundary conditions.

a) Single semitransparent layer

b) Symmetric semitransparent coatings on an opaque substrate.

Fig. 2 Effect of optical thickness on temperature distributions in a semitransparent layer symmetrically heated by radiation and cooled by convection. ($n = 1.5$, $t_g = 1$, $\tilde{q}_r^o = 1.5^4$, $H = 10$, $N = 0.1$, $\Omega = 0$).

Fig. 3 Substrate emissivity effects on temperatures in a coating on an opaque substrate with the other side of the substrate insulated. ($n = 1.5$, $t_g = 1$, $H = 10$, $N = 0.1$, $\Omega = 0$).

a) Radiative heating with convective cooling, $\tilde{q}_r^o = 1.5^4$

b) Convective heating with radiative cooling, $\tilde{q}_r^o = 0.25^4$.

Fig. 4 Effect of coating refractive index and substrate emissivity for radiative heating with convective cooling. ($t_g = 1$, $\tilde{q}_r^o = 1.5^4$, $H = 10$, $N = 0.1$, $\Omega = 0$).

Fig. 5 Scattering effects in a coating for convective heating with radiative cooling. ($n = 1.5$, $t_g = 1$, $\tilde{q}_r^o = 0.25^4$, $H = 10$, $N = 0.1$).

Fig. 6 Effect of heat conductivity in a coating for radiative heating and convective cooling, and for radiative cooling with convective heating. ($\kappa_D = 1$, $n = 1.5$, $t_g = 1$, $\tilde{q}_r^o = 1.5^4$, $H = 10$, $\Omega = 0$).

Fig. 7 External convective cooling effects on temperatures in a coating heated by radiation. ($\kappa_D = 1$ and 0.003 , $n = 1.5$, $t_g = 1$, $\tilde{q}_r^o = 1.5^4$, $N = 0.1$, $\Omega = 0$).

Fig. 8 Apparent surface temperature determined from radiation leaving the coating surface. ($t_g = 1$, $\tilde{q}_r^o = 1.5^4$, $H = 1$ and 10 , $N = 0.1$, $\Omega = 0$).

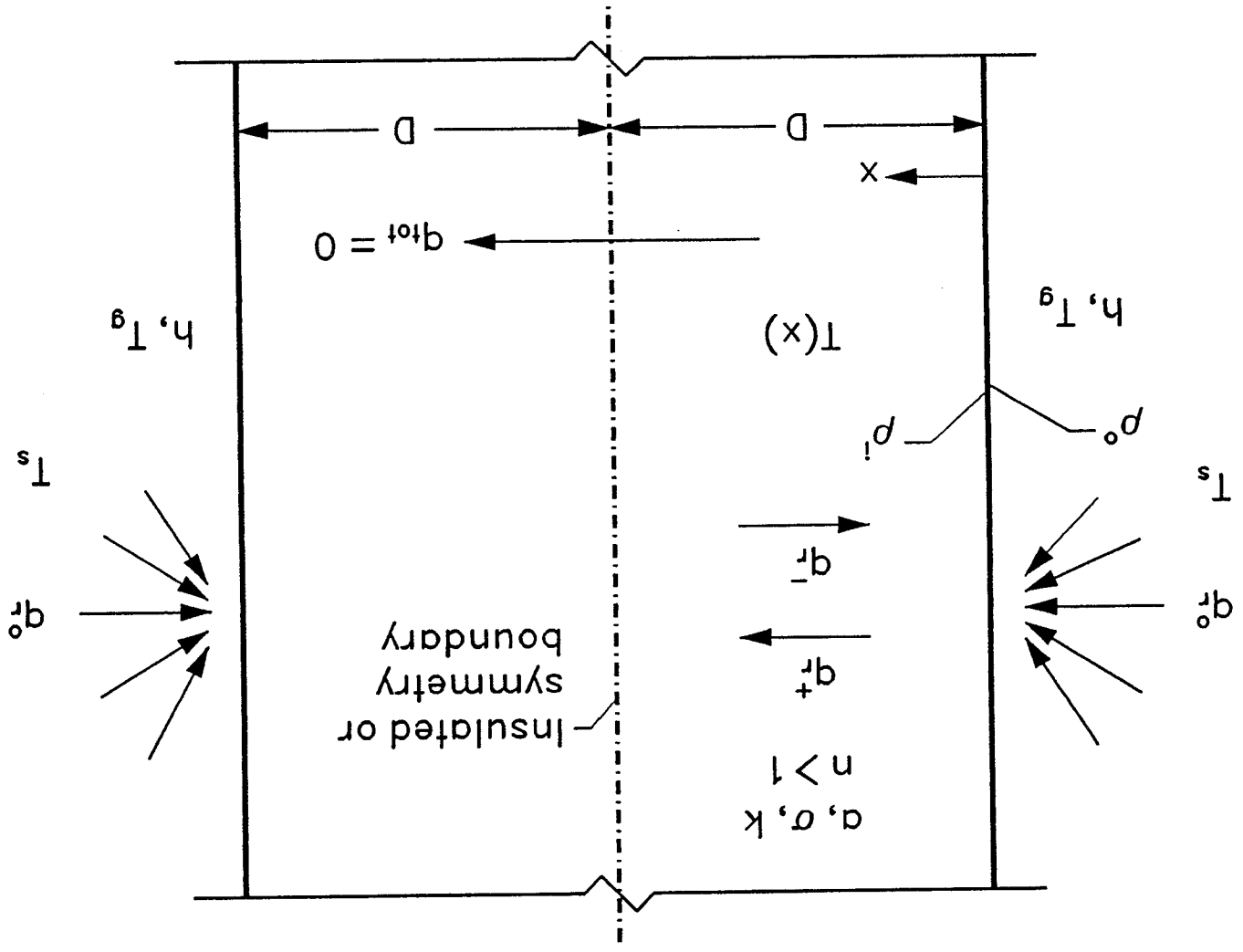


Fig. 1a

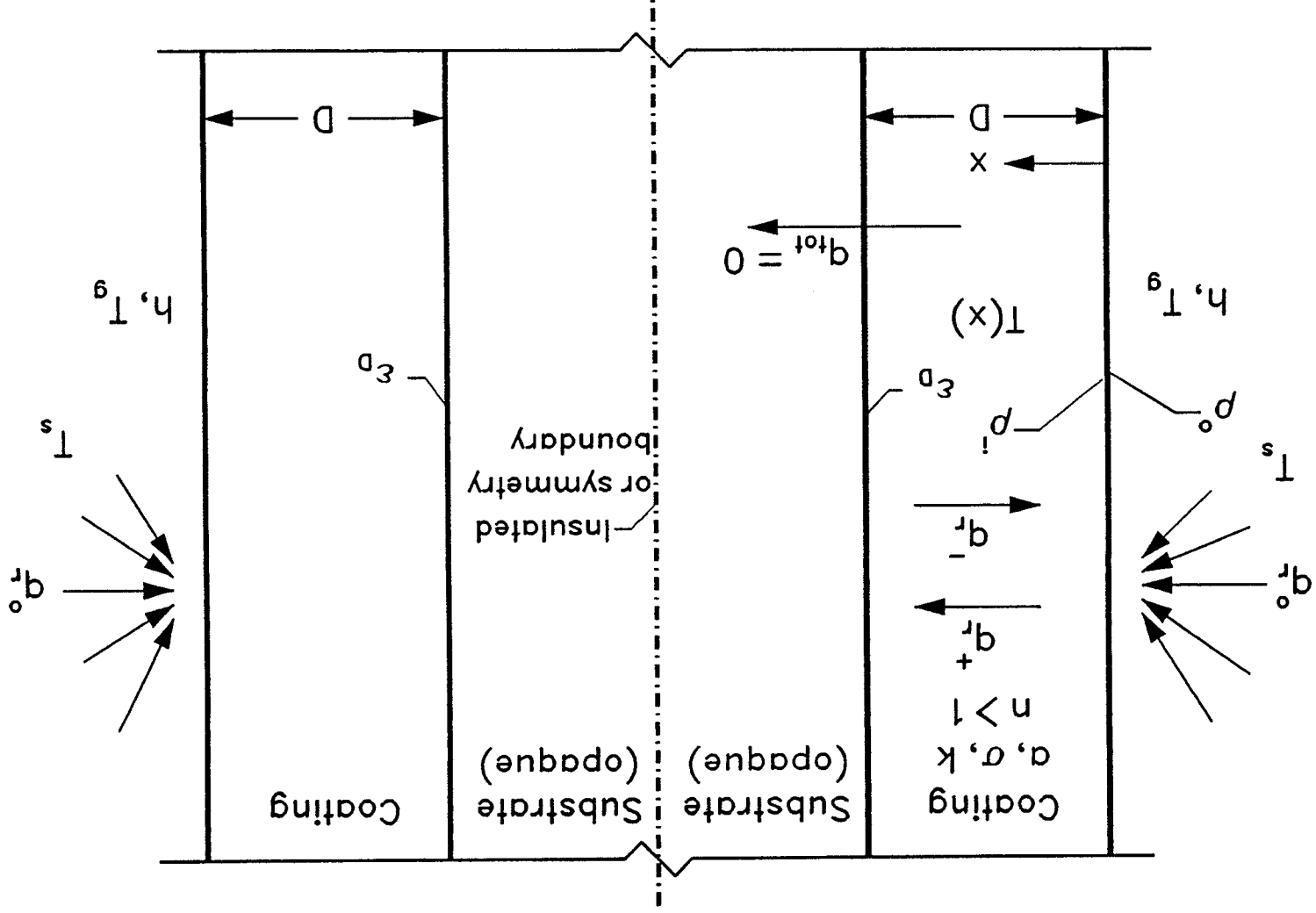


Fig. 1b

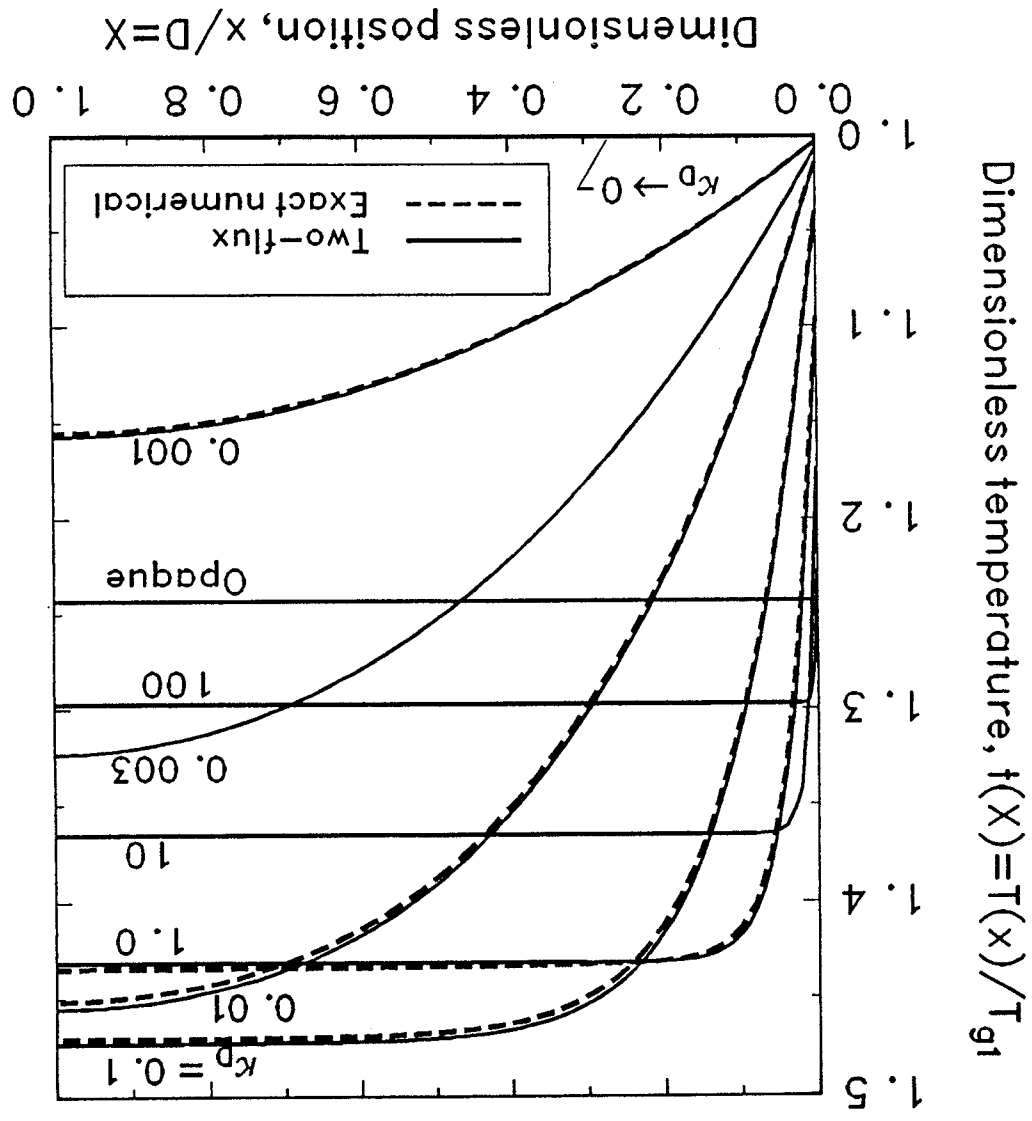


Fig. 2

Dimensionless temperature, $t(X)=T(x)/T_{g1}$

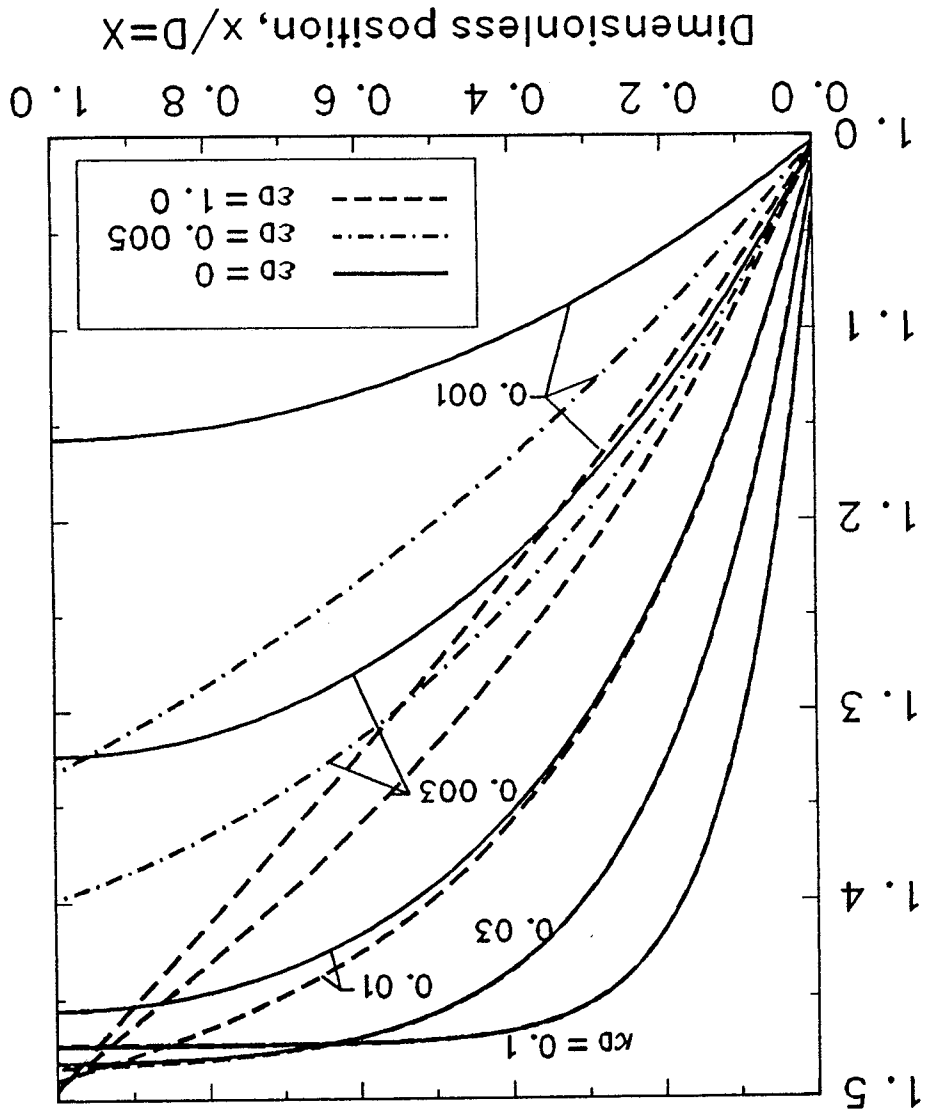


Fig. 3a

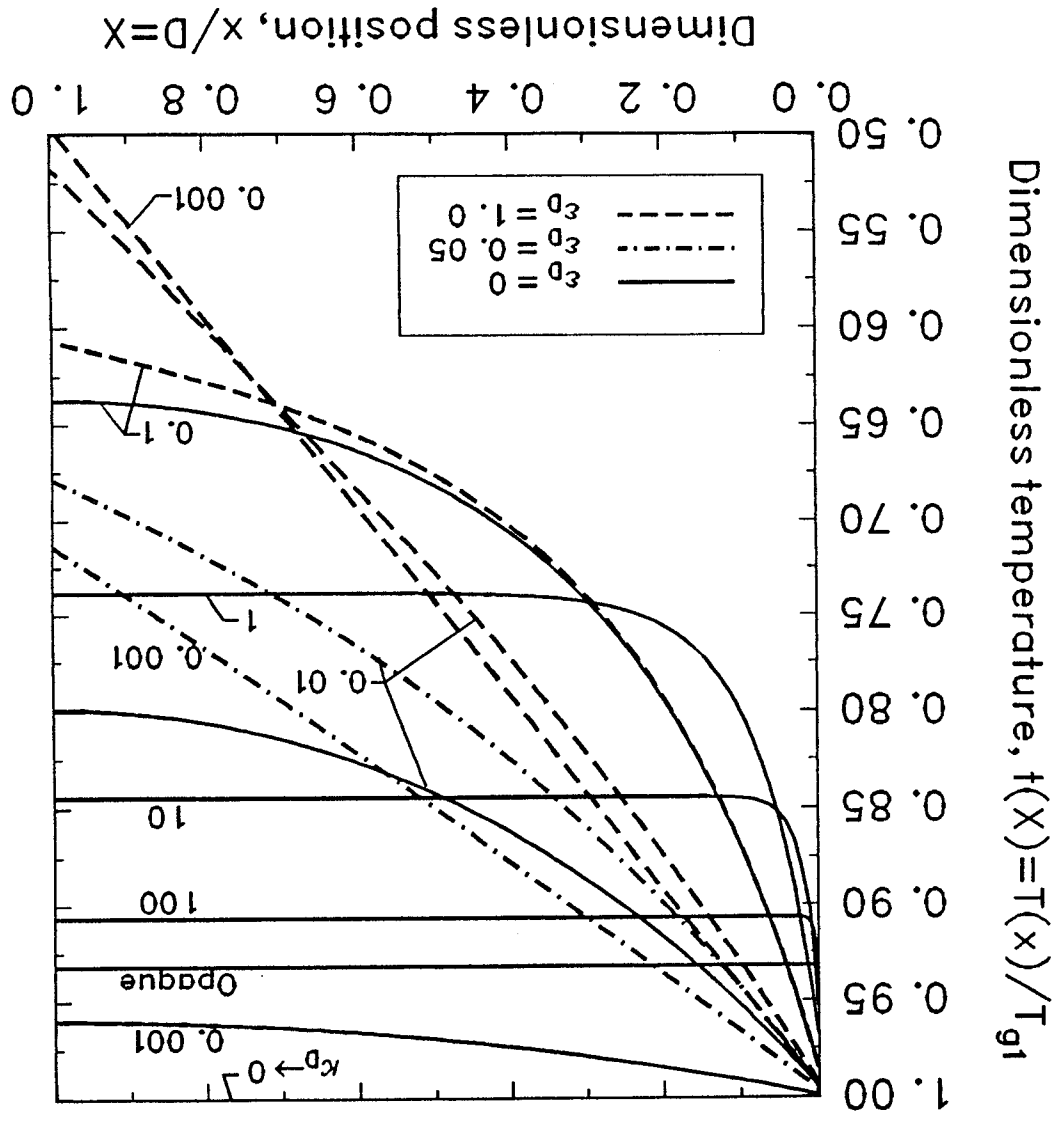


Fig. 3b

Dimensionless temperature, $t(X)=T(x)/T_{g1}$

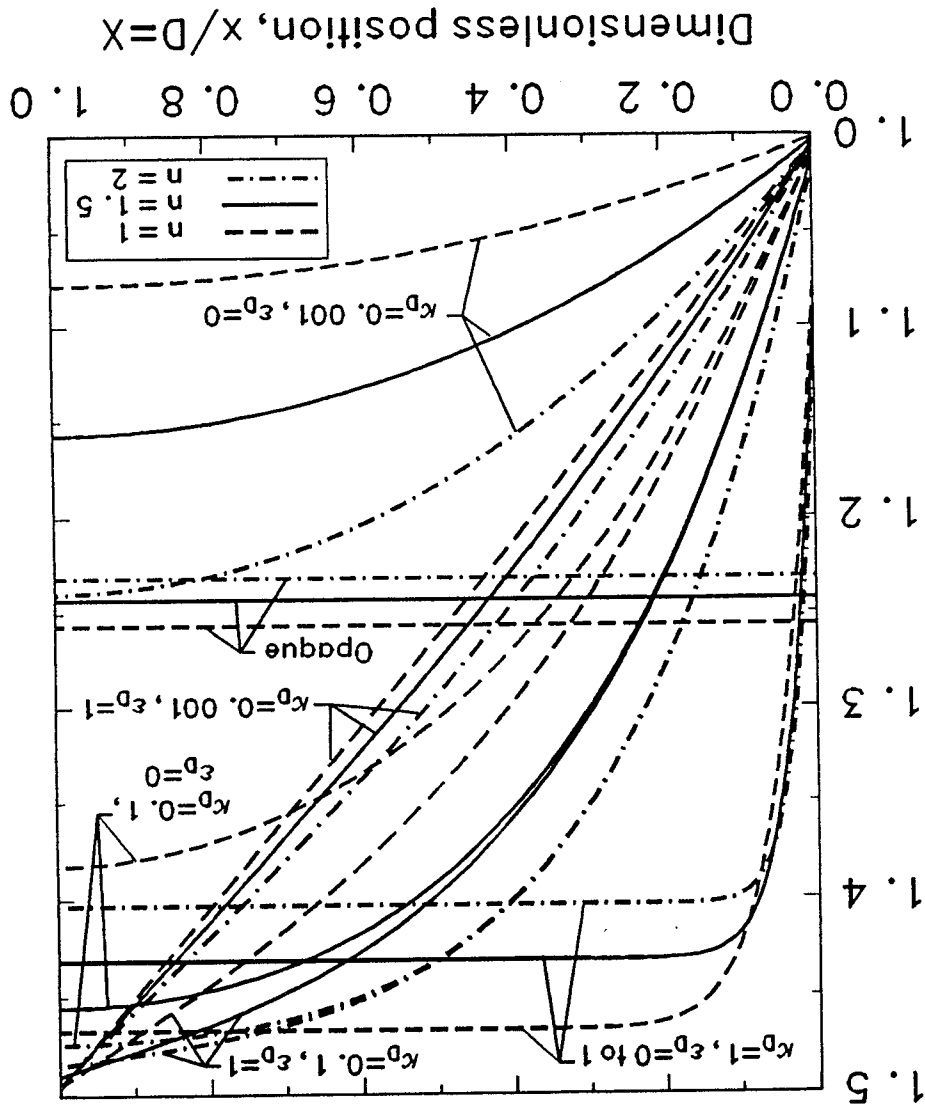


Fig. 4

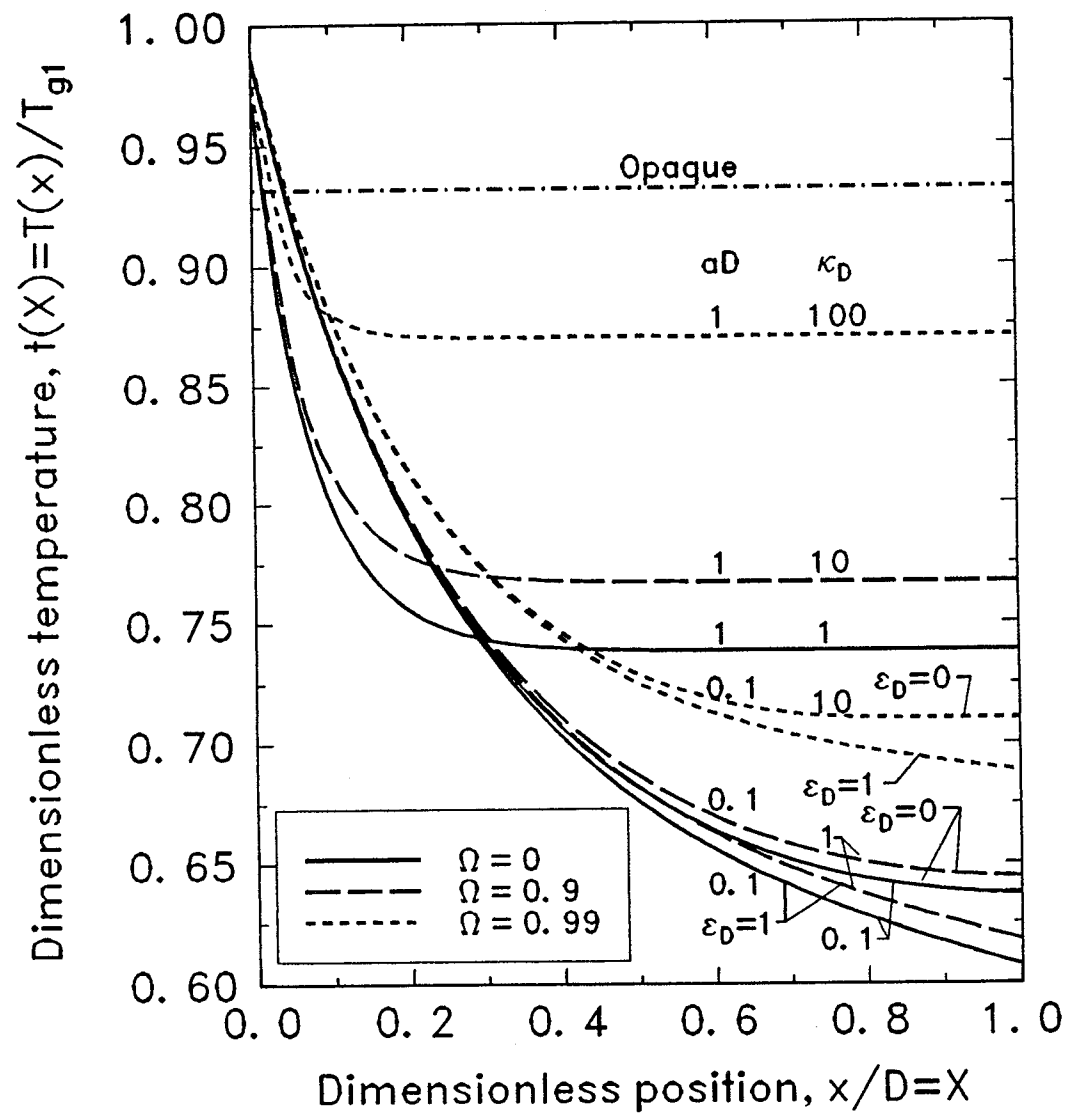


Fig. 5

Dimensionless temperature, $t(X)=T(x)/T_{g1}$

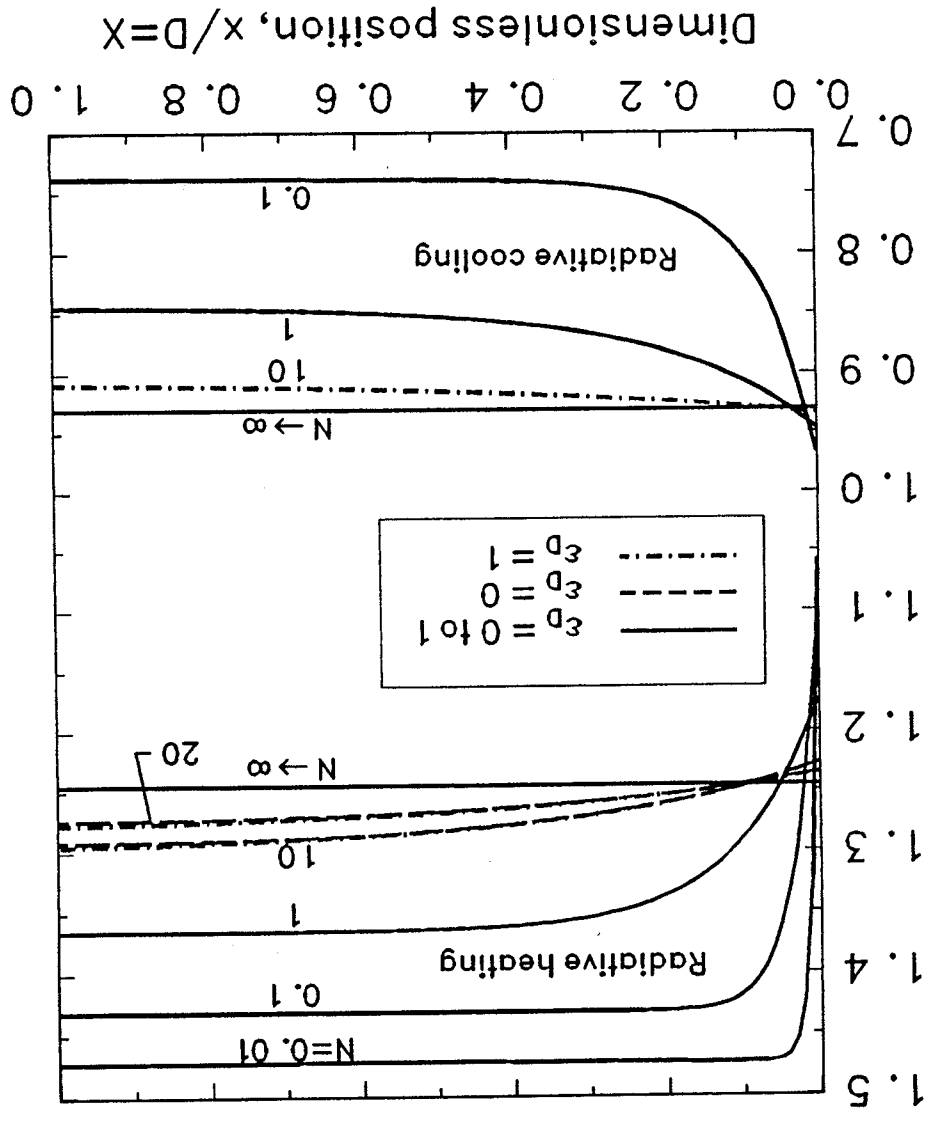


Fig. 6

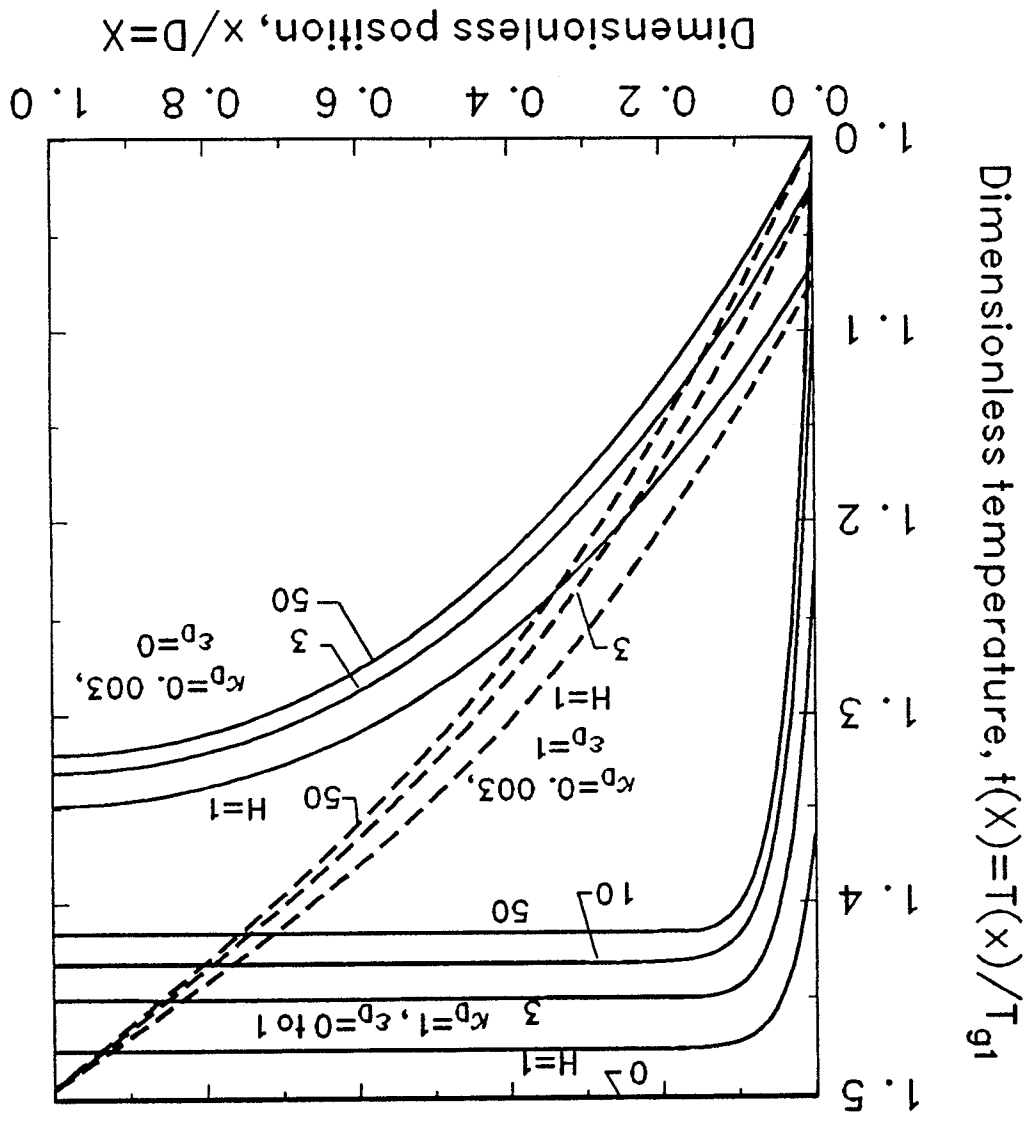


Fig. 7

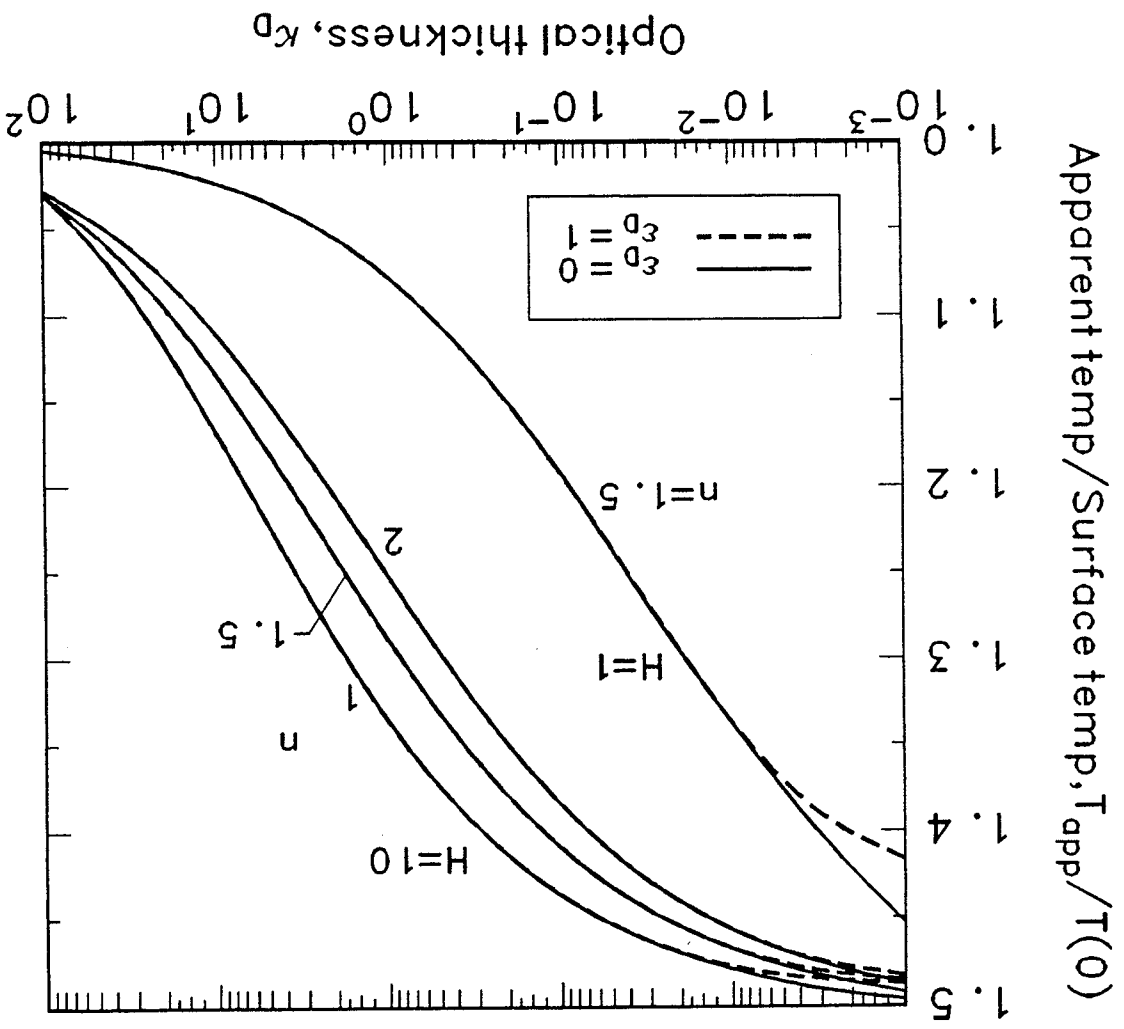


Fig. 8