

Time-Dependent Parabolic Finite Difference Formulation for Harmonic Sound Propagation in a Two-Dimensional Duct With Flow

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ABSTRACT

An explicit finite difference real time iteration scheme is developed to study harmonic sound propagation in aircraft engine nacelles. To reduce storage requirements for future large three-dimensional problems, the time dependent potential form of the acoustic wave equation is used. To insure that the finite difference scheme is both explicit and stable for a harmonic monochromatic sound field, a parabolic (in time) approximation is introduced to reduce the order of the governing equation. The analysis begins with a harmonic sound source radiating into a quiescent duct. This fully explicit iteration method then calculates stepwise in time to obtain the "steady state" harmonic solutions of the acoustic field. For stability, applications of conventional impedance boundary conditions requires coupling to explicit hyperbolic difference equations at the boundary.

The introduction of the time parameter eliminates the large matrix storage requirements normally associated with frequency domain solutions, and time marching attains the steady-state quickly enough to make the method favorable when compared to frequency domain methods. For validation, this transient-frequency domain method is applied to sound propagation in a two-dimensional hard wall duct with plug flow.

INTRODUCTION

Both steady-state (frequency domain) and transient (time domain) finite difference and finite element techniques have been developed to study sound propagation in aircraft nacelles. To date, the numerical solutions have generally been limited to moderate frequency sound and mean flow Mach numbers in two-dimensional axisymmetric nacelles. Wavelength resolution problems have prevented a broader range of applications of the numerical methods. A fine grid is required to resolve the short wavelengths associated with high frequency sound propagation with high inlet Mach numbers. Thus, application of numerical techniques to high frequency sound propagation in three-dimensional engine nacelles has yet to be attempted.

To extend numerical analysis to higher frequencies and inlet flow Mach numbers, as well as three-dimensional geometries Baumeister and Kreider (ref. 10), suggest using a pseudo-time-frequency transformation to the acoustic potential equations. Their method eliminates the large matrix storage requirements of the steady state (Fourier transforms) techniques in the frequency domain but still allows the use of conventional impedance conditions. Most importantly, their two step time marching formulation is fully explicit under flow conditions. They also suggested using an almost highly structured grid to reducing computer storage and run times. Using the same grid system and governing acoustic potential equations, the goal of the present paper is to develop a stable real time iteration scheme.

The paper begins with a brief description of the governing equations. A detailed development of the governing equations has been given earlier (ref. 10); it is therefore described only briefly here. Next, assuming a harmonic monochromatic sound field, a parabolic (in time) approximation is introduced to reduce the order of the governing equation. The bulk of the paper describes the development of a stable, explicit finite difference scheme. The scheme is iterated in time to converge to the steady-state solution associated with a Fourier transform solution.

NOMENCLATURE

\bar{c} steady speed of sound, $\bar{C}^\# / C_0^\#$, equation (2)

$D^\#$ dimensional duct height or diameter

D duct height, $D = 1$

d parameter, equation (25)

F source amplitude at duct entrance, equation (17)

$f^\#$ dimensional frequency

f dimensionless frequency, $f^\# D^\# / C_0^\#$

g parameter, equation (27)

h parameter, equation (26)

i $\sqrt{-1}$

L length of duct, $L^\# / D^\#$, figure 1

M_f Mach number at duct entrance

n unit outward normal

$P^\#$ dimensional pressure

P dimensionless fluid pressure, $P^\# / \rho_0^\# C_0^\#{}^2$

P' acoustic pressure fluctuation, equation (6)

t dimensionless time, $f^\# t^\#$

Δt time step

x dimensionless axial coordinate, $x^\# / D^\#$

Δx axial grid spacing

y dimensionless transverse coordinate, $y^\# / D^\#$

Δy transverse grid spacing

γ ratio of specific heats

- ρ dimensionless fluid density, $\rho^\#/\rho_0^\#$
- $\bar{\rho}$ steady fluid density
- Φ dimensionless time dependent flow potential, $\phi^\#/C_0^\#D^\#$
- $\bar{\phi}$ steady mean flow potential
- ϕ' transient acoustic potential
- ψ spatial potential, equation (10)
- ω dimensionless frequency, $2\pi f$
- ∇ $D^\#\nabla^\#$

Subscripts

- i axial index, see figure 1
- j transverse index, see figure 1
- o ambient of reference condition

Superscripts

- # dimensional quantity
- k time step

PROBLEM STATEMENT

The problem under consideration here is the steady-state propagation of sound, represented by the perturbation acoustic potential, through a two-dimensional rectangular duct. The source, noise emanating from fan blades in a jet engine inlet nozzle, is represented by specifying the pressure distribution at the fan face. The goal of the paper is to develop a stable, explicit finite difference scheme that incorporates the far field impedance condition applied at the duct exit and rigid body boundary conditions on the duct walls. The method is designed with the intention of extending the current two-dimensional duct formulation to general three-dimensional nacelle design problems with a variety of possible boundary conditions in the near and far fields.

GOVERNING EQUATIONS

Acoustic propagation in inlet nacelles can be reasonably modeled by an inviscid approximation. For single mode JT15D engine data, a previous finite element study (ref. 8) employing the potential formulation in the frequency domain showed good agreement with experimental data—in the far field radiation pattern as well as suppressor attenuation. Due to this success, the problem under consideration here is formulated in terms of an acoustic potential.

For inviscid, nonheat conducting and irrotational flow, the linearized acoustic equation for two-dimensional potential flow can be written in dimensionless form as (ref. 10, eq. (1))

$$0 = f^2 \phi'_{tt} - (\bar{c}^2 - \bar{\Phi}_x^2) \phi'_{xx} - (\bar{c}^2 - \bar{\Phi}_y^2) \phi'_{yy} + 2\bar{\Phi}_x \bar{\Phi}_y \phi'_{xy} + 2f\bar{\Phi}_x \phi'_{xt} + 2f\bar{\Phi}_y \phi'_{yt} + 2(\bar{\Phi}_x \bar{\Phi}_{xx} + \bar{\Phi}_y \bar{\Phi}_{yy}) \phi'_x + 2(\bar{\Phi}_x \bar{\Phi}_{xy} + \bar{\Phi}_y \bar{\Phi}_{yy}) \phi'_y - (\gamma - 1)(f\phi'_t + \bar{\Phi}_x \phi'_x + \bar{\Phi}_y \phi'_y)(\bar{\Phi}_{xx} + \bar{\Phi}_{yy}) \quad (1)$$

where

$$\bar{c}^2 = 1 - \frac{1}{2}(\gamma - 1)(\bar{\Phi}_x^2 + \bar{\Phi}_y^2) \quad (2)$$

The symbol $\bar{\phi}$ represents steady mean flow potential while ϕ' represent the time dependent acoustic potential. The speed of propagation of a disturbance is represented by c and the frequency of an acoustic source by f . The subscripts indicate partial differentiation with respect to the subscripted variables.

The conventional normalization factors used to develop these nondimensional equations are given in the NOMENCLATURE. However, the normalization of time deserves some special comment. A common choice for normalizing time is $t = C_0^{\#} t^{\#} / D^{\#}$. The superscript $\#$ designates a dimensional quantity while the subscript o indicates an arbitrary reference value. With this choice, the dimensionless frequency f would not appear in equations (1) or (2). However, in this paper, the dimensional frequency $f^{\#}$ of the forcing acoustic signal was chosen to normalize time, so that $t = f^{\#} t^{\#}$. As a result, the time t indicates the number of complete acoustic cycles that have occurred since the start of the solution process. This is advantageous because the total time of the numerical calculation can generally be set independently of the frequency of the acoustic signal.

For simplicity, the current paper will focus only on plug flow. For this case, the mean flow terms in equation (1) become

$$\bar{\Phi}_y = \bar{\Phi}_{xy} = \bar{\Phi}_{xx} = 0 \quad \bar{\Phi}_x = M_f \quad (3)$$

Substituting equation (3) into equation (1) yields

$$0 = f^2 \phi'_{tt} - (\bar{c}^2 - M_f^2) \phi'_{xx} - \bar{c}^2 \phi'_{yy} + 2fM_f \phi'_{xt} \quad (4)$$

and

$$\bar{c}^2 = 1 - \frac{1}{2}(\gamma - 1)M_f^2 \quad (5)$$

The relationship between the acoustic pressure and potential can also be expressed as (ref. 10, eq. (7))

$$\frac{1}{\rho} P'(x, y, t) = -f\phi'_t - M_f \phi'_x \quad (x = 0, \quad \Phi_y = 0) \quad (6)$$

Equation (4) is the basic equation used to establish a finite difference formulation for sound propagation in the rectangular duct shown in figure 1. However, care must be taken when discretizing derivative terms to insure that the resulting scheme is both stable and explicit. Note that it is easy to develop a stable implicit method, but this would yield a matrix formulation that is no better than a frequency domain approach. It turns out that the proper treatment of the mixed time and space derivative terms (which appear on the second line of eq. (1)) is critical for maintaining stability.

The implicit formulation is obtained by approximating the mixed partials by (ref. 6, eq. (16), and ref. 1, p. 884, eq. (25.3.27))

$$\phi'_{xt} = \frac{\phi'_{i,j}{}^{k+1} + \phi'_{i-1,j}{}^k + \phi'_{i+1,j}{}^k + \phi'_{i,j}{}^{k-1} - 2\phi'_{i,j}{}^k - \phi'_{i-1,j}{}^{k+1} - \phi'_{i+1,j}{}^{k-1}}{2\Delta x \Delta t} \quad (7)$$

$$\phi'_{yt} = \frac{\phi'_{i,j}{}^{k+1} + \phi'_{i,j}{}^{k-1} + \phi'_{i,j+1}{}^k + \phi'_{i,j-1}{}^k - 2\phi'_{i,j}{}^k - \phi'_{i,j+1}{}^{k+1} - \phi'_{i,j-1}{}^{k-1}}{2\Delta t \Delta y} \quad (8)$$

where i and j denote the space indices for the nodal system shown in figure 1, k is the time index defined by

$$t^{k+1} = t^k + \Delta t \quad (9)$$

and Δx , Δy , and Δt are the space and time mesh spacings, respectively.

Reference 6 shows that equation (7) can be used in an explicit fashion for one-dimensional plug flow problems in a duct. However, if the flow is not one-dimensional or if the region exterior to the duct is included, the scheme must be implicit. This approach is inappropriate for general three-dimensional problems. This problem can be circumvented by modifying the governing equation. The details follow in the next section.

PARABOLIC APPROXIMATION

There are several ways to develop a frequency domain formulation for the general two-dimensional acoustic wave equation (1) or the plug flow simplification equation (4). The Fourier Transform can be applied if the potential has a multifrequency content. In the monochromatic case, this is equivalent to assuming that

$$\phi'(x, y, t) = \psi(x, y)e^{-i\omega t} = \psi(x, y)e^{-i2\pi t} \quad (10)$$

which, in the case of plug flow (from eq. (4)), yields

$$0 = (\bar{c}^2 - M_f^2)\psi_{xx} + \bar{c}^2\psi_{yy} + \omega^2\psi + i2\omega M_f\psi_x \quad (11)$$

This equation would be solved numerically using a linear system of equations. However, the associated matrix is not positive definite, which can lead to numerical difficulties, and which preclude the use of iterative techniques. Therefore, it is desirable to develop an explicit finite difference scheme to avoid the use of matrices. In time-dependent form, equations (1) or (4) cannot easily be discretized in such a way that the resulting finite difference scheme is both stable and explicit in the presence of flow (it is possible to obtain reasonable results in the no-flow case).

Reference 10 resolves these difficulties by a transformation to a transient-frequency domain. Herein, however, the resolution of these difficulties is achieved by applying the transformation equation (10) only once to equation (4). Employing equation (10), the time derivatives in equation (4) can be replaced by the following relationships:

$$\phi'_t = -i2\pi\psi e^{-i2\pi t} = -i2\pi\phi' \quad (12)$$

$$\phi'_{xt} = \frac{\partial}{\partial x}(-i2\pi\psi e^{-i2\pi t}) = -i2\pi\psi_x e^{-i2\pi t} = -i2\pi\phi'_x \quad (13)$$

Under this transformation, the plug flow equation (4) becomes,

$$-i\omega\phi'_t = (\bar{c}^2 - M_f^2)\phi'_{xx} + \bar{c}^2\phi'_{yy} + i2\omega M_f\phi'_x \quad (14)$$

The mixed derivative problem term which prevented an explicit finite difference representation of equation (4) has been eliminated from equation (4). An explicit finite difference solution can now be formulated to solve equation (14).

The interest in this paper is in steady state harmonic solutions to the wave equation as represented by the variable ψ . Therefore, after sufficient time during the transient solution of equation (14), the ϕ' variable can be related to the steady state Fourier transformed solution, using equation (10).

$$\psi(x, y) = \frac{\phi'(x, y, t)}{e^{-i2\pi t}} \quad (15)$$

INITIAL AND BOUNDARY CONDITIONS

The duct is assumed to be quiescent at time 0, so that the initial condition is

$$\phi'(x, y, 0) = 0 \quad (16)$$

As the equation is iterated in time, the solution builds up to the steady state harmonic solution.

At the duct entrance, ($x = 0$), the potential is given by

$$\phi'(0, y, t) = F(y)e^{-i2\pi t} \quad (17)$$

If the pressure at $x = 0$ is specified as the boundary condition, then the potential is related to the pressure directly through equation (6).

The hard wall condition is

$$\nabla\phi' \cdot \mathbf{n} = 0 \quad (18)$$

where \mathbf{n} is the unit outward normal.

To simulate a nonreflective boundary at the duct exit in figure 1, the difference equation at the end of the computational domain can be expressed in terms of an exit impedance. In tern, this can be used to develop an exit gradient condition to simulate a nonreflecting grid termination. For plane wave propagation in the examples to follow, Baumeister and Kreider (ref. 10) have shown this gradient to be of the form:

$$\phi'_x = \frac{i\omega}{c + M_f} \phi' \quad (19)$$

FINITE DIFFERENCE EQUATIONS

The finite difference approximations determine the potential at the spatial grid points at discrete time steps $t^k = k\Delta t$. Starting from the known initial conditions at $t = 0$ and the boundary conditions, the algorithm marches the solution out to later times.

Away from the duct boundaries, as shown by the cell in figure 1, each partial derivative in equation (14) can be expressed as follows:

$$\phi'_t = \frac{\phi'_{i,j}{}^{k+1} - \phi'_{i,j}{}^{k-1}}{2\Delta t} \quad (20)$$

$$\phi'_{xx} = \frac{\phi'_{i+1,j}{}^k - 2\phi'_{i,j}{}^k + \phi'_{i-1,j}{}^k}{\Delta x^2} \quad (21)$$

$$\phi'_{yy} = \frac{\phi'_{i,j+1} - 2\phi'_{i,j} + \phi'_{i,j-1}}{\Delta y^2} \quad (22)$$

$$\phi'_x = \frac{\phi'_{i+1,j} - \phi'_{i-1,j}}{2\Delta x} \quad (23)$$

Substituting these expressions into equation (14) yields

$$\begin{aligned} \phi'_{i,j}{}^{k+1} \left(\frac{-h}{2\Delta t} \right) = & \phi'_{i,j}{}^k \left(-\frac{2d^2}{\Delta x^2} - \frac{2\bar{c}^2}{\Delta y^2} \right) + \phi'_{i+1,j}{}^k \left(\frac{d^2}{\Delta x^2} + \frac{g}{2\Delta x} \right) + \phi'_{i-1,j}{}^k \left(\frac{d^2}{\Delta x^2} - \frac{g}{2\Delta x} \right) + \phi'_{i,j+1}{}^k \left(\frac{\bar{c}^2}{\Delta y^2} \right) + \phi'_{i,j-1}{}^k \left(\frac{\bar{c}^2}{\Delta y^2} \right) \\ & + \phi'_{i,j}{}^{k-1} \left(\frac{-h}{2\Delta t} \right) \end{aligned} \quad (24)$$

where

$$d^2 = \bar{c}^2 - M_f^2 \quad (25)$$

and

$$h = i\omega f \quad (26)$$

and

$$g = 2i\omega M_f \quad (27)$$

Equation (24) is an explicit two step scheme. At $t = 0$, field values at t^{k-1} are assumed zero because the initial field is quiescent.

The expressions for the difference equations at the boundaries are complicated somewhat by the impedance conditions. However, a simple integration procedure resolves this problem. Baumeister (refs. 5 and 6) gives precise details for generating the time difference equations at the boundaries. In fact, because of problems with applying the exit termination condition, the actual hyperbolic exit difference equations used in these references will be applied. This will be fully discussed later.

STABILITY

A von Neumann stability analysis (ref. 14) indicates that the method is conditionally stable, subject to the conservative condition

$$\Delta t < \frac{1}{\frac{4}{\omega f} \left[\left(\frac{d}{\Delta x} \right)^2 + \left(\frac{\bar{c}}{\Delta y} \right)^2 \right] + \frac{2|M_f|}{f\Delta x}} \quad (28)$$

In a typical application, ω , f , and M_f are set by the engine operating conditions. Next, the grid spacing parameters Δx and Δy are set to accurately resolve the estimated spatial harmonic variation of the acoustic field. Finally, Δt is chosen to satisfy equation (28).

In the von Neumann analysis, conditional stability means that the amplification factor, which describes how errors propagate from one time step to the next, has magnitude one. Thus, when inequality (28) is satisfied, errors are not magnified or diminished in magnitude. This is a desirable property, since the numerical formulation can not distinguish between an error and a small acoustic mode.

The von Neumann stability analysis does not take into account boundary conditions. For stability, gradient boundary conditions require the use of smaller Δt than predicted by equation (28).

NUMERICAL EXAMPLES

In the three examples that follow, the parabolic transient results are compared to the exact results of the steady Fourier transformed solutions. The exact Fourier transformed solution for the problem considered is given by (ref. 10, eq. (35))

$$\psi(x) = e^{\frac{i\omega}{\bar{c}+M_f}x} \quad (29)$$

The following problem is considered: a plane wave propagates from the left into a quiescent duct of length one, and the acoustic potential field is to be computed in the duct. Note that, boundary conditions can introduce instabilities (refs. 7 and 11) into otherwise stable finite difference schemes. Therefore, it is important to test the proposed method for convergence in time to the steady state solution in the absence of the exit boundary condition (eq. (19)), and to test independently the effect of the exit boundary condition itself on the solution.

Semi-Infinite Duct

In this example, the computational boundary is set far enough away from the true boundary $x = 1$ that any artifacts arising from imperfections in the exit boundary condition do not affect the solution in $[0,1]$. The numerical solution propagates one node per time step, so setting the boundary at $x = 50$ with step $\Delta x = 0.05$ provides a sufficient number of time steps to gauge the convergence of the method before any artifacts might reflect back from the computational boundary.

The numerical and exact results are compared in figure 2, for no flow (fig. 2(a)) and for Mach number $M_f = -0.5$ (fig. 2(b)). The frequency is normalized to $f = 1$. Both cases show excellent agreement. The total calculation time was $t_T = 5.0$.

Finite Duct $L = 1$

In this example, the computational boundary is moved up to the true boundary $x = 1$ to examine the effect of the exit boundary condition (eq. (19)). The frequency is normalized to 1. Three cases were considered—no flow ($M_f = 0$) and Mach number $M_f = \pm 0.2$.

The formulation using the parabolic wave equation, equation (14), was found to be unstable. Apparently, the parabolic wave equation is not compatible with impedance boundary conditions represented by equation (19). Fortunately, the exit termination difference equation could be written in an explicit fashion using the hyperbolic equation, equation (4). The difference equation for this exit condition is given by Baumeister (ref. 5).

The results are shown in figures 3(a) to (c), respectively using a parabolic difference formulation in the interior of the duct and a hyperbolic formulation at the exit. In figure 3, the direction of the arrow indicates the direction of propagation of the acoustic wave. The numerical results again match well with the exact results. Notice also that the time step has been decreased here, which tends to increase the execution time; however, the computational domain is smaller, which tends to decrease the execution time. The total calculation time in this example was $t_T = 4.0$.

It is clear that the exit boundary condition does have a slight degrading effect on the solution particularly with flow. In fact, for Mach number greater than 0.2 the errors are unacceptable. Therefore, the parabolic solution should only be employed in problems where the exit Mach number is zero or reasonably low. Consequently, in most problems, the transient-frequency approach developed by Baumeister and Kreider (ref. 10) is preferred.

CONVERGENCE RATE

In this example, the convergence rate is studied for the region $x = 0$ to $x = 1$ using the semi-infinite duct. The results are shown in figure 4 for the magnitude of the potential. As seen in this figure, the numerical solution quickly and accurately converges to the exact steady state solution. As seen in figure 4, the parabolic solution converges to the Fourier transformed results after a time of $t = 1$ has elapsed. This is about twice as fast as the transient-frequency approach presented by Baumeister and Kreider (ref. 10).

SOLUTION METHODS

With the parabolic transient approach developed in this paper, four different solution techniques are now available to solve the hyperbolic wave equation that describes acoustic propagation in ducts and jet engine nacelles with a monochromatic source. This is illustrated in figure 5 for the zero mean flow case.

The Fourier transform of the wave equation was the first numerical approach used to study sound propagation in jet engine ducts (refs. 2 and 4). This steady state approach is outlined on the centerline of figure 5 (third column from left). The governing hyperbolic wave equation is transformed to the elliptic Helmholtz "wave" equation. Finite difference (FD) and finite element (FE) numerical formulations have been employed to solve this equation. After applications of the boundary conditions (fig. 5; [BC]), the associated finite difference or finite element global matrix is solved for the velocity potential (or pressure). Because the matrix form of the Helmholtz partial differential equation is not positive definite, matrix elimination solutions are generally employed. This requires extensive storage. Conveniently, the steady state approach allows the direct calculation of the sound pressure levels. Currently, this is the most popular approach for solving acoustic propagation in ducts.

In the inlet to a turbojet engine, the dimensionless frequencies f can be on the order of 30 to 50 for the higher harmonics of the blade passing frequency. The storage requirements and associated computer run times for these high frequencies makes computations expensive and even impossible. To make the numerical solutions more cost effective, grid saving approximations to the governing Helmholtz wave equation have been used (fig. 5; [Approx. Spatial]). Baumeister (ref. 3) employed the wave envelope theory while Hardin and Tappert (ref. 13) developed a similar approach for underwater sound propagation with the addition of a parabolic (space) approximation. Candel (ref. 12) presents an extensive discussion of the contemporary research in this area and a detailed development of the parabolic (spatial) equation method (PEM).

The transient solution to the wave equation was the second numerical approach used to study propagation in jet engine ducts, which is shown in the second column from the left in figure 5. To eliminate the matrix storage requirements, Baumeister developed time dependent finite difference numerical solutions for noise propagation in a two-dimensional duct (ref. 5).

Sound is introduced as a boundary condition at the duct entrance. The initial conditions generally assume a quiescent duct. Finite difference (FD) approximations to the hyperbolic wave equation are then solved by an iteration process. The calculations are run until the initial transient dies out and steady harmonic oscillations are established. Finally, the transient variable ϕ' is transformed into the steady state variable ψ associated with the solution of the Helmholtz equation. As with the steady state approach, approximate spatial solutions reduce computer storage and run times (ref. 9).

The third option, the transient-frequency technique (ref. 10), is illustrated in the fourth column from the left of figure 5. This fully explicit iteration method eliminates the large matrix storage requirements of steady state techniques and allows the use of conventional impedance conditions. As time increases, the iteration process directly computes the steady state variable ψ .

The fourth option, as discussed in this paper, involves a parabolic (time) approximation to the wave equation. The transient finite difference solution of this equation can be conveniently expressed in explicit form with mean flow.

CONCLUDING REMARKS

A parabolic transient numerical solution of the potential acoustic equations has been developed. The potential form of the governing equations has been employed to reduce the number of dependent variables and their associated storage requirements. The method eliminates the large matrix storage requirements of steady state techniques in the frequency domain and the formulation is fully explicit under flow conditions. The field is iterated in time from an initial value of 0 to attain the steady state solution. In each example provided, the numerical solution quickly and accurately converges to the exact steady state solution.

Application of impedance boundary conditions to this parabolic formulation causes an instability. To circumvent this problem, the termination boundary conditions are developed in terms of the hyperbolic acoustic equations. However, with high Mach numbers this termination procedure leads to large errors. Thus, the method should only be applied to problems with a zero mean flow far field boundary condition.

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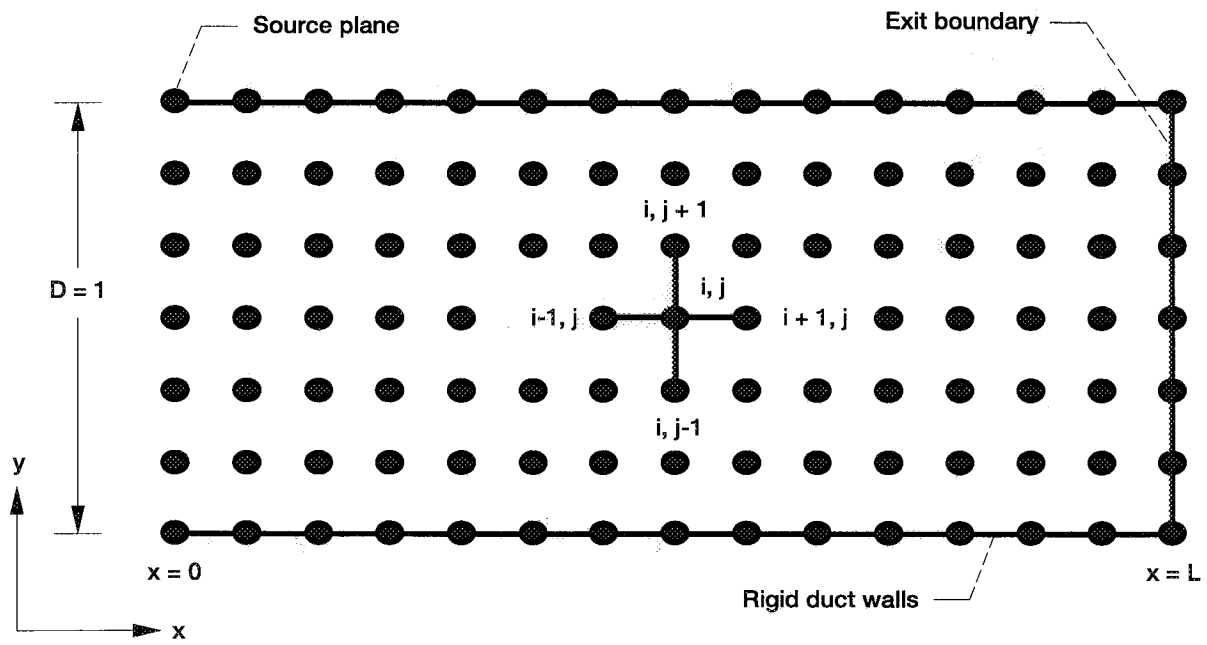


Figure 1.—Structured FD-TD mesh for rectangular duct.

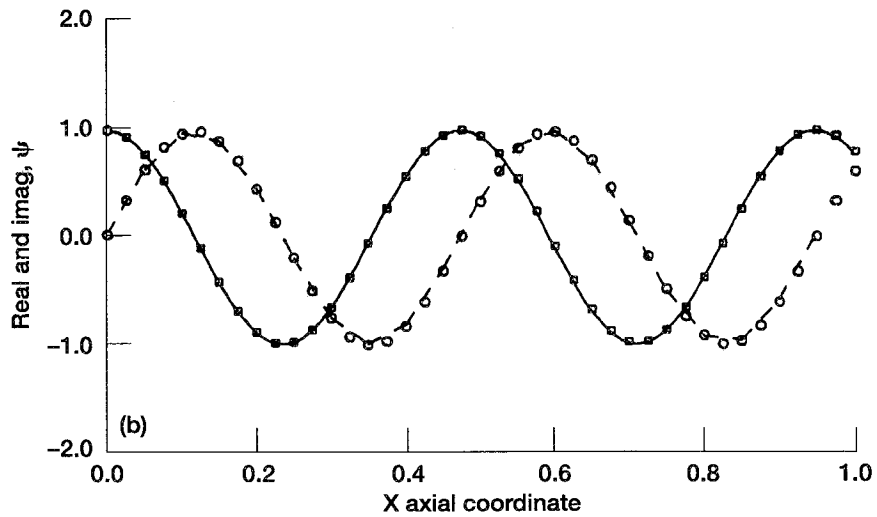
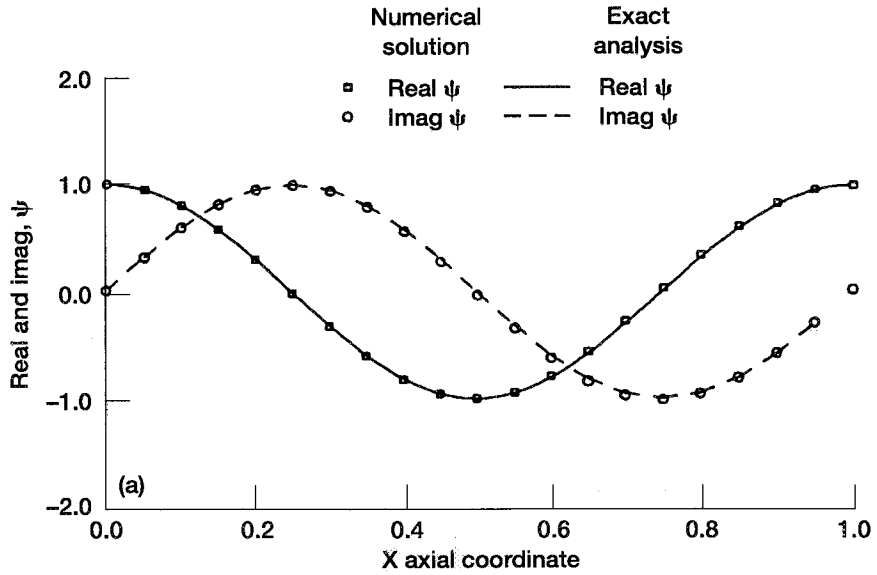


Figure 2.—Analytical and numerical potential profiles along wall for plane wave propagating in a semi-infinite hard wall duct ($f = 1$). (a) $M_f = 0$ ($\Delta x = 0.05$, $\Delta t = 0.003$ $t_T = 5.0$). (b) $M_f = -0.5$ ($\Delta x = 0.025$, $\Delta t = 0.001$ $t_T = 5.0$).

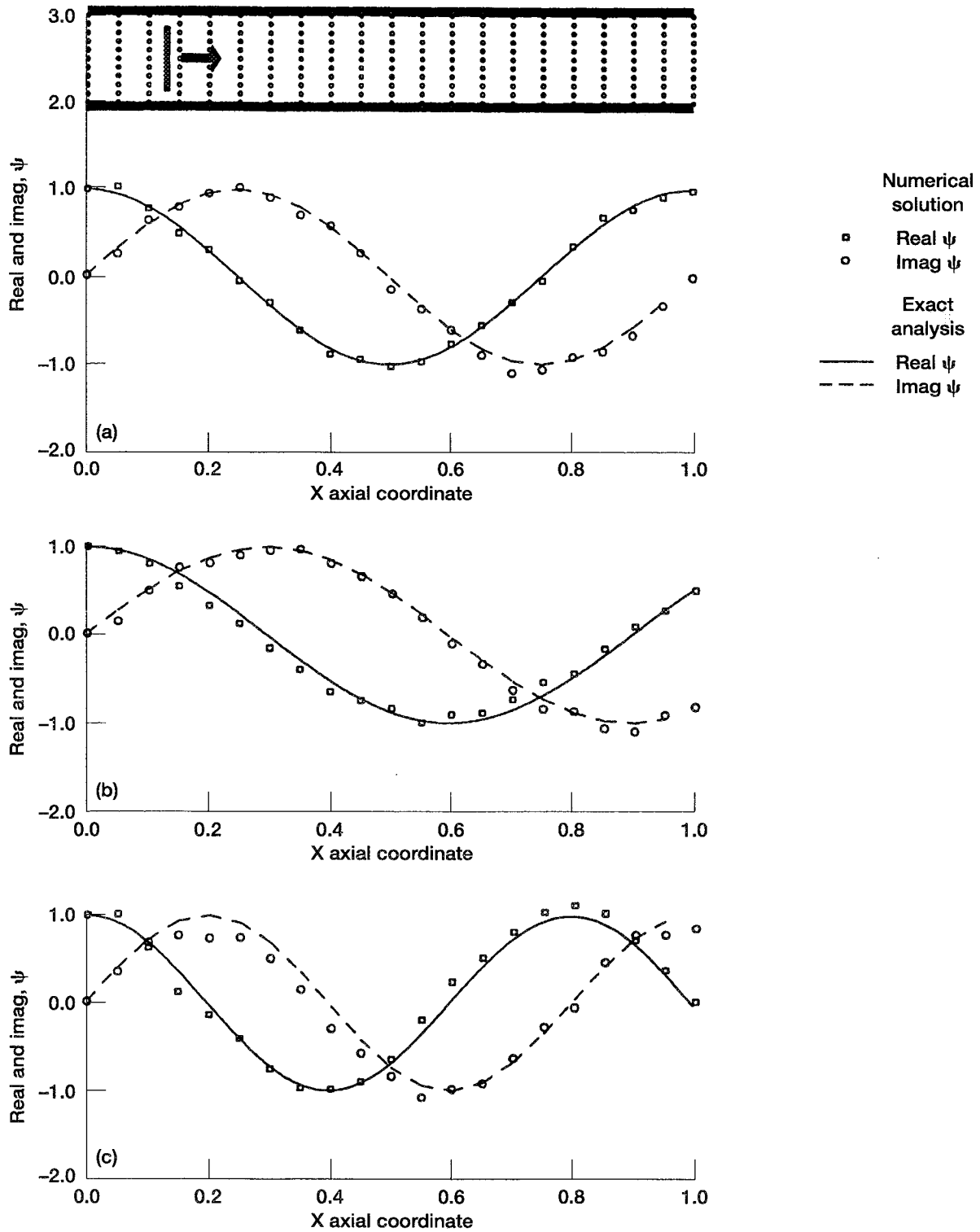


Figure 3.—Analytical and numerical potential profiles along wall for plane wave propagating in a hard wall duct of unit length and a non-reflecting exit condition ($f = 1$). (a) $M_f = 0$ ($\Delta x = 0.05$, $\Delta t = 0.001$ $t_T = 4.0$). (b) $M_f = +0.2$ ($\Delta x = 0.05$, $\Delta t = 0.001$ $t_T = 4.0$). (c) $M_f = -0.2$ ($\Delta x = 0.05$, $\Delta t = 0.001$ $t_T = 4.0$).

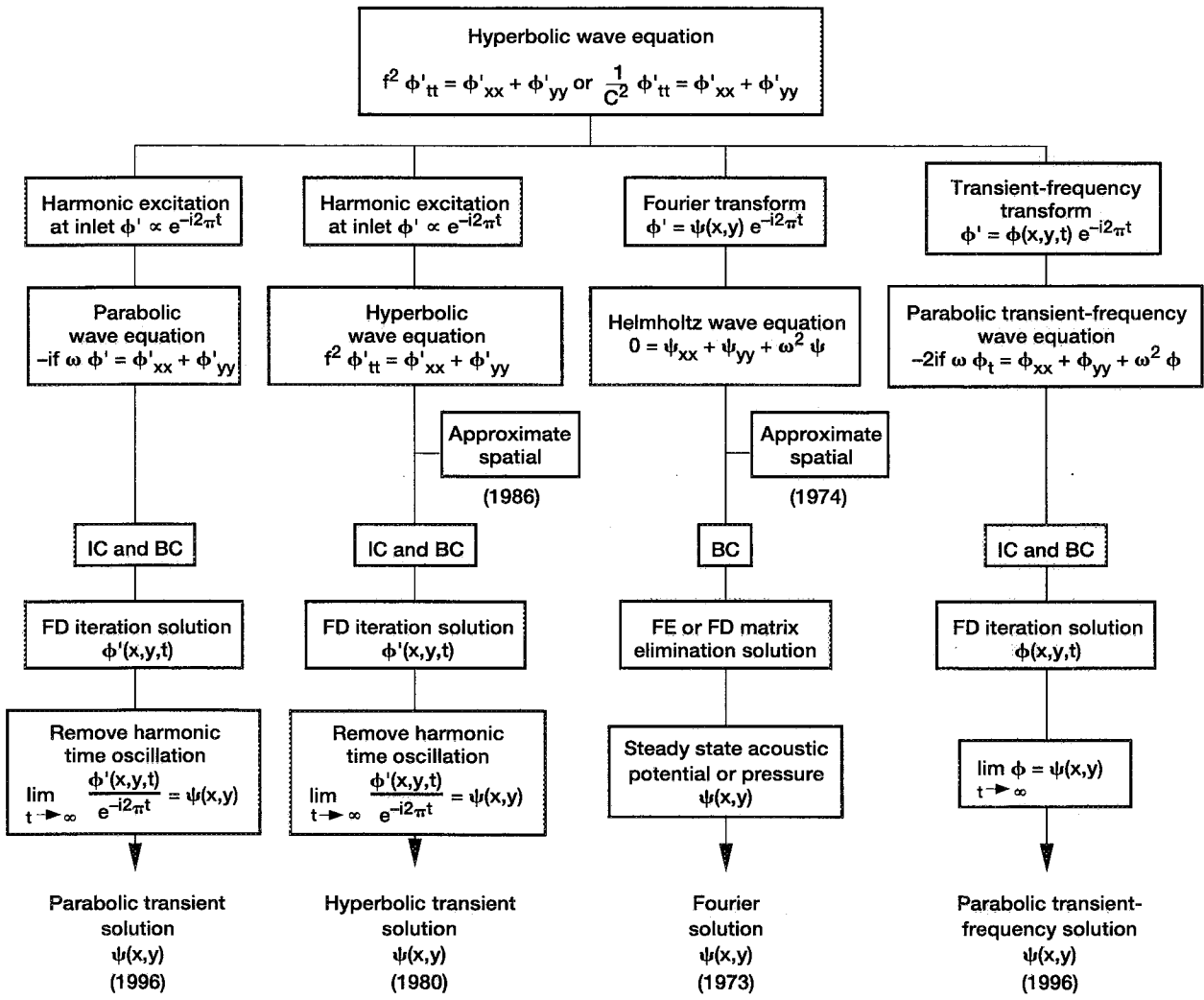


Figure 5.—Alternate finite difference/element methods in solving wave equation with monochromatic source. (IC = initial condition, BC = boundary condition).

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