Progress Report on

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JET AEROACOUSTICS: NOISE GENERATION MECHANISM AND PREDICTION

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This is the first year of the project. The research plan consists of two main tasks. They are:

(a) Physics and prediction of turbulent mixing noise from supersonic jets.

(b) Numerical simulation of supersonic jet noise.

We would like to report that much progress and accomplishments have been made on both tasks during the year. Part of the research results are reported in the following papers.

Tam, C.K.W., Golebiowski, M. and Seiner, J.M. "On two components of turbulent mixing noise from supersonic jets", AIAA paper 96–1716, 1996.

Tam, C.K.W. "Jet noise: since 1952", Lighthill Symposium, November 6–8, 1996.

Tam, C.K.W., Fang, J. and Kurbatskii, K.A. "Inhomogeneous radiation boundary conditions simulating incoming acoustic waves for computational aeroacoustics", Proceedings of the International Congress on Fluid Dynamics & Propulsion, pp. 332–339, 1996.

Tam, C.K.W. and Auriault, L. "Time-domain impedance boundary conditions for computational aeroacoustics" AIAA Journal, vol. 34, 917–923, 1996.

Tam, C.K.W., Kurbatskii, K.A. and Fang, J. "Numerical boundary conditions for computational aeroacoustics benchmark problems" Second Computational Aeroacoustics Workshop on Benchmark Problems, November 4–5, 1996.

Tam, C.K.W. and Hao, S. "Screech tones of supersonic jets from bevelled rectangular nozzles" AIAA paper 97–0143, 1997.

Copies of these papers are attached at the end of this report.

Our work has established that there are two components of turbulent mixing noise. One component is generated by the large turbulence structures of the jet in the form of Mach wave radiation. The other component is generated by the fine scale turbulence of the jet flow. It has a very broad spectrum and is the dominant noise component around 90 degrees and in the forward directions. The fact that turbulent mixing noise of supersonic jets consists of two independent components is new. The classical Acoustic Analogy Theory attributes quadrupoles as the noise source of jets. This implies that there is only one component of noise. Our results, based on experimental measurements, thus suggests that the classical theory is inadequate.

In our work, presented at the Lighthill Symposium, we pointed out that the scaling formula $I \sim v_j^3$ derived by Ffowcs Williams for high speed jets was not consistent with experimental measurements over the velocity range of $1 < v_j/a_\infty \le 2.5$. Data shows that the noise intensity depends on v_j to a much higher power; at inlet angle 160 degrees the velocity exponent is larger than 9.0. Furthermore the velocity exponent is jet temperature dependent whereas the Ffowcs Williams theory has no jet temperature dependence at all. The classical $I \sim v_j^3$ formula was developed primarily based on dimensional analysis. In light of our finding, it appears that it is likely that the dimensional argument is defective. This is not surprising for when the formula was derived, understanding of turbulence was primitive. Even the concept and the existence of large turbulence structures were unknown at that time.

In relation to the second task of the research objectives of the project, we have developed a set of improved numerical radiation boundary conditions. The case with incoming disturbances is reported in the Proceedings of the International Congress on Fluid Dynamics & Propulsion (see the full paper at the end of this report). The more general case is reported in the Second Computational Aeroacoustics Workshop on Benchmark Problems. In the presence of impedance boundaries existing numerical boundary conditions are not applicable. We have reported in the AIAA Journal, vol. 34, 1996, how a set of time-dependent impedance boundary conditions can be developed. Further we are able to establish that the proposed boundary conditions are numerically stable.

Imperfectly expanded supersonic jets inevitably contain a quasi-periodic shock cell structure in the jet plume. This is so even for bevelled nozzles (these nozzles have certain favorable aerodynamic and noise characteristics). The presence of the shock cell structure leads to the generation of screech tones and broadband shock associated noise. For bevelled nozzles the screech frequencies as functions of jet Mach number exhibit unusual characteristic band structures. Our study (AIAA paper 97–0143) provides an explanation for the phenomenon. Also in this work, a tone frequency prediction formula is established. Excellent agreement between the predicted tone frequencies and experimental measurements are found.

Currently we are focussing our work on direct numerical simulation of supersonic jet noise. A working computer code capable of simulating the axisymmetric screech mode of these jets has been developed. We anticipate we will have a good deal of new results to report in the next progress report.

Jet Noise: Since 1952[1]

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Abstract. Jet noise research was initiated by Sir James Lighthill in 1952. Since that time, the development of jet noise theory has followed a very tortuous path. This is, perhaps, not surprising for the understanding of jet noise is inherently tied to the understanding of turbulence in jet flows. Even now, our understanding of turbulence is still tenuous. In the fifties, turbulence was regarded as consisting of a random assortment of small eddies. As a result, the primary focus of jet noise research was to quantify the noise from fine-scale turbulence. This line of work persisted into the eighties. The discovery of large turbulence structures in free shear flows in the early seventies led some investigators to begin questioning the validity of the then established theories. Some went further to suggest that, for high speed jets, it was the large turbulence structures/instability waves of the flow that were responsible for the dominant part of jet mixing noise. Development of a quantitative theory of noise from large turbulence structures/instability waves took place during the next fifteen years. Precision instrumentation and facilities for jet noise measurements became available in the mid-eighties. This permitted a large bank of high-quality narrow band jet noise data to be gathered over the subsequent years. Recent analysis of these data has provided irrefutable evidence that jet noise, in fact, is made up of two basic components; one from the large turbulence structures/instability waves, the other from the fine-scale turbulence. This is true even for subsonic jets. In this paper, some of the crucial research results of the past 44 years, that form the basis of our present understanding of jet noise generation and propagation, are discussed.

1. Introduction

1952 was a very special year for jet noise research, for it was this year when Sir James Lighthill published the first of his two-part paper (Lighthill 1952, 1954) on aerodynamic sound in the *Proceedings of the Royal Society of London.* This paper has since been

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regarded as marking not only the beginning of jet noise research but also the birth of the research area 'Aeroacoustics'. Forty-four years has now elapsed. During that time, the development of jet noise theory has followed a very tortuous path. This is, perhaps, not surprising, for the understanding of jet noise is inherently tied to the understanding of the turbulence in jet flows. Even to-day, our understanding of turbulence is still tenuous.

The aim of this paper is to highlight the important developments of jet noise theory since Lighthill's original work. Special emphasis is given to the more recent findings that appear to offer a new perspective on jet noise characteristics and generation mechanisms.

2. The Fifties to the Seventies

2.1. The Acoustic Analogy Theory

To develop a jet noise theory, intuitively, it seems that the first thing to do is to identify the sources of noise. In his 1952 and 1954 papers, Lighthill tackled this problem by establishing the renowned Acoustic Analogy Theory. The basic idea is to cast the compressible equations of motion into a form representing the propagation of acoustic waves. Whatever terms that are left are then moved to the right side of the equation. The result is an inhomogeneous wave equation of the form,

$$\frac{\partial^2 \rho}{\partial t^2} - a_\infty^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{1}$$

where ρ is the density, a_∞ is the ambient sound speed and $T_{ij} = \rho v_i v_j + (p - a_\infty^2)\delta_{ij} - \tau_{ij}$ is known as the Lighthill stress tensor. v_i, p and τ_{ij} are the velocity, pressure and viscous stresses. δ_{ij} is the Kronecker delta. Within the framework of the Acoustic Analogy Theory, the following reasoning is advanced. By design, the left side of equation (1) represents acoustic wave propagation. It follows, therefore, that the right side of the equation must be the sources that generate the noise field. The source terms involve second spatial derivatives. They are referred to as quadrupoles.

During the fifties, the prevailing view of turbulence was that it consisted of a random assortment of small eddies. Thus, although no formal relationship between quadrupoles and small turbulent eddies was ever established, the implication was that the quadrupoles were related in some way to the small eddies. In present day terminology, we call the small eddies the fine-scale motion of the turbulent flows.

One important result that can be derived from the Acoustic Analogy Theory is the noise scaling laws. By using the Green's function of the wave equation, Lighthill obtained a formal solution of equation (1). On applying dimensional analysis to the formal solution, he established that the acoustic power radiated by a jet should vary as the eighth power of the jet velocity V_j. This became the celebrated V_j^8 Law.

2.2. The Source Convection Effect

In jets, the quadrupoles are convected downstream by the mean flow at a relatively high speed. That is, these are moving sources. It is easy to verify that moving sources tend to radiate more noise in the direction of motion. This is somewhat analogous to synchrotron radiation. Lighthill recognized immediately the significance of the source convection effect on the directivity of jet noise. At higher jet speed, this effect is more pronounced. This was investigated by Ffowcs-Williams (1963). By extending Lighthill's dimensional argument including the effect of source convection, Ffowcs-Williams found that for very high speed jets, the power of the radiated noise should vary as the third power of the jet velocity. That is $P \sim V_j^3$ where P is the acoustic power radiated. This and the Lighthill V_j^8 Law are the two most important results of the classical Acoustic Analogy Theory.

2.3. Effect of Refraction

Once sound is generated by the quadrupoles, the acoustic waves have to propagate through the jet flow to reach the far field. The mean flow of the jet is highly nonuniform. Thus, the radiated sound undergoes refraction in its passage through the jet.

Figure 1 illustrates the refraction of a ray of sound emitted by a point source S located in the mixing layer of a jet. To see why the ray bends outward, one needs only to consider the propagation of the wave front AB. The point A moves at a speed equal to the local sound speed plus the local flow velocity of the jet. So is the point B. If the jet is nearly isothermal, the speed of sound is the same at A and B. But the flow velocity at B is higher. As a result, as the wave front propagates, it becomes tilted as $A' B'$. Obviously, this effect of refraction is even more severe for hot jets. In this case, the sound speed at B is higher than that at A. One of the important consequences of mean flow refraction is that less sound can be radiated in the direction of jet flow. This creates a relatively quiet region around the jet axis commonly known as the 'cone of silence'. Experimentally, the presence of a cone of silence in which the noise intensity drops by more than 20 dB has been demonstrated by Atvars, Schubert and Ribner (1965).

2.4. Variants of the Acoustic Analogy Theory

In the years after the work of Lighthill, there have been many attempts to modify or improve the Acoustic Analogy Theory. Many of these efforts involved modifying the wave propagation operator on the left side of equation (1). One immediate result is that each new theory produces a slightly different set of noise source terms. Just as in the original theory, the noise source terms of all the modified theories are themselves unknowns. In other words, these theories are not self-contained. A good deal of turbulence information has to be input into the theory before a prediction can be made.

One good reason to modify the wave propagation operator is to account for the mean flow refraction effect. Lilley (1974) suggested that the correct wave propagation operator was the linearized Euler equations. In his paper, the linearization was performed over the mean flow of the jet. In this way, a version of the Acoustic Analogy Theory designed to include mean flow refraction effect was developed. Although it is not clear why the full Euler equations, which, in principle, not only account for the mean flow refraction effect, but also the nonlinear propagation effects, are not the more appropriate wave propagation operators. This must have been rejected because it would leave almost no noise source terms. Lilley's proposal received immediate acceptance. It has been followed by numerous subsequent investigators. This approach formed the main direction of jet noise research throughout the seventies and early eighties. Further discussions and references to other jet noise theories built around the Acoustic Analogy concept of Lighthill may be found in a review article by Lilley (1991).

3. The Seventies and the Eighties

Turbulence research took an abrupt turn in the early seventies when it was discovered independently by Crow and Champagne (1971), Brown and Roshko (1973) and Winant and Broward (1973) that turbulence in jets and free shear layers is made up of large turbulence structures as well as fine-scale turbulence. The large turbulence structures are somewhat more deterministic than the fine-scale turbulence motion. They dominate the dynamics and overall mixing processes of jets and free shear layers.

Soon after the discovery of the large turbulence structures, Crow and Champagne, who studied subsonic jets, and others began to propose that they were important jet noise sources. It took a few years of investigation before it became clear that the large turbulence structures are definitely important direct noise sources of supersonic jets. However, they are not as important for subsonic jets. For imperfectly expanded supersonic jets, a shock cell structure automatically develops in the jet plume. In the presence of the shock cells, the jets emits two additional components of noise. They are referred to as screech tones and broadband shock associated noise. In this paper, we will not consider these shock-related noise components. Our attention is confined to turbulent mixing noise alone.

3.1. Large Turbulence Structures Model

To predict the noise radiated by the large turbulence structures in jets, a mathematical description of these entities is necessary. The familiar turbulence modeling approach (see e.g., Speziale 1991) is inappropriate. It has no large structures in its formulation.

The first statistical description of the large turbulence structures in free shear flows in the form of a stochastic instability wave model was proposed by Tam and Chen (1979). This stochastic model approach has since been used and extended, by Plaschko (1981), Morris *et al.* (1990), Viswanathan and Morris (1992), Tam and Chen (1994) and others, in their mixing layers, jet flows and jet noise studies. The crucial observation that forms the basis of the model is that the turbulent jet flow spreads out very slowly. This means that the flow variables as well as the turbulence statistics change only very slowly in the downstream direction; i.e., the turbulence statistics are nearly constants locally. If, indeed, all the turbulence statistics are true constants, the flow is statistically stationary in time and in the flow direction. This implies that dynamically the turbulence fluctuations are locally in quasi-equilibrium. For a system in dynamical equilibrium, statistical mechanics theory suggests that the large-scale fluctuations of the system can be represented mathematically by a superposition of its normal modes. In the case of high-speed jets, the large-scale fluctuations are the large turbulence structures while the dominant normal modes are the instability wave modes of the mean flow. The turbulence statistics and mean flow of jets and mixing layers are well-predicted by the stochastic instability wave model. More detailed descriptions and references of large turbulence structure models can be found in the two recent reviews by Tam (1991, 1995).

3.2. Mach Wave Radiation

It may seem natural and reasonable to represent large turbulence structures by the instability waves of the mean flow. But it is well-known that instability wave solutions decay to zero exponentially away from a jet or mixing layer. In other words, there is no acoustic radiation associated with instability waves. Tam and Morris (1980) recognized this difficulty. They correctly pointed out that the difficulty arose because of the locally parallel flow assumption. This assumption is routinely used in classical hydrodynamic stability analysis. They showed that to determine sound radiation, a global solution of the entire instability wave propagation phenomenon along the jet column is necessary. Earlier, Saric and Nayfeh (1975), Crighton and Gaster (1976) and others had succeeded to account for the slow divergence of the mean flow by using a multiple-scales expansion method. Tam and Morris, however, demonstrated that the multiple-scales expansion instability wave solution is not uniformly valid outside the jet or the mixing layer. Thus, it is not surprising that the multiple-scales type solution still produces no acoustic radiation.

To construct a uniformly valid instability wave solution inside and outside the jet, Tam and Burton (1984) employed the method of matched asymptotic expansions. The inner solution is the multiple-scales instability wave solution. This solution is valid inside and in the neighborhood immediately outside the jet. The outer solution is essentially a solution of the acoustic wave equation taking into consideration the growth and decay of the instability wave amplitude in the flow direction. The outer solution is valid in the near field outside the jet all the way to the far field.

The matched asymptotic expansions solution reveals the basic mechanism by which sound in the form of Mach wave radiation is generated by the large turbulence structures/instability waves of the flow. The inner solution, which is the instability wave solution, is valid out to the near field immediately outside the jet. This means that the influence of the instability waves extends beyond the mixing layer of the jet flow. The change-over from instability wave solution to acoustic wave solution takes place near the edge of the jet and is contained in the outer solution. One may, therefore, regard the sound field of the Mach wave radiation to be generated near the edge of the jet flow. Physically an instability wave behaves like a wavy wall moving at a high speed in the downstream direction. When the wave speed is supersonic relative to the ambient sound speed, Mach waves are generated (see figure 2). This Mach wave radiation is highly directional. Since they are generated near the edge of the jet, they are, therefore, not subjected to the mean flow refraction effect. In other words, there is no cone of silence for large turbulence structures noise. The near field sound pressure level contours predicted by the matched asymptotic expansions solution of Tam and Burton for a Mach 2.1 moderate Reynolds number jet agree well with the measurements of Troutt and McLaughlin (1982). Recent work by Hixson, Shih and Mankbadi (1995) using direct numerical simulation clearly shows strong Mach wave radiation associated with the instability waves of the jet. Their computed sound pressure level contours are in good agreement with the analytical results of Tam and Burton and the experimental measurements of Troutt and McLaughlin.

Tam and Burton (1984) pointed out that the wavy wall analogy must be modified to account for the growth and decay of the instability wave as it propagates downstream. The growth and decay of the wave amplitude are important to the noise radiation process. For a fixed frequency wave of constant amplitude, the wave spectrum is discrete. With a single wavenumber there is only a single wave speed, so the Mach waves are radiated in a single direction. The growth and decay of the instability wave amplitude lead to a broadband wavenumber spectrum. This results in Mach wave radiation over large angular directions. Furthermore, a single frequency subsonic wave of constant amplitude would not radiate sound according to the Mach wave radiation mechanism. However, with growth and decay of the wave amplitude, a part of the broadband wavenumber spectrum could have supersonic phase velocity. These supersonic phase disturbances will lead to noise radiation. The single instability wave matched asymptotic expansions solution has recently been extended by Tam and Chen (1994) to include a broad frequency spectrum in a stochastic model theory of supersonic jet noise from the large turbulence structures. Their calculated noise directivities for Mach 2 jets at different temperatures are in good agreement with the measurements of Seiner *et al.* (1992).

The Mach wave radiation mechanism discussed above relies on the existence of supersonic phase components (relative to ambient sound speed). For highly supersonic jets, especially at high temperature, this is an extremely efficient noise generation process. But if the jet speed is subsonic, the efficiency is greatly reduced. Thus, for subsonic jets the fine-scale turbulence is probably the more dominant noise source, except in the cone of silence.

The Mach wave radiation by the large turbulence structures/instability waves of high-speed jets discussed above is not to be confused with eddy Mach wave radiation considered by a number of investigators in the sixties (e.g., Phillips (1960), Ffowcs-Williams and Maidanik (1965)). The eddy Mach waves proposal is that eddies moving supersonically relative to ambient sound speed would generate strong Mach wave radiation. However, a closer examination reveals that this is physically an untenable process. Eddies are, by definition, localized entities with limited range of influence spatially. While an eddy may be moving supersonically relative to the ambient gas outside the jet, it is definitely moving subsonically relative to the fluid in its immediate surroundings (eddies are convected downstream by the mean flow). That being the case, no Mach waves can be produced.

4. The Ninties

Until the late eighties, jet noise was, invariably, measured in $\frac{1}{3}$-octave bands. The experimental facilities available were generally incapable of producing high-speed jets at a high jet temperature. A $\frac{1}{3}$-octave band spectrum artificially enhances the importance of the high frequency noise component. This inevitably complicates the physical interpretation of the data. With improvements in instrumentation and experimental facilities beginning near the end of the eighties, jet noise data, for research purposes, has since been routinely processed in narrow bands. Also, very high temperature data, up to a temperature ratio of five, have been measured at the Jet Noise Laboratory of the NASA Langley Research Center. This new data offers the aeroacoustics community an unprecedented opportunity not only to study the characteristics of the noise of high-speed jets but also to test the validity of jet noise theories developed since Lighthill's original work.

With the coming of the nineties, a number of investigators (e.g., Tam and Chen (1994), Seiner and Krejsa (1989), Tam (1995) and others) suggested that turbulent mixing noise from supersonic jets actually consists of two distinct components. One component is produced by the large turbulence structures/instability waves of the jet flow in the form of Mach wave radiation. The other component is generated by the fine-scale turbulence of the jet. Figure 3 shows the noise directivities measured by Seiner *et al.* (1992) at selected Strouhal numbers of a Mach 2 perfectly expanded jet. It is clear in this figure that the dominant part of jet noise is radiated in the downstream direction in the sector with inlet angle, χ, larger than 125 deg. Tam and Chen (1994) showed that this highly directional noise component was generated by the large turbulence structures of the jet flow. They also observed that for inlet angle, χ, less than 110 deg (see figure 3) the jet noise radiation was almost uniform without a strongly preferred direction. They suggested that this low-level, almost uniform, background noise was generated by the fine-scale turbulence of the jet flow. In other words, the fine-scale turbulence noise is dominant over inlet angles smaller than 110 deg for the experiment of figure 3. By implication, in the intervening angular directions 110 deg $< \chi <$ 125 deg , both noise components are important.

4.1. The Similarity Spectra

In the mixing layer of a turbulent jet, there is no inherent geometrical length scale. Also, it is well-known that at high Reynolds number, viscosity is not a relevant parameter. Based on these observations, it is easy to see that there are no intrinsic length and time scales in the mixing layer in the core and the transition regions of a jet flow (see figure 1). As a result, the mean flow as well as all the turbulence statistics of the flow must exhibit self-similarity. Over the years, that the mean flow and turbulence statistics of a high-speed turbulent jet possesses a similarity profile has been well-verified experimentally. Since noise is generated by the turbulence of the jet, the above facts and reasonings strongly suggest that the noise spectra of the two independent turbulent mixing noise components should also exhibit similarity. In the absence of an intrinsic time or frequency scale, the frequency f must be scaled by f_L, the frequency at the peak of the large turbulence structures/instability waves noise spectrum or f_F, the frequency at the peak of the fine-scale turbulence noise spectrum.

The noise of a high-speed jet naturally depends on the jet operating parameters V_j (the fully expanded jet velocity), T_r (the reservoir temperature), D_j (the fully expanded jet diameter), T_∞ (the ambient temperature), χ (the direction of radiation), and r (the distance of the measurement point from the nozzle exit). On accounting for the contributions of the two independent noise components, the jet noise spectrum, S, may, therefore, be expressed in the following similarity form,

$$S = \left[AF\left(\frac{f}{f_L}\right) + BG\left(\frac{f}{f_F}\right)\right]\left(\frac{D_j}{r}\right)^2 \tag{2}$$

where $F\left(\frac{f}{f_L}\right)$ and $G\left(\frac{f}{f_F}\right)$ are the similarity spectra of the large turbulence structures noise and the fine-scale turbulence noise. These spectrum functions are normalized such that $F(1) = G(1) = 1$. In equation (2), A and B are the amplitudes of the independent spectra; they have the same dimensions as S. The amplitudes A and B and the peak frequencies f_L and f_F are functions of the jet operating parameters, $\frac{V_j}{a_\infty}$, $\frac{T_r}{T_\infty}$ and inlet angle χ.

To provide concrete experimental evidence that turbulent mixing noise from supersonic jets is, indeed, made up of two distinct components, Tam, Golebiowski and Seiner (1996) performed a careful analysis of all the jet noise spectral data (1,900 spectra in all) measured in the Jet Noise Laboratory of the NASA Langley Research Center. By means of this set of data, they were able to identify the similarity spectrum functions $F\left(\frac{f}{f_L}\right)$ and $G\left(\frac{f}{f_F}\right)$. Figure 4 shows the shapes of the empirically determined spectrum functions in dB scale versus $\log\left(\frac{f}{f_{\text{peak}}}\right)$, where $f_{\text{peak}} = f_L$ for the large turbulence structures noise and $f_{\text{peak}} = f_F$ for the fine-scale turbulence noise. (Note: In a $\log\left(\frac{f}{f_L}\right)$ plot, the graph of $10 \log F$ should fit the entire measured spectrum, if it is dominated by the large turbulence structures noise, when the peak of the graph is aligned with the peak of the measured spectrum. The same is true for the fine-scale turbulence noise.) The two spectrum shapes are distinctly different. The 10 log F function has a relatively sharp peak and drops off linearly as shown. The 10 log G function, on the other hand, consists of a very broad peak and rolls off extremely gradually.

Figure 5 shows typically how well the spectrum function 10 log F fits the measured data. In these examples, the jet Mach numbers, M_j, are 1.5 and 2.0. The jet to ambient temperature ratio increases from 1.11 to 4.89. The direction of radiation, χ, varies from 138 deg to 160 deg. As can be seen, there is good agreement in all the cases. Figure 6 illustrates typical comparisons between the spectrum function 10 log G and the measured data for perfectly expanded supersonic jets at $M_j = 1.5$ and 2.0 in directions for which the noise from fine-scale turbulence dominates. The jet to ambient temperature ratio covers the range of 0.98 to 4.89. The inlet angle χ, varies from 83.3 deg to 120 deg. Clearly, there is good agreement over the entire measured frequency range.

For angular directions neither too far upstream nor downstream both mixing noise components are important. Figure 7 shows examples of how the two noise spectra can be added together to reproduce the measured spectra. To obtain a good fit to the data, the amplitude functions A and B as well as the peak frequencies f_L and f_F are adjusted in each case. The separate contributions from each of the two noise components are shown in the figure.

The similarity spectra have been checked by comparing the entire data bank (1,900 spectra). They fit all the measured spectra over the entire range of Mach number and temperature ratio of the NASA Langley data.

4.2. Turbulent Mixing Noise from Subsonic Jets

Tam, Golebiowski and Seiner (1996), in their data analysis effort, found that M_j, the fully expanded jet Mach number, is not a useful parameter for characterizing turbulent mixing noise. But if M_j is unimportant, then the finding that turbulent mixing noise consists of two distinct components should be true regardless of M_j. In other words, it must be valid for supersonic jets ($M_j > 1$) as well as subsonic jets ($M_j < 1$).

It has been discussed before that the mean flow refraction effect creates a cone of silence for the fine-scale turbulence noise around the direction of the jet flow. On the other hand, as pointed out above, this is the principal direction of Mach wave radiation by the large turbulence structures. Thus, the noise spectrum and characteristics inside the cone of silence of the fine-scale turbulence noise of a subsonic jet should be those associated with

Mach wave radiation. They would, therefore, be distinctly different from noise radiated at angles outside the cone of silence. The experimental measurements of Lush (1971) and Ahuja (1973) prove that this is, indeed, the case. Inside the cone of silence, their measured noise intensity is not only not small; it is the highest. Moreover, the spectrum shape is distinctly different from those in directions at larger exhaust angles.

To provide concrete experimental evidence that turbulence mixing noise from subsonic jets, just as their supersonic counterparts, consists of two distinct components, comparisons between the measured data of Ahuja (1973) and the two similarity noise spectra have been carried out. Figure 8 shows the noise spectrum of a Mach 0.98 jet measured by Ahuja at $\frac{T_r}{T_\infty} = 1.0$ and $\chi = 160$ deg (inside the cone of silence). The smooth curve in this figure is the similarity spectrum 10 log F. It is evident that the curve is an excellent fit to the data providing irrefutable evidence that this is, in fact, the noise from the large turbulence structures of the subsonic jet. Figure 9 shows the corresponding measured noise spectrum at $\chi = 90$ deg. In this direction, the noise is from the fine-scale turbulence. Here the smooth curve is the similarity noise spectrum of 10 log G. The agreement between the data and the similarity noise spectrum is very good, giving strong support to the contention that the second component of turbulent mixing noise from subsonic jets is, as in the case of supersonic jets, fine-scale turbulence noise.

4.3. Noise Intensity Scaling Formulas

In decibel scale, the directional dependence of the large turbulence structures noise amplitude turns out to be quite simple. A typical case is given in figure 10. Here SPL is effectively $10 \log \left(\frac{A}{p_{\text{ref}}^2}\right) - 40$ dB ($p_{\text{ref}} = 2 \times 10^{-5} \frac{N}{m^2}$ is the reference pressure for the decibel scale). This quantity increases linearly with χ until a plateau is reached where the noise amplitude is practically constant. In the plateau region, the noise intensity is maximum. This maximum intensity is a function of the jet operating parameters, $\frac{V_j}{a_\infty}$ and $\frac{T_r}{T_\infty}$. Figure 11 shows a typical dependence of the amplitude of the fine-scale turbulence noise, $10 \log \left(\frac{B}{p_{\text{ref}}^2}\right) - 40$ dB, on directivity. Again, there is a linear increase with χ. The slope of the straightline relationship is a function of the jet operating parameters.

To obtain an idea of how the intensity of the large turbulence structures noise varies with the jet velocity and temperature, Tam, Golebiowski and Seiner concentrated their attention on $\chi = 160$ deg. Figure 12 is a plot of 10 log S versus $\log \frac{V_j}{a_\infty}$ with $\frac{T_r}{T_\infty}$ as a parameter at $\chi = 160$ deg and $\frac{r}{D_j} = 100$. One obvious feature of this figure is that data corresponds to the same jet to ambient temperature ratio align themselves along nearly parallel straight lines. A good fit to the entire set of data is (in dB per 1 Hz band)

$$10 \log\left(\frac{A}{p_{\text{ref}}^2}\right) = 75 + \frac{46}{\left(\frac{T_r}{T_\infty}\right)^{0.3}} + 10 \log\left(\frac{V_j}{a_\infty}\right)^n \tag{3}$$

where

$$n = 10.06 - 0.495\frac{T_r}{T_\infty}. \tag{4}$$

For cold jets, the velocity exponent n is approximately equal to 9.5. That is significantly larger than 8 or 3, the velocity exponents predicted by the Acoustic Analogy Theory. (Note: Although the velocity exponent is large, the noise power radiated by a jet expanding to its maximum velocity, i.e., into a vacuum, is still only a small fraction of its mechanical power.)

To assess the correct scaling formula for fine-scale turbulence noise Tam, Golebiowski and Seiner focussed on the noise radiated at $\chi = 90$ deg. In this direction, there is practically no large turbulence structures noise. Figure 13 shows a plot of 10 log S versus $\log\left(\frac{V_j}{a_\infty}\right)$ with $\frac{T_r}{T_\infty}$ as a parameter at $\frac{r}{D_j} = 100$. Again the data corresponding to different jet temperature ratio can be adequately approximated by straight lines. A good fit to the data is (in dB per 1 Hz band)

$$10 \log\left(\frac{B}{p_{\text{ref}}^2}\right) = 83.2 + \frac{19.3}{\left(\frac{T_r}{T_\infty}\right)^{0.62}} + 10 \log\left(\frac{V_j}{a_\infty}\right)^n \tag{5}$$

where

$$n = 6.4 + \frac{1.2}{\left(\frac{T_r}{T_\infty}\right)^{1.4}}. \tag{6}$$

According to this empirical fit, the velocity exponent is equal to 7.6 for cold jets. This is close to the well-known subsonic jet value of 8. However, the jet temperature exerts a fairly strong effect. At a jet temperature ratio of 2, the value of n drops to 6.85.

It is worthwhile to point out that in figures 12 and 13, the subsonic jet data of Ahuja (1973) forms a natural extension of the supersonic jet noise data from the NASA Langley Research Center. This reinforces the belief that the noise generation mechanisms are the same regardless whether the jet is subsonic or supersonic.

5. The Future

It is clear that considerable progress has been made in our understanding of jet mixing noise since the work of Lighthill. As yet we are still unable to predict, even with a good deal of empirical input, the noise spectra of the two basic components. Given our limited understanding of jet turbulence at the present time, the prospect of successfully formulating a first principle noise prediction theory is not very encouraging. However, it seems possible, perhaps, in a few years time, that a semi-empirical prediction theory based on the turbulence modeling approach could be developed. At the present time, the noise generation processes of the large turbulence structures of the jet flow appears to be reasonably well understood. It is hoped that future work will clarify how fine-scale turbulence produces noise.

Recently, computational aeroacoustics has made impressive advances. This new methodology should be able to assist in the simulation of large turbulence structures in jet flows. It would not be surprising, before too long, that jet noise from the large turbulence structures can be accurately predicted by direct numerical simulation. However, the same may not be true for fine-scale turbulence noise. To be able to resolve this type of turbulent motion accurately, exceedingly large computer memory is required. It seems that the requirement far exceeds what could become available in the near future. Thus, it is likely that jet noise, just as turbulence, will remain an unfinished business for quite some time to come.

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