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Reliability, Risk and Cost Trade-Offs for Composite Designs

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RELIABILITY, RISK AND COST TRADE-OFFS FOR COMPOSITE DESIGNS

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ABSTRACT

Risk and cost trade-offs have been simulated using a probabilistic method. The probabilistic method accounts for all naturally-occurring uncertainties including those in constituent material properties, fabrication variables, structure geometry and loading conditions. The probability density function of first buckling load for a set of uncertain variables is computed. The probabilistic sensitivity factors of uncertain variables to the first buckling load is calculated. The reliability-based cost for a composite fuselage panel is defined and minimized with respect to requisite design parameters. The optimization is achieved by solving a system of nonlinear algebraic equations whose coefficients are functions of probabilistic sensitivity factors. With optimum design parameters such as the mean and coefficient of variation (representing range of scatter) of uncertain variables, the most efficient and economical manufacturing procedure can be selected. In this paper, optimum values of the requisite design parameters for a predetermined cost due to failure occurrence are computationally determined. The results for the fuselage panel analysis show that the higher the cost due to failure occurrence, the smaller the optimum coefficient of variation of fiber modulus (design parameter) in longitudinal direction.

2. INTRODUCTION

Aerospace structures are complex assemblages of structural components that operate under severe and often uncertain service environments. They require durability, high reliability, light weight, high performance, at an affordable cost. Composite materials are potential candidates for meeting these requirements. Composite materials possess outstanding mechanical properties with excellent fatigue strength and corrosion resistance. Their mechanical properties are derived from a wide variety of variables such as constituent material properties and laminate characteristics (fiber and void volume ratios, ply orientation, and ply thickness). These parameters are known to be uncertain in nature.

In order to account for various uncertainties and to satisfy design requirements, knockdown (safety) factors are used extensively. These knockdown factors significantly reduce the design load of composite structures which result in substantial weight increase, but without a quantifiable measure of their reliability. This paper describes an alternate method which determines the structural reliability. This method is embedded in the computer code IPACS (Integrated Probabilistic Assessment of Composite Structures) [ref. 1] for a comprehensive probabilistic assessment of composite structures. The schematic of IPACS is shown in Figure (1).

Since cost is a major driver for a structural design, optimization techniques should be sought to achieve the balance between maximum reliability and minimum cost. In this paper, reliability-based cost optimization is conducted to assess the risk and cost trade-offs.

3. IPACS COMPUTER CODE FOR RELIABILITY CALCULATION

The IPACS Computer Code [ref. 1] has evolved from extensive research activities at NASA Lewis Research Center to integrate probabilistic structural analysis methods [ref. 2] and computational composite mechanics [ref. 3]. The composite micromechanics, macromechanics and laminate theory are embodied in ICAN [ref. 3]. IPACS consists of two stand-alone computer modules: PICAN and NESSUS. PICAN simulates probabilistic composite mechanics [ref. 4]. NESSUS uses the information from PICAN to simulate probabilistic structural responses [ref. 5]. Direct coupling of these two modules makes it possible to simulate uncertainties in all inherent scales of the composite - from constituent materials to the composite structure including its boundary and loading conditions as well as environmental effects. It is worth noting that special reliability algorithm FPI (Fast Probability Integrator) (ref. 6) is used instead of the conventional Monte Carlo simulation [ref. 7], to achieve substantial computational efficiencies which are acceptable for practical applications. Therefore, probabilistic composite structural analysis becomes feasible which can not be done traditionally, especially for composite structures which have a large number of uncertain variables. The results from

IPACS analysis include probability distribution function of structural response, reliability for a design criterion, and the probabilistic sensitivity factors of the primitive variables to the structural response and structural reliability.

4. PROBABILISTIC SENSITIVITY INFORMATION FOR OPTIMIZATION

The commonly used sensitivity in a deterministic analysis is the performance sensitivity, $\partial Z/\partial X_i$, which measures the change in the performance Z due to the change in a design parameter X_i. This concept is extended to probabilistic analysis to define the probabilistic sensitivity which measures the change in reliability relative to the change in each random variable. The failure probability for a given performance is defined in equation 1 (ref. 8).

$$P_f = \Phi(-\beta) \qquad 1$$

where β is the reliability index; Φ is the cumulative distribution function of a normally distributed random variable. Probabilistic sensitivity factor (SF_i) for i^{th} random variable is defined in equation 2.

$$SF_i = \frac{\partial \beta}{\partial X_i} = \frac{u_i^*}{\beta} \qquad 2$$

where u^* is the most probable failure point of a limit state function in a unit normal probability space. The sensitivity of design parameters to structural reliability is another useful information for controlling and adjusting design parameters from manufacturing to obtain the "best" benefit with minimum alteration. The sensitivity of the reliability to the mean of a normally distributed random variable X_i can be represented by equation 3 (ref. 8).

$$\frac{\partial \beta}{\partial m_i} = - \frac{SF_i}{\sigma_i} \qquad 3$$

where m_i and σ_i are the mean and standard deviation of random variable X_i respectively; Similarly, the sensitivity of the reliability to the standard deviation of a normally distributed random variable X_i can be computed from equation 4.

$$\frac{\partial \beta}{\partial \sigma_i} = - \frac{SF_i \; u_i^*}{\sigma_i} = - \frac{u_i^{*2}}{\beta \sigma_i} \qquad 4$$

With this information, optimization with reliability considerations can be achieved as will be demonstrated later.

5. MINIMIZATION OF RELIABILITY-BASED COST

A major design goal is to achieve the balance between maximum reliability and minimum cost. The criterion which addresses both reliable component and cost simultaneously is the cost function shown in equation 5.

$$C_T = C_I + P_f \; C_F \qquad 5$$

where C_T is total cost; C_I is manufacturing cost; P_f is failure probability; C_F is the cost incurred due to structural failure. Manufacturing cost can be represented by equation 6.

$$C_I = \sum_{j=1}^{N} C_j(p_j) + C_0 \qquad 6$$

where N is the number of distribution parameters; $C_j(p_j)$ is manufacturing cost for j^{th} distribution parameter; C_0 is other unrelated constant cost (assume zero in this paper). Total cost can be minimized when

$$\frac{\partial C_T}{\partial p_j} = 0 \qquad j = 1, N \qquad 7$$

Therefore, optimization can be achieved by solving a system of nonlinear algebraic equations as shown in equation 8.

$$\frac{\partial C_j(p_j)}{\partial p_j} + C_F \frac{\partial \Phi(-\beta)}{\partial \beta} \frac{\partial \beta}{\partial p_j} = 0 \qquad j = 1, N \tag{8}$$

For a normally distributed random variable, $\partial\beta / \partial p_j$ can be calculated by equations 3 and 4.

6. DEMONSTRATION FOR OPTIMIZATION OF COMPOSITE FUSELAGE PANEL STRUCTURES

Reliability-based cost minimization is demonstrated using a lower side panel of a composite fuselage structure shown in Figure (2). The fuselage panel is 60 in. x 90 in. with a radius of 122 in. There are four longitudinal stringers, 1.49 in. x 0.304 in., four J-frames, 5.10 in. x 0.140 in., and a 0.096 in. thick skin with 0.191 in. thick pads under the stringers and 0.124 in. thick pads under the frames. The width of the pad for stringer and frame are 3.57 in. and 2.85 in. respectively. The panel is made of graphite/epoxy composite. The constituent material properties, their assumed probabilistic distribution and coefficient of variations (representing range of the scatter) are summarized in Table 1. The corresponding fabrication variables used to make the composite panel are summarized in Table 2. Although only normally distributed random variables are considered in this example, the computer code IPACS can handle random variables with other distribution types such Lognormal or Weibull. The reliability of the structure is evaluated for design criterion that requires the buckling load be greater than 30% of the ultimate design load.

The probability density function of the buckling load is shown in Figure 3. Based on a reliability of 0.999, the ultimate design load will be 5000 lb/in. (1500 lb/in. divided by 0.30). The probabilistic sensitivity factors of the uncertain variables to 0.001 probability of the buckling load are listed in Figure 4. It is found that fiber volume ratio has the highest influence on the scatter of the buckling load, followed by the skin thickness, pad thickness under stringer and fiber modulus in longitudinal direction.

The probability of failure for different coefficients of variation of E_{f11} with the rest of distribution parameters unchanged is shown in Figure 5. It is noticed that probability of failure increases while coefficient of variation increases. It means that lesser quality product has higher probability of failure.

To minimize the total cost in equation 5, initial cost C_I is defined to be

$$C_I = A(p_1) * p_3 + B(p_2) * D * (1 - p_3) \qquad 9$$

where p_1 and p_2 are the coefficients of variation of fiber modulus (E_{f11}) and matrix modulus (E_m) respectively; p_3 represents the mean value of fiber volume ratio. $A(p_1)$ is the cost for fiber defined by equation 10.

$$A(p_1) = A_0 * (1.25 - p_1^2) \qquad 10$$

where A_0 is set to be \$35/lb to represent typical cost for graphite fiber. $B(p_1)$ in equation 9 is the cost for epoxy matrix defined by equation 11.

$$B(p_1) = B_0 * (1.25 - p_2^2) \qquad 11$$

where B_0 is set to be \$25/lb to represent typical cost for epoxy matrix. D in equation 9 is the ratio of the weight densities between epoxy matrix and graphite fiber. The cost function defined by equation 9 only accounts for the cost to manufacture fiber and matrix material. Other manufacturing cost is assumed to be constant and is set to be zero in this assessment. To simplify the optimization procedure and for demonstration purposes, only the coefficient of variation of fiber modulus in longitudinal direction is chosen to be the design parameter. Substituting equation 9 into equation 8, optimization can be achieved by solving the following equation.

$$-7000\ p_1\ p_3 + C_F \frac{\partial\Phi(-\beta)}{\partial\beta} \frac{\partial\beta}{\partial p_1} = 0 \qquad \mathbf{12}$$

Letting C_F be \$15000/lb, the manufacturing cost C_I and the weighted failure cost $P_f{*}C_F$ at different coefficient of variation (COV) for E_{f11} are plotted in Figure 6. It is noticed that C_I decreases while COV of E_{f11} increases because low quality fiber (wider scatter of E_{f11}) costs less to manufacture. On the other hand, the weighted failure cost increases since more weight for the failure cost is considered. Optimum p_1 (COV of E_{f11}) is calculated using equation 12. The minimized reliability-based total cost is calculated by equation 5. In Figure 7, total cost is computed at different COV of E_{f11}. It is seen that at optimum COV of E_{f11} (0.05), the manufacturing cost and weighted cost due to failure is balanced. However, the increase in the weighted cost due to failure is greater than the reduction in the manufacturing cost when COV of E_{f11} is greater than 0.05. The optimum COV of E_{f11} for different cost due to failure occurrence is computed as shown in Figure 8. It is seen that the larger the cost due to failure, the smaller the optimum COV of E_{f11}.

From this assessment, it demonstrates that selection among possible arrangements for optimum cost with reliability consideration can be achieved using the probabilistic method adopted in the computer code IPACS.

7. CONCLUSIONS

Risk and cost trade-offs for polymer composite structures are assessed and demonstrated. The assessment is accomplished by integrating optimization of reliability-based cost concept into the IPACS (Integrated Probabilistic Assessment of Composite Structures) computer code. The optimization is achieved by solving a system of nonlinear algebraic equations whose coefficients are functions of probabilistic sensitivity factors. Optimum design parameters such as the mean and coefficient of variation (representing range of scatter) of uncertain variables are essential information for the selection of the most efficient and economical manufacturing procedure. In this paper, the reliability-based cost for a composite fuselage panel is minimized with coefficient of variation of fiber modulus in longitudinal direction as the design parameter. The minimum total cost for a 15000 failure cost is found when COV of E_{f11} is equal to 0.05. Therefore, the balance between the maximum reliability and minimum cost is achieved. The optimum COV of E_{f11} for different failure cost is also computed. It is found that the larger the failure cost, the smaller the optimum COV of E_{f11}.

8. REFERENCES

1. C. C. Chamis and M. C. Shiao, "IPACS--Integrated Probabilistic Assessment of Composite Structures: Code Development and Application", Third NASA Advanced Composites Technology Conference, Long Beach, CA, June, 1992
2. C. C. Chamis, "Probabilistic Structural Analysis Methods for Space Propulsion System Components", NASA TM 88861, June 1986.
3. P. L. N. Murthy and C. C. Chamis, "Integrated Composite ANalyzer (ICAN), Users and Programmers Manual", NASA TP 2515, 1986.
4. G. T. Mase, et. al, "Probabilistic Micromechanics and Macromechanics of Polymer Matrix Composites", NASA TM 103669, January 1991.
5. "Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion Systems Components (4th Annual Report)", by Southwest Research Institute, NASA CR-187198, October 1991.
6. Y.-T. Wu, "Demonstration of a New Fast Probability Integration Method for Reliability Analysis", Advances in Aerospace Structural Analysis, O. H. Burnside and C. H. Parr, ed. , ASME, New York, pp. 63-73, 1985
7. G. Gordon, "System Simulation", Prentice-Hall, Inc., Inglewood, N.J. 1969
8. H.O. Madsen, S. Krenk, and N.C. Lind, "Methods of Structural Safety", Prentice-Hall, Inc., 1986

9. SYMBOLS

E_{f11}: fiber modulus in longitudinal direction
E_{f22}: fiber modulus in transverse direction
G_{f12}: in-plane fiber shear modulus
G_{f23}: out-of-plane fiber shear modulus
ν_{f12}: in-plane fiber Poisson's ratio
ν_{f23}: out-of-plane fiber Poisson's ratio
E_m: matrix elastic modulus
G_m: matrix shear modulus
ν_m: matrix Poisson's ratio
fvr: fiber volume ratio
pdf: probability density function
Φ : cdf of Normal Random Variable
Z : performance function
β : Reliability Index
P_f : Probability of Failure
SF : Safety Factor
u^* : Most Probable Point
N : Number of Distribution Parameters
C_o : Constant Cost
P_j : j^{th} Distribution Parameter
C_T : Reliability-based Total Cost
C_I : Manufacturing Cost
C_F : Cost due to Failure
X_i : i th random variable
m_i : mean of i th random variable Failure
cov: coefficient of variation
p_1: COV of E_{f11}
p_2: COV of E_m
p_3: mean value of fiber volume ratio
A_0: typical manufacturing cost for graphite fiber
B_0: typical manufacturing cost for epoxy matrix
D : ratio of weight densities between matrix and fiber
σ_i : standard deviation of i th random variable
$C_j(P_j)$: Manufacturing Cost for j^{th} Distribution Parameter

Table 1. The Statistics of Fiber and Matrix Properties for Graphite-Epoxy Composite

PROPERTY SYMBOLS	UNIT	DISTRIBUTION TYPE	MEAN	COEFFICIENT OF VARIATION (percentage of scatter)
E_{f11}	Msi	Normal	31.0	0.05
E_{f22}	Msi	constant	2.0	0.00
G_{f12}	Msi	constant	2.0	0.00
G_{f23}	Msi	constant	1.0	0.00
ν_{12}	—	constant	0.2	0.00
ν_{23}	—	constant	0.25	0.00
E_m	Msi	Normal	0.5	0.05
G_m	Msi	constant	0.185	0.00
ν_m	—	constant	0.35	0.00

Table 2. The Statistics of Fabrication Variables

PROPERTY SYMBOLS	UNIT	DISTRIBUTION TYPE	MEAN	COEFFICIENT OF VARIATION (percentage of scatter)
Fiber Volume Ratio	–	Normal	0.600	0.05
Void Volume Ratio	–	constant	0.020	0.00
Ply Misalignment Angle	deg.	constant	0.000	0.00
Skin Thickness	in	Normal	0.096	0.04
Stringer Thickness	in	Normal	0.304	0.04
Frame Thickness	in	Normal	0.140	0.04
Pad Thickness (Stringer)	in	Normal	0.191	0.04
Pad Thickness (Frame)	in	Normal	0.124	0.04

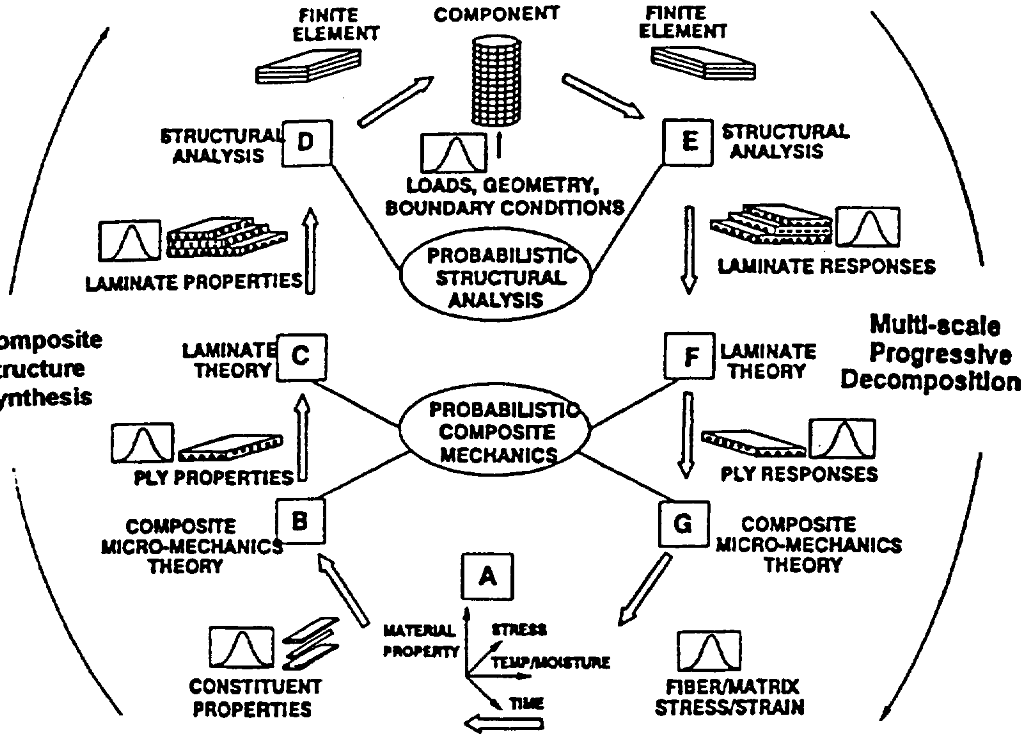

Figure 1. - Concept of Probabilistic Assessment of Composite Structures

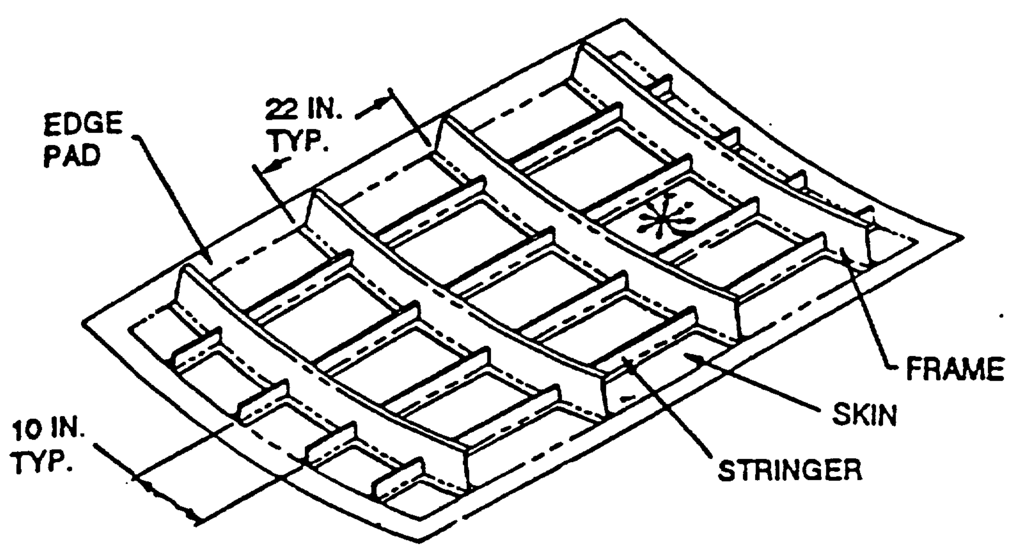

Figure 2. - Lower Side Panel Component

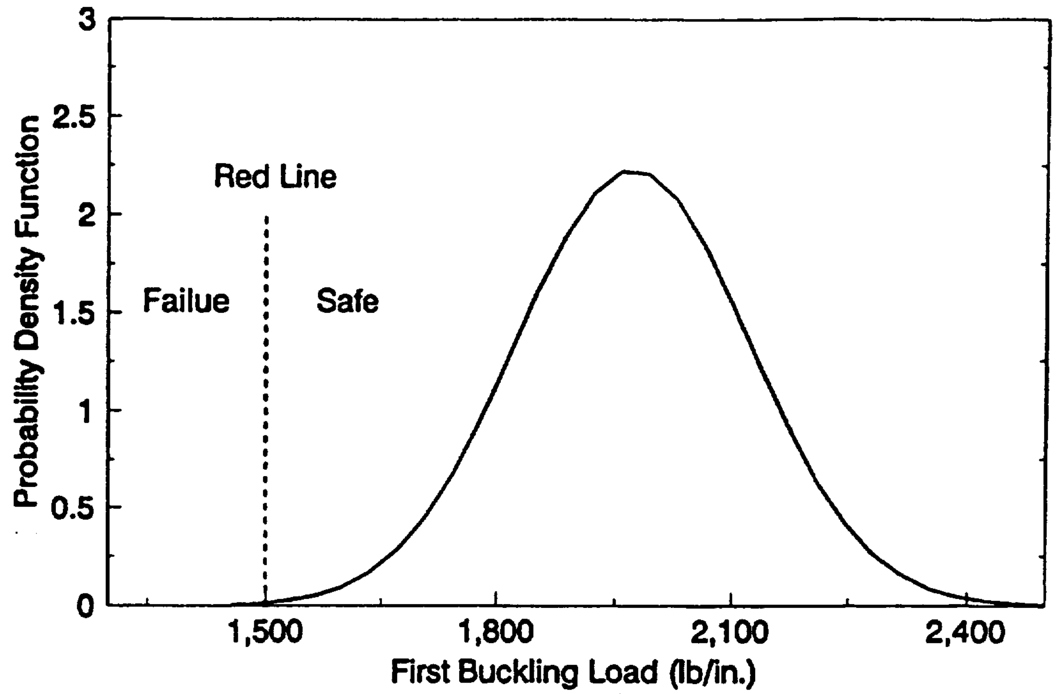

Figure 3. - PDF of First Buckling Load

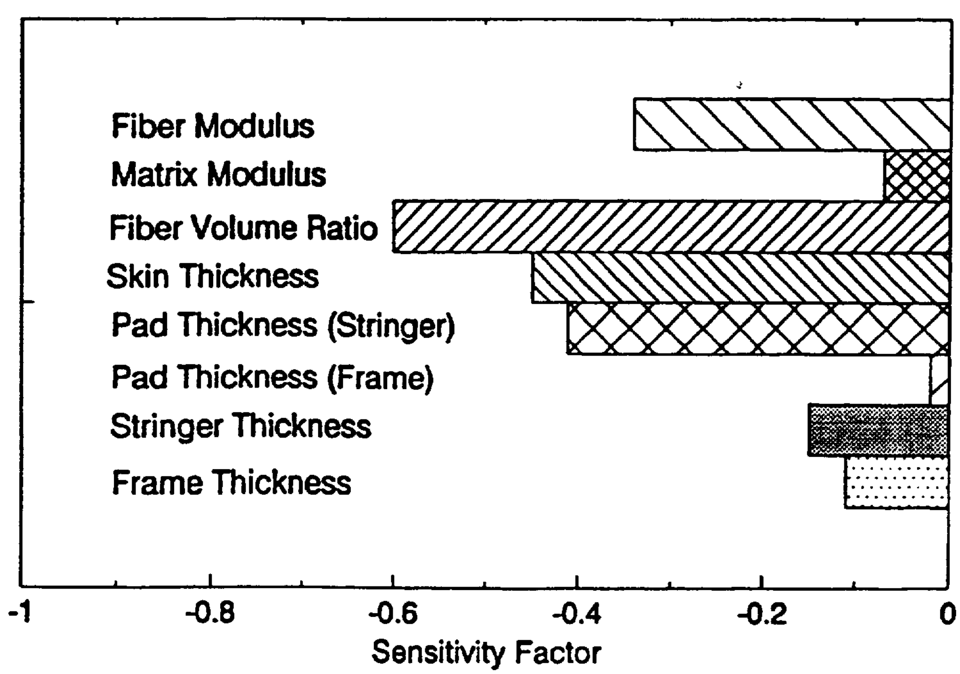

Figure 4. - Sensitivity Factors of Uncertain Variables to 0.001 Probability of First Buckling Load

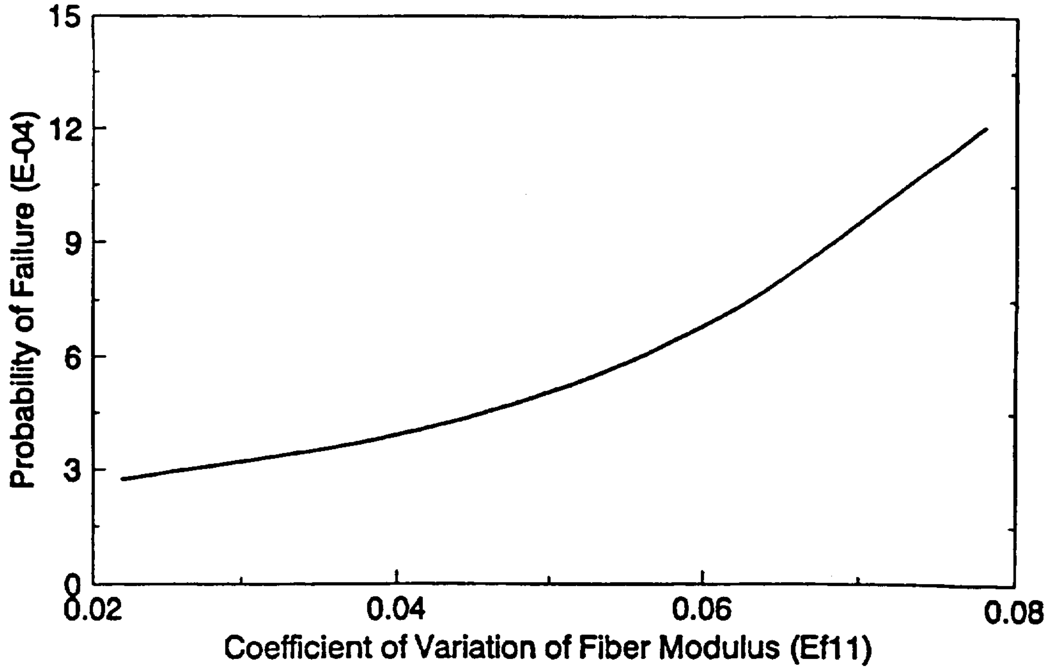

Figure 5. - Probability of Failure as a Function of COV of Ef11

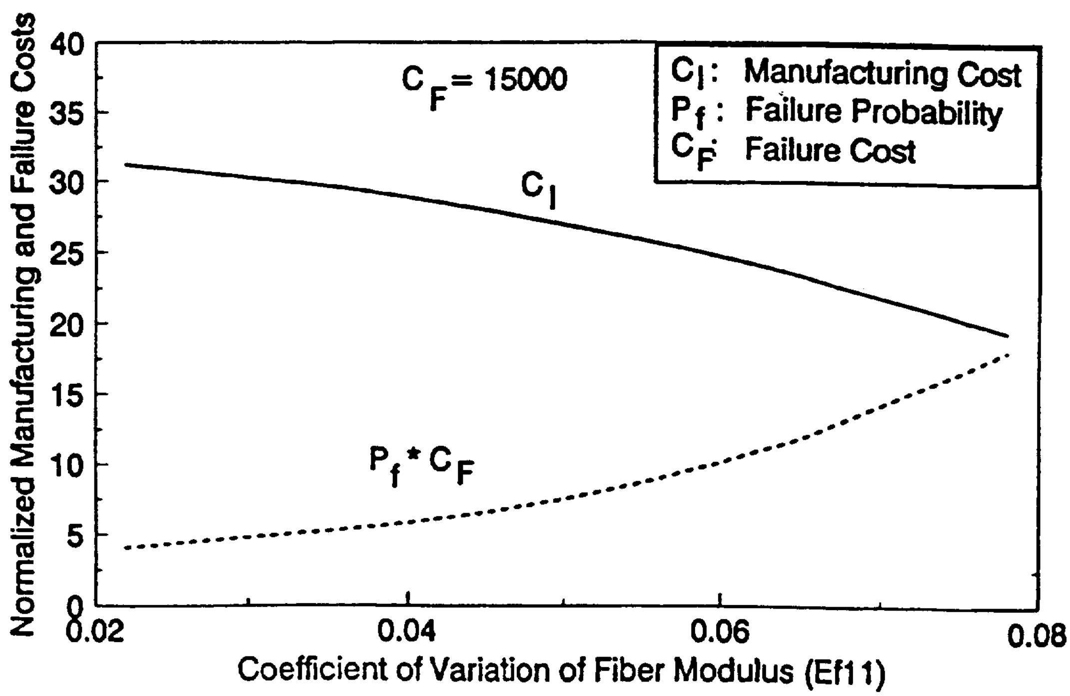

Figure 6. - Normalized Partial Cost as a Function of COV of Ef11

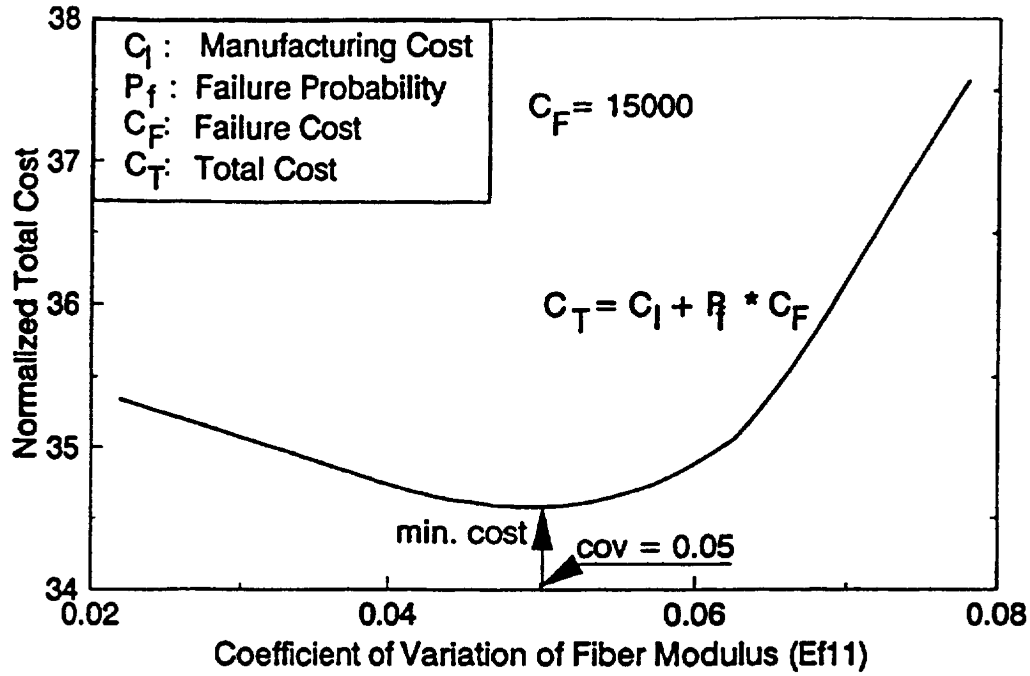

Figure 7. - Normalized Total Cost as a function of COV of Ef11

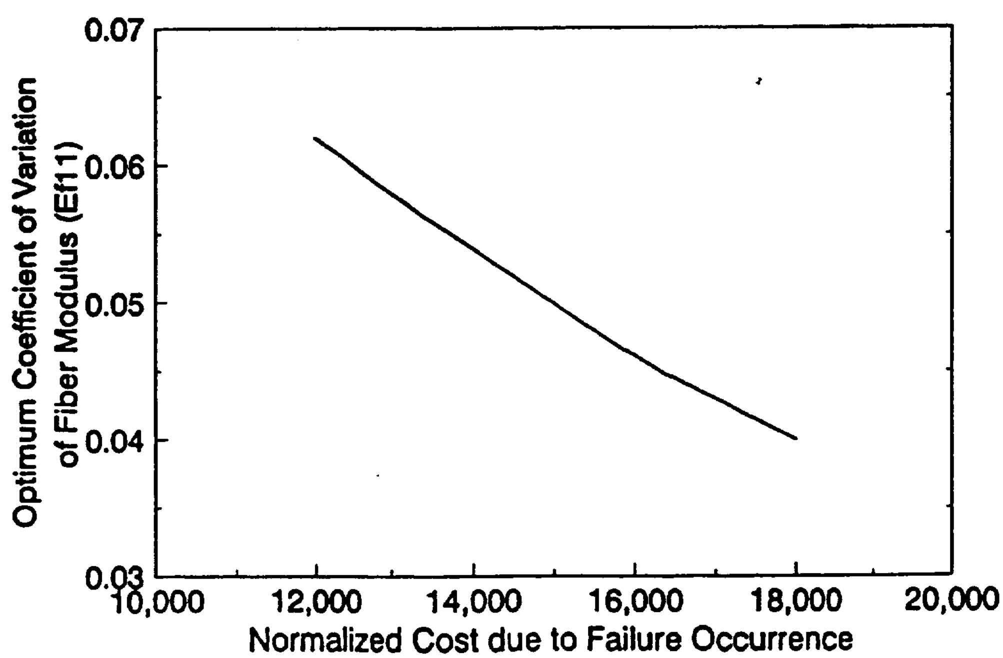

Figure 8. - Optimum COV of Fiber Modulus Ef11 as a Function of Normalized Cost due to Failure Occurrence

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13. ABSTRACT ***(Maximum 200 words)***

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14. SUBJECT TERMS
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