/N-24
025 307

NASA Technical Memorandum 110328
U. S. Army Research Laboratory Technical Report 1340

A Statistical Simulation Approach to Safe Life Fatigue Analysis of Redundant Metallic Components

William T. Matthews
U.S. Army Research Laboratory
Vehicle Technology Center
Langley Research Center, Hampton, Virginia

Donald M. Neal
U.S. Army Research Laboratory
Materials Directorate
Watertown, Massachusetts

March 1997

National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23681-0001

A Statistical Simulation Approach To Safe Life Fatigue Analysis Of Redundant Metallic Components

William T. Matthews[1] and Donald M. Neal[2]

ABSTRACT

This paper introduces a dual active load path fail-safe fatigue design concept analyzed by Monte Carlo simulation. The concept utilizes the inherent fatigue life differences between selected pairs of components for an active dual path system, enhanced by a stress level bias in one component. The design is applied to a baseline design; a safe life fatigue problem studied in an American Helicopter Society(AHS) round robin. The dual active path design is compared with a two-element standby fail-safe system and the baseline design for life at specified reliability levels and weight. The sensitivity of life estimates for both the baseline and fail-safe designs was examined by considering normal and Weibull distribution laws and coefficient of variation levels.

Results showed that the biased dual path system lifetimes, for both the first element failure and residual life, were much greater than for standby systems. The sensitivity of the residual life-weight relationship was not excessive at reliability levels up to R=0.9999 and the weight penalty was small. The sensitivity of life estimates increases dramatically at higher reliability levels.

Key Words: reliability, fatigue life, metals

[1]U.S. Army Research Laboratory, Langley Research Center, Hampton, Virginia

[2]Formerly, U. S. Army Laboratory, Materials Directorate, Watertown, Massachusetts

INTRODUCTION

Recent studies by the authors [1,2,3] have identified excessive uncertainties related to high statistical reliability estimates in structural design. These studies indicate that the computation of high statistical reliability may have little or no association with actual engineering high reliability[3]. Excessive sensitivity is inherent in the application of a very high reliability criterion that is based on the extreme tails of probability density functions (PDFs). True PDFs are rarely known and there is no assurance that the simple PDFs commonly assumed, based on central data, properly represent rare events such as unusually high applied stresses or excessively weak components. Although rare, these events are the primary concern of structural integrity and these events may occur within the range of these high reliability estimates. Despite these potential difficulties, there is currently considerable interest in computing high reliability values and applying them in a statistically based structural design process[4,5].

This paper addresses two issues: reliability based structural fatigue design concepts for fail-safety and reliability prediction sensitivity as a function of the magnitude of structural design reliability goal. A fail-safe fatigue design provides protection against catastrophic failure in a qualitative sense. In addition, for a reliability based design, a fail-safe design may provide an adequate specified system reliability goal while the required element reliability level is reduced. A more stable and meaningful estimate of reliability may be obtained at the lower reliability associated with the element level of the design. This study will investigate the potential of dual active path and standby element fail-safe concepts and will assess the sensitivity of reliability estimates for fail-safe systems compared with the single element baseline system.

In this paper fail-safe design configuration concepts are applied to the uniaxially stressed structure (sketched in Figures la and lb). One system is a dual active load path structure, Figure 1a. A design concept is introduced using this configuration which takes advantage of the inherent variability in material fatigue strength of the dual elements that is enhanced by a biased stress condition where the stress in one element is slightly decreased relative to the other element. The stress reduction is obtained by increasing the element cross section and its corresponding weight. The analysis utilizes a Monte Carlo simulation method to represent statistically varying applied stress and material strength states in the fatigue life computations and to provide an array of lifetime values that express the structural reliability. The primary design objective is to provide a statistically based adequate fail-safe life in the surviving element, after the first element failure. This residual life is the critical attribute of a fail-safe structural system. Additional design objectives are to provide adequate system life while limiting the system weight increase.

The second fail-safe system, Figure 1b, is the "standby system." The standby system features a relatively lightweight "standby element" that is not loaded until the element called the "primary element" fails. This standby element life provides the system fail-safety. The standby design concept was described by Leonard Da Vinci for the design of flying machines[7], "In constructing wings one should make one cord to bear the strain and a loose one in the same position so that if the one breaks under strain the other is in position to serve the same function."

The fail-safe design concepts are applied to a baseline design that is derived from the fatigue life problem defined by the American Helicopter Society(AHS) Subcommittee on Fatigue and Damage Tolerance.[6] The goal in applying the simulation process to the simple configurations is to provide insight concerning the potential of these reliability based fail-safe fatigue design concepts and to identify design situations where reliability estimates can add value to the design process. The designs are evaluated with respect to whether adequate residual and total fatigue lives can be achieved without excessive weight penalties. Element lifetimes and the structural weight in the fail-safe systems are compared to the weight of the baseline single element AHS problem system.

The second major issue considered is the sensitivity of reliability based fatigue life estimates to the effect of the PDF assumptions. The effects of Weibull versus normal probability law assumptions and changes in the coefficient of variation upon reliability estimates will be examined. Sensitivity issues in both single element structures and fail-safe systems will be investigated. For single element systems the goal is to identify a generic reliability level at which reliability estimates may be obtained that are not excessively sensitive to uncertainties in computing statistical reliability. At that reliability level structural performance criteria of candidate designs, such as fatigue life, may be compared analytically in order to assess the relative merit of the designs. The uncertainty in reliability based life estimates of the fail-safe fatigue designs will be assessed to indicate whether meaningful stable estimates of reliability can be obtained at the reduced element reliability levels selected for the fail-safe designs.

In the following the application of the Monte Carlo simulation method, using normal and Weibull PDFs, to the safe life helicopter fatigue analysis is described. The method is applied to dual active path and standby two-element redundant systems. The biased stress design concept is introduced for the dual path system reliability based life analysis. Results for this design are examined to assess the benefit of redundant versus single element design in terms of system life, fail-safety and the associated weight penalty. The sensitivity of lifetime estimates will be examined by comparing results for normal and Weibull PDFs for both single element and redundant systems and for changes in the coefficient of variation of the normal PDF.

FATIGUE LIFE COMPUTATION

The following safe life fatigue computation procedures were those used in a round robin study conducted by the AHS, Reference 6. The form of the material S-N curve is assumed to be,

$$N = C\left(S^* - S_E\right)^D \tag{1}$$

where N = number cycles to failure, S_E = fatigue limit strength or "endurance limit" (the fatigue strength, under constant amplitude cycling where the minimum stress is equal to zero, for which the number of cycles to failure is very large, or approaches infinity), S^* = maximum cyclic stress range where the minimum stress in the cycle is equal to zero, and C and D are parameters from a regression least squares analysis.

In order to apply the S-N curve in Equation 1 using the assumed operating stress spectrum means and ranges in the AHS problem where the minimum cyclic stress does not equal zero, the following relation for S^* is required,

$$S^* = \frac{\alpha \cdot S_u \cdot S_L}{S_u - \alpha \cdot S_m + \alpha \cdot S_L/2} \, . \tag{2}$$

This equation represents a form of the Goodman correction factor used in Reference 6, which converts a specified spectrum mean stress and stress range to an equivalent stress range, with a minimum stress equal to zero, which causes equal fatigue damage. S_u represents the ultimate strength of the material. S_m and S_L, tabulated in Table 1, are the mean stress and the stress range, respectively, obtained from a rainflow count of the standard Felix 28 spectrum, Reference 6. The α value is a scaling parameter for the spectrum stress values S_L and S_m which provides a representation of an effective stress severity, averaged over the lifetime of a component. This parameter can provide changes in the applied spectrum stress in order to account for differences in mission usage and component loading due to differences in pilot technique, weather, weight, etc. Let the fatigue life NP represent the number spectrum repetitions, or passes, prior to the component failure. Then from Miner's Law [8],

$$NP = \frac{1}{DF} \quad , \tag{3}$$

where the damage fraction per spectrum pass (DF) is,

$$DF = \sum_{k=1}^{NK} \frac{n(k)}{N(k)} \quad . \tag{4}$$

The $N(k)$ are the number of cycles at failure for the material from Equation 1 at the effective stress range (S^*) from Equation 2. The $n(k)$ represent the number of cycles for each of the discrete k stress combinations in Table 1 and NK represents the total number of discrete spectrum stress combinations from Table 1, (NK= 50). The range and mean pairs represent the fatigue damaging content of the Felix 28 standard spectrum for fixed or semi-fixed helicopter blade configurations. The discrete level representation is convenient for application of Miner's Damage Law. The Felix 28 spectrum is specified in relative values where the maximum stress equals 100. In Table 1 the stresses are absolute values, with the maximum stress equal to 70.44 KSI.

MONTE CARLO SIMULATION PROCESS

A key feature of this dual path design and analysis methodology is the application of a simulation method of analysis to characterize the fail-safety of the design. The fail-safety is derived from the statistical differences in fatigue strength of elements that are selected in pairs from the total population. The Monte Carlo Method is convenient and it assists in understanding fail-safe design behavior by producing data sets where individual extreme values that govern reliability can be examined directly to identify reliability behavior patterns.

The following procedure was applied in order to obtain a relationship between reliability (R) and life for a single element. In the simulation process, the cycles to failure, N from Equation 1, as a function of the discrete stress levels k in Table 1 is written as,

$$N_{ij}(k) = C\left(S^*(j) - S_E(i)\right)^D, \tag{5}$$

where i and j identify the individual simulations of fatigue limit strength and stress. The $S^*(j)$ values from Equation 2 are determined from,

$$S^*(j) = \frac{\alpha(j)\cdot S_u \cdot S_L(k)}{S_u - \alpha(j)\cdot S_m(k) + \alpha(j)\cdot S_L(k)/2}, \tag{6}$$

where an M random set of normally distributed α values are obtained from,

$$\alpha(j) = \overline{\alpha}\left(1 + V_S \cdot Z_j\right), j = 1,2,\ldots,M. \tag{7}$$

The Z_j values are obtained from the cumulative distribution function (CDF) of the standard normal distribution for a random set of values selected from a uniform (0,1) function. The V_S value is an assumed coefficient of variation (CV) and $\overline{\alpha}$ is the statistical mean of the stress scaling factor. The $S_E(i)$ values are obtained independent of the α as,

$$S_E(i) = \overline{S}_E(1 + V_S \cdot Z_i), i = 1,2,\ldots,M, \tag{8}$$

where $\overline{S}_E$ is the statistical mean of the fatigue strength S_E and the Z_i values are obtained from another random sampling of the standard normal CDF.

The above simulation process can also be performed using a Weibull PDF. The same mean and CV's used in the normal PDF application can be converted to Weibull parameters. For example, the strength value (S_E) can be obtained from a Weibull PDF,

$$f(S_E) = \frac{\beta}{\eta}\cdot\left(\frac{S_E}{\eta}\right)^{\beta-1}\cdot\exp\left[-\left(\frac{S_E}{\eta}\right)^{\beta}\right]. \tag{9}$$

The shape parameter (β) and the scale parameter (η) are determined from [9],

$$\beta = \frac{1.27}{CV} - .56, \tag{10}$$

and,

$$\eta = \frac{\bar{S}_E}{\Gamma\left(\frac{1}{\beta}+1\right)} \quad , \tag{11}$$

where Γ = Gamma function. The Weibull distributed random set of S_E values are then obtained from the Weibull cumulative distribution function, F (S_E) as,

$$S_E(i) = \eta \cdot \left\{-\log\left[1-U(i)\right]\right\}^{(1/\beta)}, i = 1,2,3\ldots M, \tag{12}$$

where F (S_E) are taken equal to U(i), a random set of values selected from a uniform (0,l) function.

The number of passes (*NP)* lifetime estimate, Equation 3, is obtained from the following application of the spectrum stress data $\{S_L(k)\}_1^{NK}, \{S_m(k)\}_1^{NK}, \{n(k)\}_1^{NK}$ in Table 1, where *NK* is the number of discrete spectrum stress combinations. The damage fraction value for random $S_E(i)$ and α (j) selections, can be determined from Equation 4 and written as,

$$DF_{ij} = \sum_{k=1}^{NK} \frac{n(k)}{N_{ij}(k)}, \tag{13}$$

where $n(k)$ represents the number of cycles for each of the discrete k stress combinations in Table 1 and *NK* represents the total number of discrete spectrum stress values from Table 1. The number of passes associated with failure for the randomly selected S_E (i) and α (j) values are then computed from,

$$NP_{ij} = 1 / DF_{ij} \ . \tag{14}$$

A corresponding number of helicopter operational flights to failure can then be obtained from,

$$FLT_{ij} = NP_{ij} \cdot FSC \ , \tag{15}$$

where *FSC* is the scale factor of 140 flights per spectrum pass, Reference 6. From the *M* random combinations of S_E and α values introduced in the above fatigue life model, a *M* random set of *FLT* values is obtained. *FLT* values are then ordered from the smallest to largest and assigned corresponding ordered integer values h, that is, $FLT(h)$, h = 1,2,3, ... M, which defines the array of *ordered FLT* values. *FLT(1)* and *FLT(M)* would represent the smallest and largest *FLT* flight to failure values. The corresponding reliability (R) values associated with $FLT(h)$ are obtained from,

$$R(h) = 1 - h/M, \ h = 1,2,3,.... M . \tag{16}$$

A computed life $FLT\ (R)$ is obtained for a specified R value by taking the ordered $FLT(h)$ where h determined from Equation 16. A reliability value $R(1)$ and $FLT(1)$ flights represents the highest reliability and least number of flights to failure from the M simulations.

DETERMINING LIFETIME IN DUAL PATH SYSTEM ELEMENTS

The dual path system concept assumes that each element is subjected to one half of the system total fatigue load. The order of element failure is unknown for either a biased or unbiased system due to the statistical variability of the material fatigue strength of the elements. In the following, the element with the higher fatigue endurance limit strength (S_E), called the "stronger element," accumulates damage at a slower rate than the weaker element. When the weaker element fails, the stronger element is then required to sustain the total stress until its failure, (see Figure la). The remaining life of the stronger element is now determined by applying the total load and by accounting for the damage that occurred during the original loading prior to the weaker element failure.

In applying the simulation process, identification of each element with respect to a randomly selected paired set of α and S_E values, is as follows: let element El be represented by the first of the two selected random pairs of $\alpha(j)$ and $S_E(i)$ values and element E2 by the second random pair selected. Note, in order for both elements to experience similar stresses the second selected α value must be the same as the first. For example, the damage fraction involving element El would be obtained from using $\alpha(j{=}a)$ and $S_E(i{=}a)$ in the analysis and the E2 element damage fraction would involve $\alpha(j{=}a)$ and $S_E(i{=}b)$ where a and b are integer values identifying particular simulations. The stronger element may be either El or E2. If the values obtained from Equation 13 are such that,

$$DF_{a,a} < DF_{b,a} , \tag{17}$$

then El would represent the stronger element. If this inequality is reversed, then E2 would be the stronger element.

The damage fraction and the lifetime, $FLT(R)$, of the weaker element are determined directly from the methodology described in the Fatigue Life Computation section. The total lifetime of the stronger element includes both its $FLT(R)$ value at failure of the weaker element and its residual life, $FLT^{r}(R)$, the additional number of flights after first element failure. In the residual life calculation the total load is applied to the damaged element. If $DF_{a,a} \leq DF_{b,a}$ then the residual lifetime $FLR^{r}(R)$, of the element El is obtained from,

$$FLT^{r}(R) = \frac{1 - DF_{a,a}/DF_{b,a}}{DF^{*}} \times FCS, \tag{18}$$

where DF^{*} represents the computed damage fraction of element El subjected to the total system load (double the amount of original load) and the numerator is the unconsumed portion of the damage summation. For each element the above analysis is repeated M times where the value for α chosen for the first element is also applied to the corresponding second element along with an independently chosen S_E value.

BIASED STRESS LEVEL DESIGN IN DUAL PATH SYSTEMS

The $FLT^{r}(R)$ values can be increased if the difference between damage fractions $DF_{a,a}$ and $DF_{b,a}$, Equation 17, is increased. The maximum benefit of a biased, reduced stress that reduces the damage fraction of the stronger element cannot be achieved in all members of a fielded dual path system population since the fatigue strength of specific fielded elements is unknown. The material in all elements is nominally the same. However a greater difference can be achieved on a statistical basis. Random strength simulations for individual elements are independent of the biased stress conditions. Thus for some members of the dual path system population the damage fraction of the lower stressed element may be greater than the damage fraction of its companion higher stressed element.

The stress bias in an element is implemented by defining an internal stress scaling factor α_I as,

$$\alpha_I = \frac{\alpha}{F_S}\ , \tag{19}$$

where F_S , a stress modification factor, has a value greater than 1 and α is the spectrum stress scaling factor in the single element in the AHS problem. The model assumes that the load in each element remains at $P/2$ while the stresses in one element are decreased by the factor $1/F_S$. The design configuration is such that the element loads are independent of their stiffness. In the analysis, the internal stress scaling factor F_S can be varied to account for changes in the element cross section in the unixially stressed element. The cross section of the uniformly loaded element is proportional to F_S. The weight of a uniform element is proportional to its cross section.

The lifetime for the first element to fail, the larger *DF* in Equation 17, is *FLT1*. The *FLT1* for a specified reliability at first failure, *R1*, is obtained by taking the ordered value *FLT1* (*h*) associated with the value of $h = h1$ from Equation 16 with $R(h) = R1$, where $h = 1,2,3,...M$. The lifetime $FLT^{r}(R)$ is obtained for a specified reliability (*R2*), for the second element failure, by taking the ordered value *FLT2* (*h*) associated with the value of $h = h2$ in Equation 16, where $h = 1,2,3,...h1$. When obtaining *h2*, only those simulations for life $FLT^{r}(R)$ for which the first element has failed at a life less than the life associated with $R = R1$ are used, rather than the total set of *M* simulations. Therefore the largest value of *h* for the *R2* simulations is *h1*. In this study the value of *R2* is always taken equal to *R1*.

DETERMINING LIFETIME IN STANDBY SYSTEM ELEMENTS

The standby system concept in Figure lb, the primary element Ep is designed to support the total system load until this primary element fails. After the Ep failure, the standby element E_S is subjected to the total load for the period of residual life specified for fail-safety. The standby element has not been subjected to any loading prior to the failure of the primary element. In computing the number of flights, *FLT(R)*, for the primary element Ep and standby element E_S , a separate analysis is applied for each element using the method outlined in Fatigue Life Computation section. The weights and cross sections of elements Ep and E_S for potentially viable standby designs will differ significantly from the elements in the dual path system. The element stresses can be obtained from Equation 19.

RELIABILITY FOR TWO ELEMENTS

The equation for system reliability (*RS*), where system failure requires failure of both elements is the following,

$$RS = 1 - P_f(\mathrm{E1}) \cdot P_f(\mathrm{E2}), \qquad (20)$$

where the probability of failure, $P_f = 1 - R(h)$, and the $R(h)$ is obtained from Equation 16. In the dual path system, the probability of failure of the second element to fail is computed from simulations of size $h1$, that is only including elements that are coupled with the set of first elements which fail.

RESULTS

In this section, results are obtained for fail-safe systems where the baseline design is derived by simplifying the geometry of the AHS safe life fatigue round robin problem, Reference 6. A thin AISI 4340 steel plate is uniformly loaded by the Felix 28 spectrum. The material fatigue S-N curve coefficients are those for the ASTD curve in Reference 6: $C = 3.5\text{x}\ 10^6$, $D = -1.47164$ and $\overline{S}_E = 54.5$ KSI. The spectrum scaling factor $\overline{\alpha} = .70$. Two CV levels of .07 and .10 are assumed for both α and S_E. The number of simulations M, used in the analysis, was more than one million.

Initially the simulation process was applied to the single element case in order to compare reliability and lifetime estimates from normal and Weibull PDF applications. A comparison of the two PDFs is shown in Figure 2 for $S_E = 54.5$ and a $CV = .07$. The reliability versus lifetime results show a substantial difference in life estimates for high reliability, Figure 3. For example, with "six nines" reliability, lifetime ranges from 500 to 10,000 flights. This result indicates that there is a potential problem in characterizing structural integrity on the basis of high reliability since the selection of the PDF to best represent the extremes of behavior is uncertain. Examples of possible errors caused by small differences in PDFs were shown in Reference 1. However, a characterization of the relative merit of candidate designs can be obtained at lower reliability levels where differences due to PDF uncertainties are minimal. For the single element design an assessment of relative merit could be made at R=.9.

Biased dual path and standby systems lifetime results from simulations using normal PDFs are shown as a function of element reliability in Table 2 and of system reliability in Figures 4 to 7. The relative system weight (W) is with respect to the single element baseline system weight of 1.00. The dual path system results for W= 1.00 are for unbiased equal element systems. The results shown are the average of 5 independent reruns of the M simulations. In view of the reliability sensitivity results in Figure 3, a system with each element reliability specified as 0.99 was chosen for investigation to assess whether results could be obtained without excessive sensitivity to probability laws and their parameters. The system reliability is equal to 0.9999, from Equation 20. In the dual path and standby systems both elements must fail in order to have complete system failure. It is assumed that the failure of the first element is always detected.

Dual Path System

A comparison of the single element baseline system life of 29540 flights for system R equal to .9999 with the equal weight (W=1.00) dual path system of 63224 flights (Figure 4), demonstrates the potential advantage of the dual path system. This advantage is obtained by determining life on the basis of an element R= .99 while a system reliability of 0.9999 is provided by the surviving element. However, for the unbiased system the second element life, determined from Equation 18, is only 2 flights. This residual life is probably not adequate for a practical fail-safe design. Increasing the weight of one element in the biased system is intended to improve residual life and a relatively large percentage increase in life is obtained, Figure 5. The life of the first element to fail increases moderately when $W > 1.06$, Figure 4. When $W > 1.06$ the biased stress level is sufficiently reduced so that the first element R= .99 life is influenced only by the failure of higher stressed elements and life is not improved by increasing the system weight (which further decreases the stress in the lower stressed element). For $W < 1.06$, the first element R= .99 life is influenced by failure of lower stressed elements. Therefore increasing the weight and further decreasing the stress of lower stressed elements does improve the first element R= .99 life up to W= 1.06.

If the weight of one of the elements is increased so that the system W=1.06, the estimated residual life is increased to 10, Figure 5, which is a 500 percent increase in the life of the second element. The life of the first element to fail also has increased from 63224 to 75264 flights. The benefit of the unequally stressed system on residual life increases as the system weight increases. For a system weight of W=1.09, the expected residual life for an unequally stressed system is 24 times greater than the expected life of the equal element baseline system and 6 times greater than the W= 1.06 system, Table 2 and Figure 5. Also shown in Table 2 and Figures 4 and 5 is a result for an unbiased equal element dual path design for W= 1.09. The biased unequal element system has a much shorter life to first element failure but the residual life, which is the critical attribute of fail-safety, is 6 times longer than for the equal element system. This result demonstrates the advantage of the biased element fail-safe fatigue design. A specific residual flight requirement is not adopted in this study, rather a range of residual lives is presented. These results suggest that the unequal element stress design may have the potential of providing an improved fail-safe design concept.

Standby System

Simulation results for the standby concept assuming normal PDFs are shown in Table 2 and Figures 6 and 7. The total system weight represents the combined weight of both the primary and standby elements. The element sizes for results shown were obtained from a series of design trials. The standby element was designed to provide adequate static strength and residual life similar to those for dual path systems, while limiting its weight. The primary element was sized to avoid excessive system weight. As a result, the primary element weight is significantly less than baseline system weight and its life is much shorter. In the results shown, the primary element weight is held constant and the increased system weight arises totally from the increases in standby element weights. This approach was used in order to obtain the maximum benefit of improved fail-safety provided by the standby element. The standby elements are very small compared to the primary elements and they also weigh much less than the nominal weight of the dual path elements.

For example, with a total relative weight, W= 1.00, the weights of the E_P and E_S elements are 0.79 and 0.21 respectively. The life of E_P is 11200 flights, which is approximately 18 percent of the life at first failure in the dual path system. The standby element life (E_S) was nearly equal to zero. The limited amount of E_P and E_S life is the result of the nature of the standby concept which requires that each element act alone in a system where a weight constraint, W= 1.00, is imposed. If both elements weigh the same in a W= 1.00 standby system the life of each element would be an extremely low value of 49 flights. Since each element acts alone and its size is approximately the size of the surviving element in a dual path system, the element lives are on the order of the relatively short residual life in a dual path system. The comparison of results for dual path and standby systems, Table 2 and Figure 6, show that the life to first element failure in a dual path system is almost 7 times greater than for the standby system primary element. If the weight of the standby element is increased, the improvement in residual life of the standby system is much less than the corresponding dual path systems, Figure 7.

Fail-Safe Design Reliability Prediction Sensitivity

In order to examine the sensitivity of reliability estimates the simulation results from application of the Weibull PDF in generating S_E values are shown in Table 3. The differences in element life of redundant systems resulting from using normal versus Weibull PDFs can be obtained by comparing Tables 2 and 3 and are shown in Figures 8 and 9.

In the dual path system first element failure there is a reduction in number of flights for the Weibull PDF compared with the normal PDF. However, the dual path system life of 46973 flights under the Weibull assumption is still much greater than the equivalent life for the baseline system of 7430 flights, Table 3. The dual path second element life for the Weibull PDF is also reduced for the heavier systems, but there is a slight increase in the residual life for the lightweight systems, shown in Figure 9. As W increases above 1.06 the smaller residual life for the Weibull assumption is apparently related to the absolute value of the life to failure of the first element. For W increasing above 1.06, the ratio of the residual life to the first element life for both the normal and Weibull assumption appear to be approaching similar values. The results suggest that as W increases the residual life is dominated by the deterministic effect of the differences between the biased stress levels and that differences associated with probabilistic strength assumptions have a diminishing effect. Since first element life for the Weibull assumption is approximately 3/4 times the life for the normal assumption, the residual life for the Weibull assumption appears to approach similar ratios as W increases, reaching 2/3 of the residual life for the normal assumption at $W= 1.27$. For $W< 1.06$, the slightly larger residual life for the Weibull PDF is apparently caused by the greater dispersion of the "heavier" lower tail of the Weibull function relative to the normal as shown in Figure 2. In the lower weight region, since the biased stress effect is small, a major contribution to residual life is the statistical fatigue strength difference between the dual path elements. Since the $R= .99$ life is determined by simulations drawn from the lower tail region of the assumed distribution, the greater Weibull dispersion apparently leads to longer residual lives than for the normal PDF.

For the standby system with the Weibull PDF, the first element life was reduced substantially and the residual life was reduced slightly over the system weight range compared with the normal PDF. The general reduction in life for a Weibull PDF is similar to the results for a single element system, Figure 3.

Simulation results for normal PDFs with the CV increased to 10 percent are shown in Table 4 for element $R = .99$ and in Figures 10 and 11 for system $R = .9999$. Lifetime estimates for the first element to fail are reduced by approximately a factor of 2 when compared to the 7 percent CV results, Figure 10. The effect of this CV change is substantially less than for prior analyses, Reference 3, of the single element design at $R= .9_6$, where life reduction factors of 5 to 10 occurred. The effect of this CV difference on residual life changes considerably as the weight is increased, Figure 11. For the lighter weight systems the difference in residual life for a CV increase from 7 to 10 percent is small. The same residual life of 10 flights is obtained, Figure 11, for $W= 1.06$, $CV= .07$ as for the slightly heavier $W= 1.09$, $CV= .10$.

These results indicate that for a dual path system with a specified element reliability of $R= .99$, the uncertainty of reliability estimates may be tolerable. Residual life estimates are the most important aspect of fail-safe designs with respect to structural integrity. These residual life estimates were not excessively sensitive to the assumed differences in PDF laws or coefficient of variation values over a range of potentially viable system weights. The estimates of first element failure, which essentially governs system life, are more sensitive to the assumed variations. However, conservative choices of PDF and CV seem to be practical since even with conservative estimates the life of a dual path system is much longer than for the equivalent dual path system.

DISCUSSION

The results show that the biased dual active load path system can provide a satisfactory reliability based fail-safe fatigue design with a rather small system weight penalty in the range of 6 to 9 %. Reliability based element design appears to be possible for element reliability of .99, when a suitably conservative residual life is specified. The reliability estimates appear to be adequately robust for a relatively high system reliability of .9999. The results for the redundant dual path system are for an axially loaded configuration in which the load carrying capability of an element is independent of the stiffness of the element. Thus the stress level in an element can be independently reduced. These results have been obtained for a fail-safe system assuming that a warning of failure is provided.

It is beyond the scope of this initial study to attempt to identify the number of flights that should be obtained analytically in order to demonstrate a specified residual life. Particular attention should be given to the issue of appropriate spectrum stress levels in a small number of flights relative to the total spectrum length of 140 flights. Other sources of uncertainty should be considered, including those cited by the authors in Reference 2, such as uncertainties in the S-N curve parameters and the Miner's Law failure criteria.

CONCLUSIONS

Redundant fail-safe design concepts were applied to a reliability based fatigue problem that was defined for a prior American Helicopter Society Fatigue and Damage Tolerance Subcommittee round robin study. A plain uniform steel specimen is loaded by the standard Felix 28 helicopter spectrum. Dual active path and standby redundant systems were analyzed for a specified reliability of 0.9999 using Monte Carlo simulations. The sensitivity of life estimates for a specified reliability was examined by considering normal and Weibull PDFs and coefficient of variation (*CV*) values of 0.07 and 0.10 for the normal PDF. The following conclusions were drawn from this investigation.

(1) Fail-safe fatigue designs

A. Biased dual active path system

- The results also show that the biased dual active load path system has the potential of providing a fail-safe system with adequate residual life.
- The weight penalty for a dual path system with adequate fail-safety is 6 to 9 %.
- The system fatigue life of a dual path system is 6 times longer than the life for a single element baseline design of equal reliability and weight.

B. Standby system

- The standby fail-safe design concept does not appear to be advantageous for weight sensitive airborne structures.
- The fatigue life of a standby design is substantially less than an equal weight dual active path system for both residual life and system total fatigue life.
- A standby system design with adequate strength and fail-safety requires a large weight penalty.

(2) Reliability Prediction Sensitivity

- For the single element baseline system high reliability life estimates are very sensitive to differences in assumed probability density laws and their parameters.
- A characterization of the relative merit of the structural integrity of reliability based single element designs, insensitive to probability assumptions, could be made at $R = 0.9$.
- Residual life of the dual path system over a moderate range of system weights shows little sensitivity to the choice of normal versus Weibull PDF and only a mild sensitivity to CV changes.
- The sensitivity of reliability estimates of a dual path fail-safe system to probability density function uncertainties at a system reliability equal to 0.9999 are much less than for very high reliability estimates for systems that are not fail-safe.

(3) In summary, an active dual load path redundant system with stress biased elements may provide a satisfactory reliability based fail-safe fatigue design without excessive reliability uncertainties.

REFERENCES

1. Neal, D.M., Matthews, W.T. and Vangel, M.G., *Model Sensitivity In Stress-Strength Reliability Computations,* U.S. Army Materials Technology Laboratory, MTL TR 91-3, January 1991.

2. Neal, D.M., Matthews, W.T., Vangel, M.G. and Rudalevige, T., A *Sensitivity Analysis On Component Reliability From Fatigue Life Computations,* U.S. Army Materials Technology Laboratory MTL TR 92-5, February 1992.

3. Matthews, W.T. and Neal, D.M., *Assessment of Helicopter Component Statistical Reliability Computations,* U. S. Army Material Technology Laboratory, MTL TR 92 71, September 1992.

4. Arden, R.W. and Immen, F.H., *U.S Army Requirements For Fatigue Integrity,* Proceedings of American Helicopter Society National Technical Specialists Meeting On Advanced Rotorcraft Structures, Williamsburg, VA, October 1990.

5. Gray, P.M. and Riskalla, M.G. *Probabilistic Design Of Advanced Composite Structures,* DOD/NASA/FAA Conference on Fibrous Composites in Structural Design, Lake Tahoe, NV, November, 1991.

6. Everett, R.A., Bartlett, F.D., and Elber, W., *Probabilistic Fatigue Methodology For Six Nines Reliability,* AVSCOM Technical Report 90-B-009, NASA Technical Memorandum 102757, December 1990.

7. Lewis, W.H., *Introduction to, Prevention of Structural* Failures, American Society For Metals, 1978.

8. Miner, M.A., *Cumulative Damage in Fatigue,* Journal Of Applied Mechanics, v. 127 1945, P A150 -- A164

9. Freudenthal, A.M. and Gumbel, E.J., *Distribution Functions For The Prediction Of Fatigue Life And Fatigue Strength,* International Conference On Fatigue Of Metals, The Institution Of Mechanical Engineers, London, 1956.

Table 1: Felix 28 range and mean pair cycle count derived by rainflow analysis

k	S_L	S_m	$n(k)$	k	S_L	S_m	$n(k)$
1	2.80	25.59	354	26	42.63	29.21	207
2	2.80	32.83	354	27	42.63	36.45	1274
3	6.42	29.21	416	28	46.25	21.97	274
4	10.04	29.21	609	29	46.25	25.59	6239
5	10.04	36.45	1228	30	46.25	29.21	4274
6	10.04	40.07	810	31	46.25	40.07	604
7	13.66	36.45	2	32	49.87	3.86	268
8	17.28	18.35	140	33	49.87	25.59	956
9	17.28	32.83	78	34	49.87	29.21	2179
10	20.91	32.83	2061	35	53.49	25.59	2
11	20.91	36.45	90	36	53.49	29.21	116
12	24.53	-7.00	140	37	57.12	25.59	5
13	24.53	18.35	140	38	57.12	29.21	185
14	24.53	36.45	2040	39	60.74	29.21	25
15	28.15	29.21	833	40	64.36	25.59	7
16	31.77	25.59	346	41	64.36	29.21	8
17	35.39	25.59	7904	42	64.36	32.83	75
18	35.39	29.21	56	43	67.98	29.21	9
19	35.39	32.83	71072	44	71.60	29.21	16
20	35.39	43.69	2529	45	75.22	25.59	7
21	39.01	21.97	3014	46	78.84	18.35	5
22	39.01	25.59	42825	47	78.84	25.59	1
23	39.01	29.21	6393	48	82.46	21.97	128
24	39.01	43.69	252	49	82.46	29.21	16
25	42.63	25.59	480	50	89.70	25.59	8

Table 2: Relative System Weight and Element Life in Flights
Normal PDFs, CV = .07

Relative System Weight	Element Life (Flights)					
	Biased Dual Path System				Standby System	
	First		Second		Primary (E_P)	Standby (E_S)
	.99	.9	.99	.9	.99	.99
1.00	63224 #	147280#	2#	19#	11200	→ 0
1.03	71442	181860	4	30	11200	3
1.06	75264	201600	10	78	11200	4
1.09 *	75316	203920	48	245	11200	6
1.13	75410	209160	146	696	11200	9
1.17	75684	209860	481	1539	11200	13
1.21	75026	209960	1226	4029	11200	21
1.27	76306	210098	3079	12065	11200	38

Unbiased Equal Element System

Reference Lifetimes :
Single Element Baseline System (Relative Weight = 1.00): R = .9999, 29540 Flights
: R = .99 , 75888 Flights
* Equal Element System (Relative Weight = 1.09), R = .99 : E1 = 135,000, E2 = 8

Table 3: Relative System Weight and Element Life in Flights
Weibull PDF For Strength, Normal PDF For Stress, CVs = .07

Relative System Weight	Element Life (Flights)					
	Biased Dual Path System				Standby System	
	First		Second		Primary (E_P)	Standby (E_S)
	.99	.9	.99	.9	.99	.99
1.00	46970#	134204#	3#	26#	7670	→ 0
1.03	52808	165760	5	40	7670	3
1.06	56154	186620	12	82	7670	4
1.09	59724	195580	21	200	7670	6
1.13	59751	203000	71	693	7670	8
1.17	59800	205660	225	1497	7670	13
1.21	59820	205691	757	4277	7670	20
1.27	59835	205782	2068	12853	7670	35

Unbiased Equal Element System

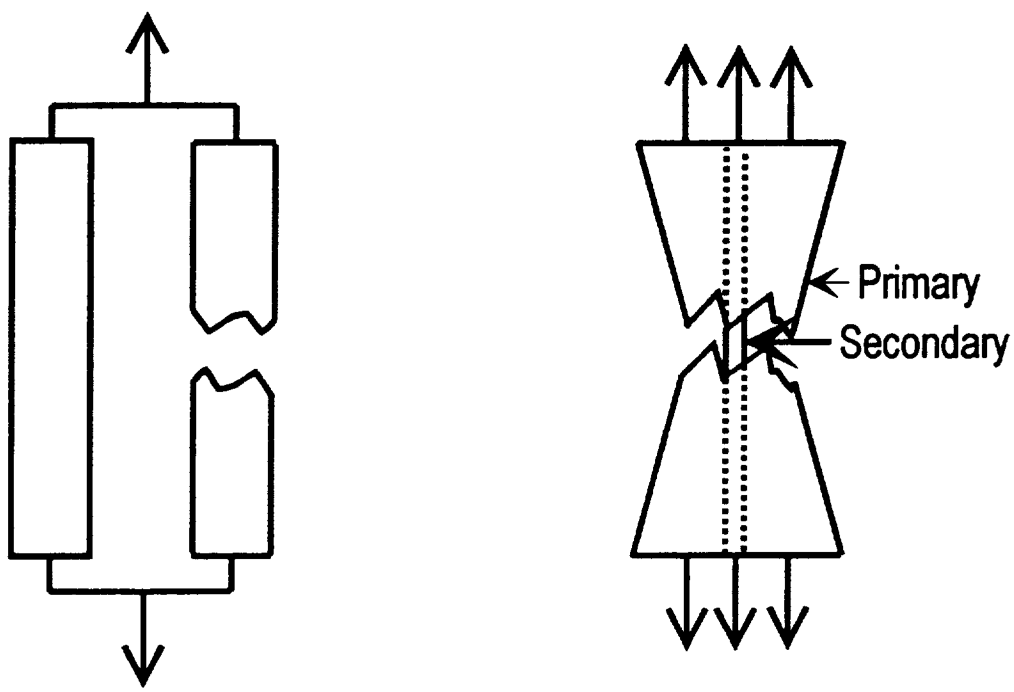
Single Element Baseline System Life (Relative Weight = 1.00) : R = .9999, 7430 Flights
R = .99 , 61250 Flights

Table 4: Biased Dual Path System Relative Weight and Life in Flights, Normal PDFs , CV = .10

Relative System Weight	Element Life (Flights), R = .99	
	First	Second
1.00	30352 #	2#
1.03	34552	3
1.06	36736	4
1.09	37674	10
1.13	37828	26
1.17	37874	74
1.21	37884	217
1.27	37889	722

Unbiased Equal Element System

Single Element Baseline System Life (Relative Weight = 1.00) : R = .9999, 4690 Flights
R = .99 , 37520 Flights



(a) Dual Path System

(b) Standby System

Figure 1. Fail - safe design concepts

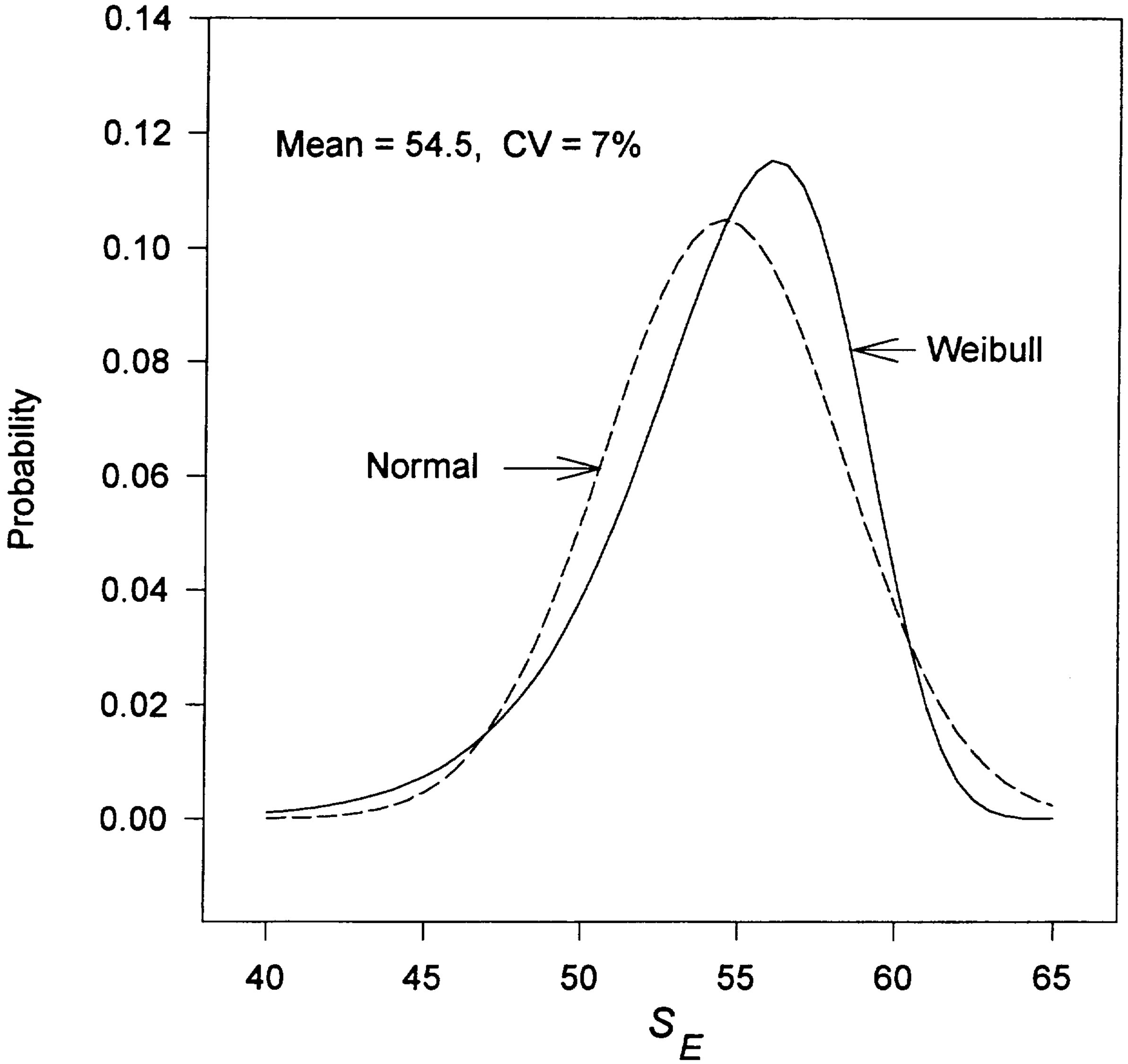


Figure 2. Normal vs. Weibull PDF for fatigue limit strength

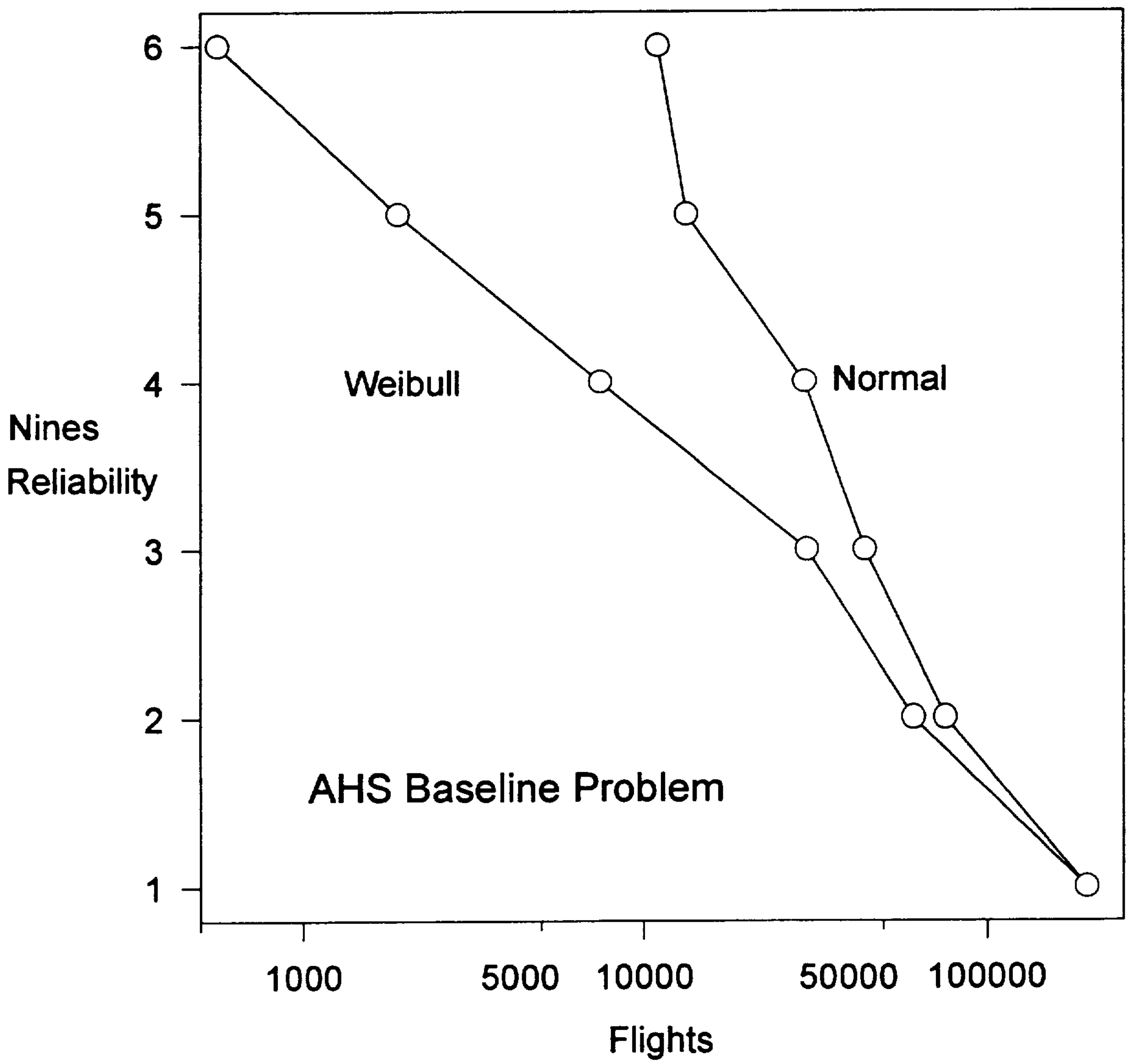


Figure 3. Reliability vs. life for normal and Weibull PDFs

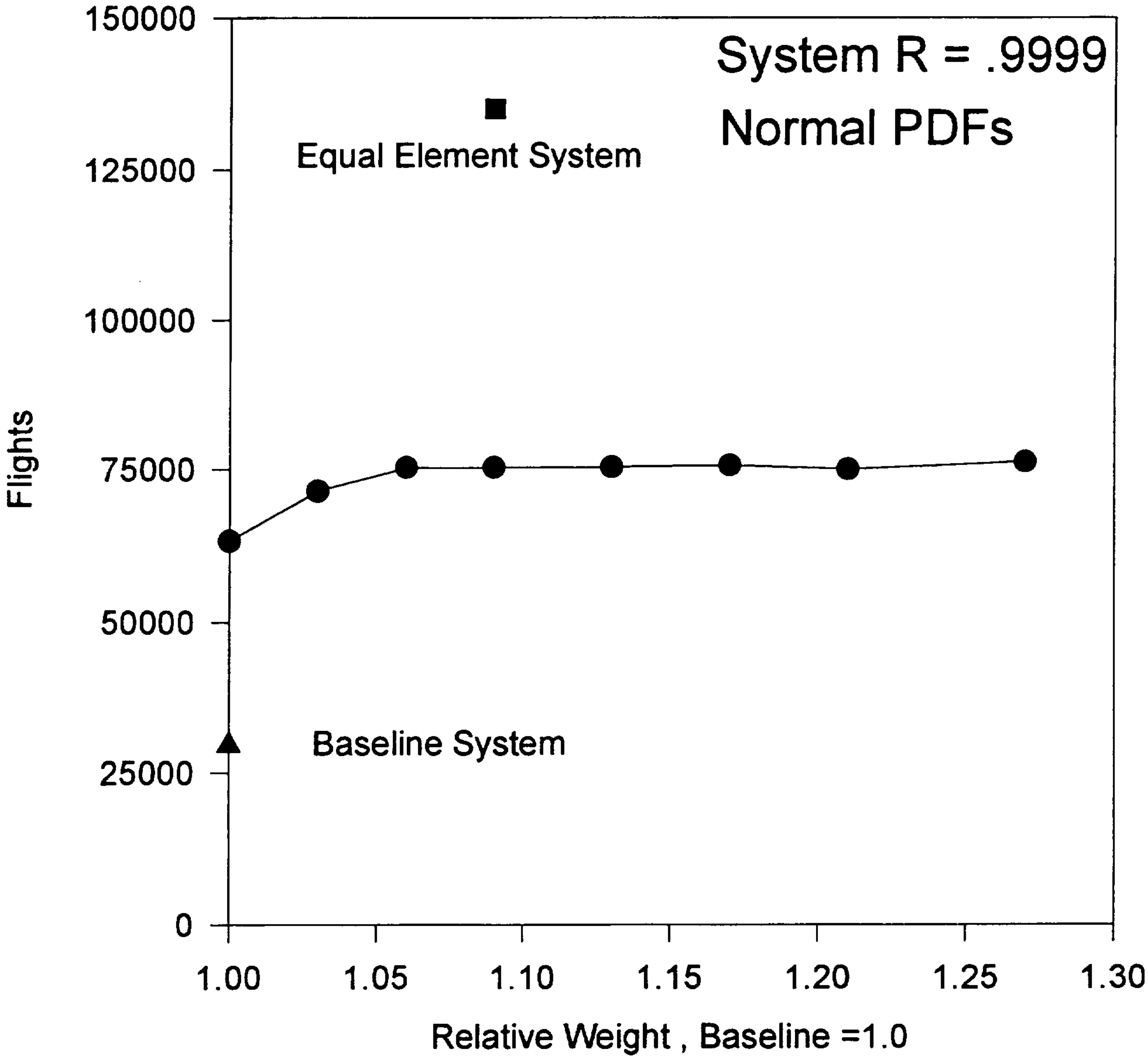


Figure 4. Biased dual path system first element life vs. relative weight

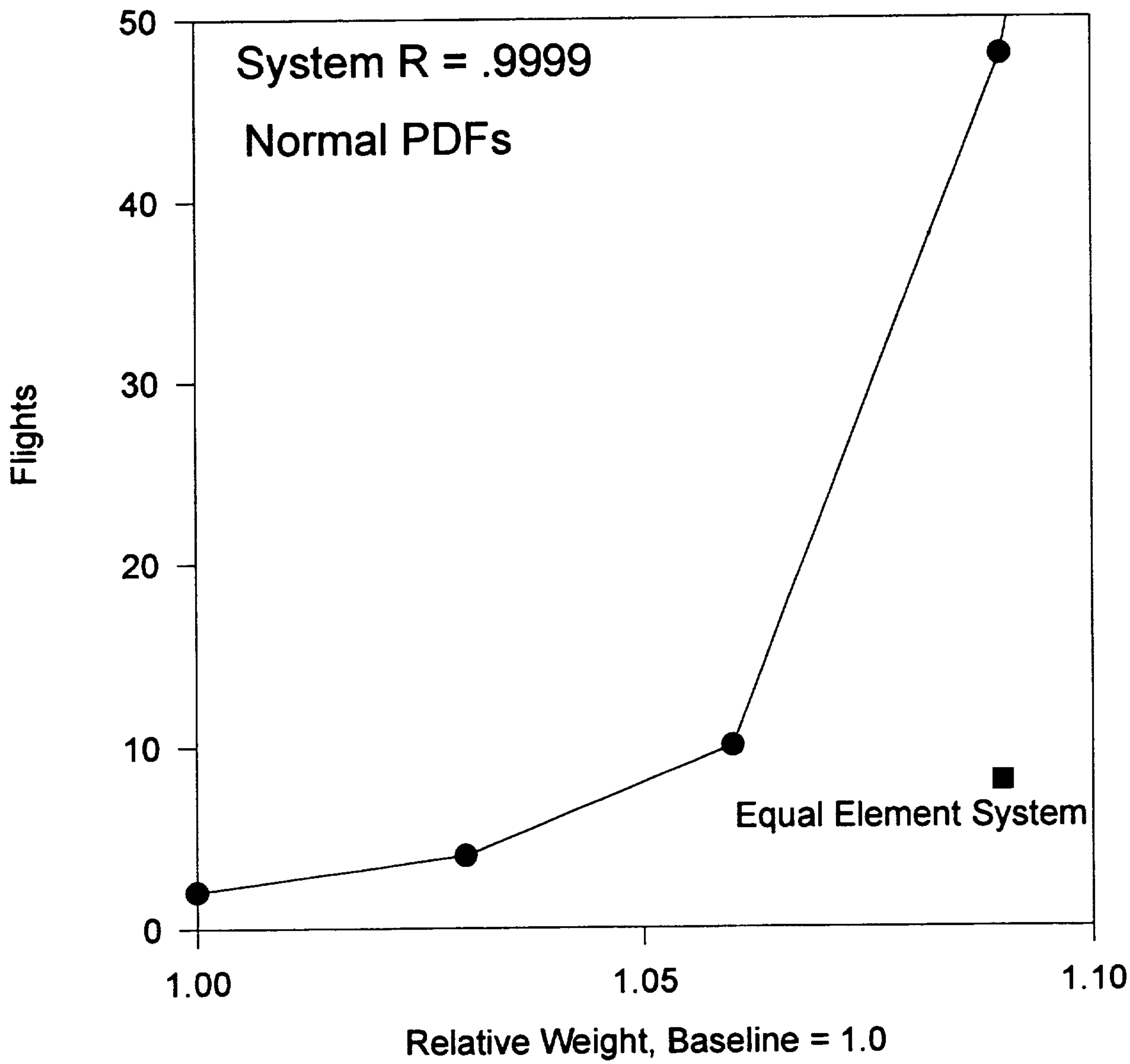


Figure 5. Biased dual path system residual life vs. relative weight

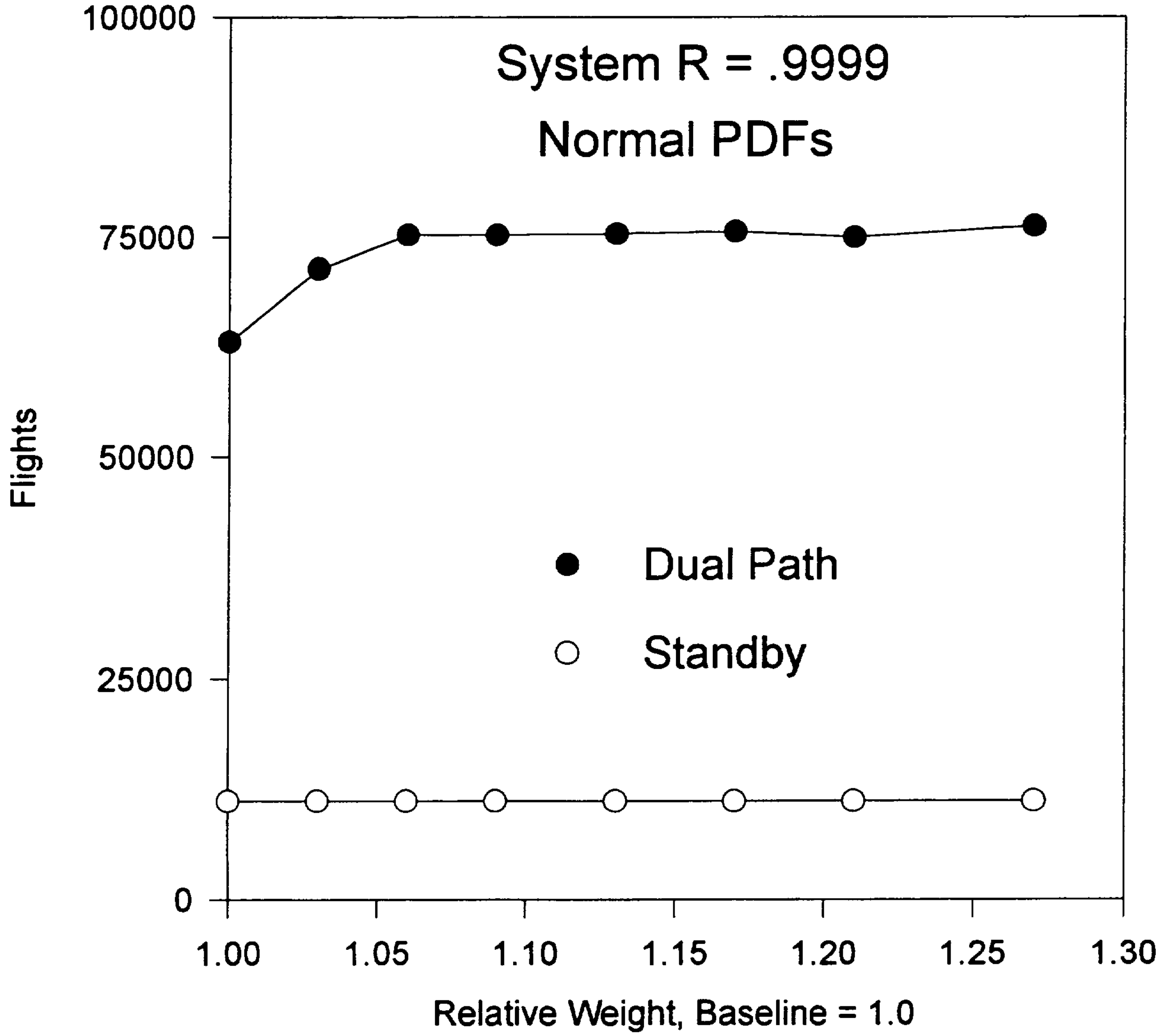


Figure 6. Biased dual path and standby system first element life vs. relative weight

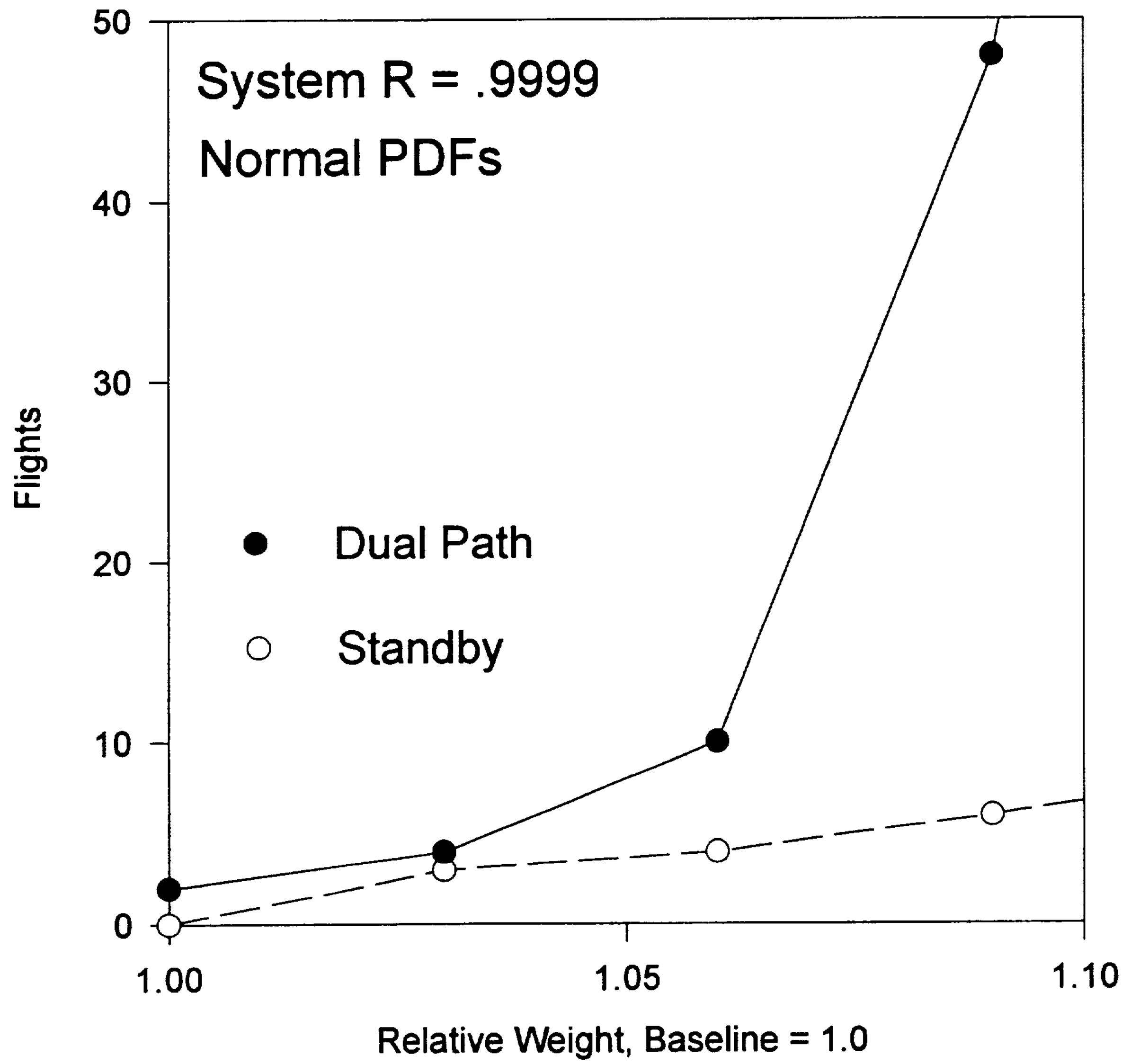


Figure 7. Biased dual path and standby system residual life vs. relative weight

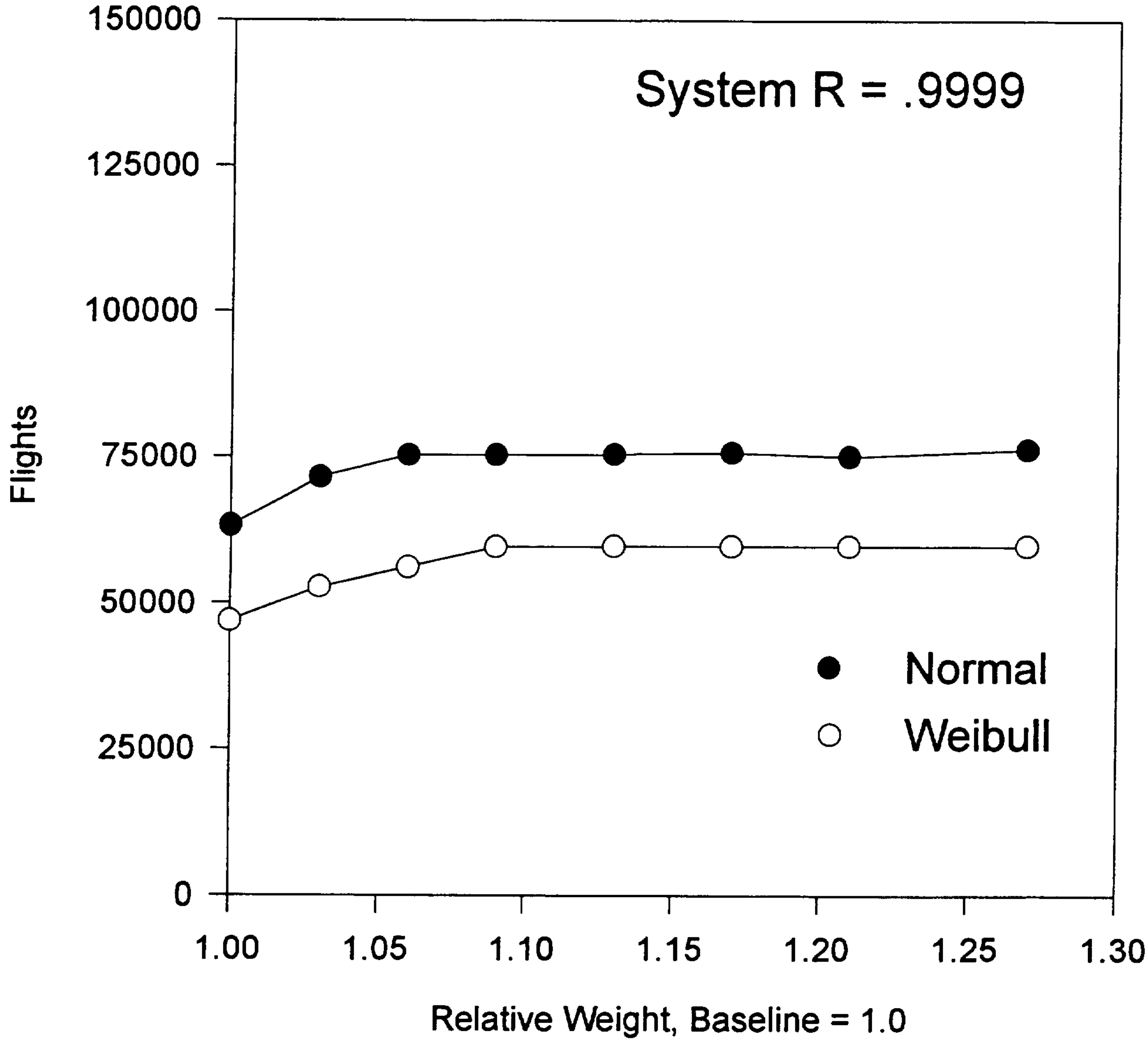


Figure 8. Biased dual path first element life vs. relative weight, normal and Weibull

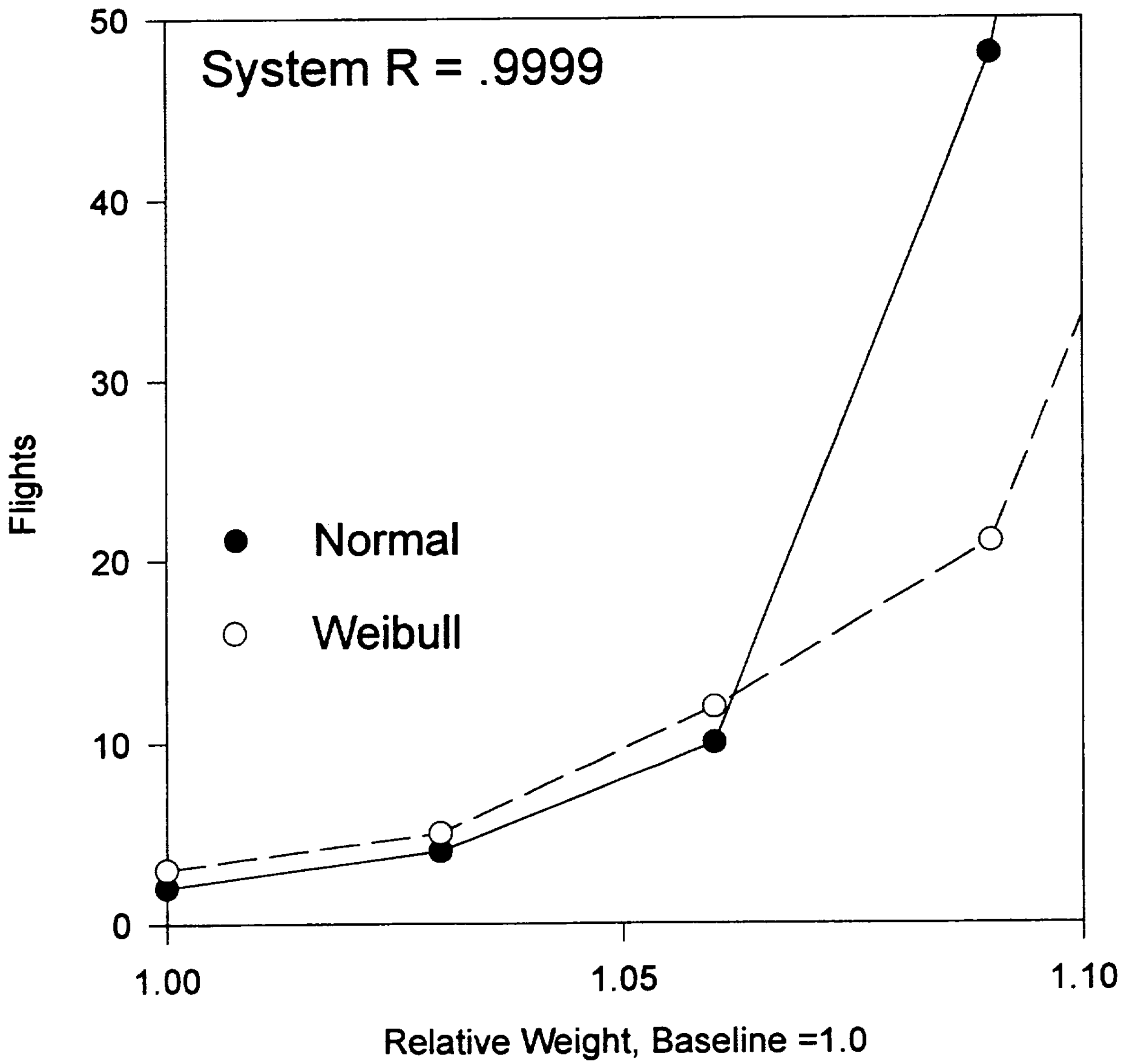


Figure 9. Biased dual path residual life vs. relative weight, normal and Weibull

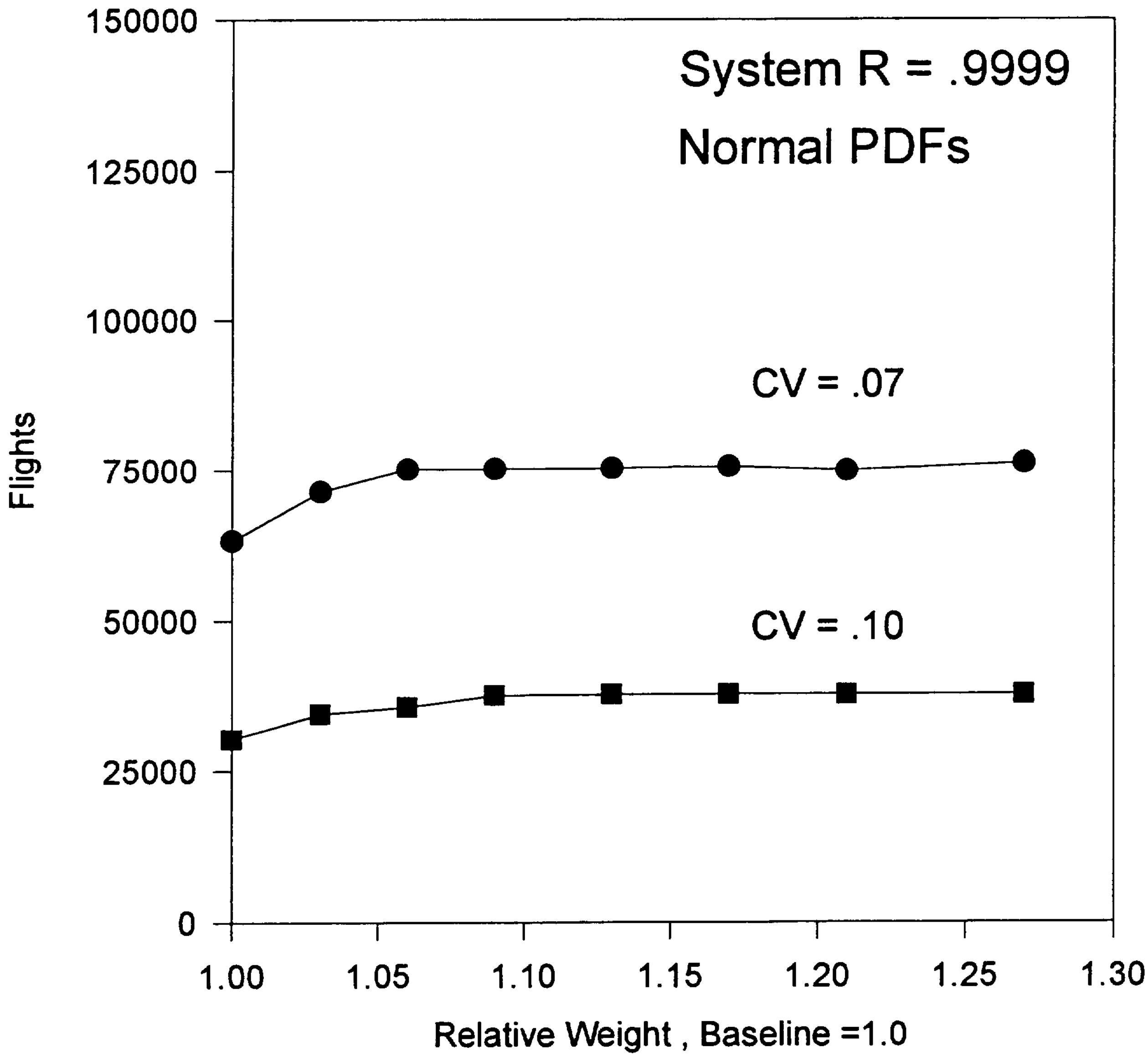


Figure 10. Biased dual path first element life vs. relative weight for CV equal to .07 and .10

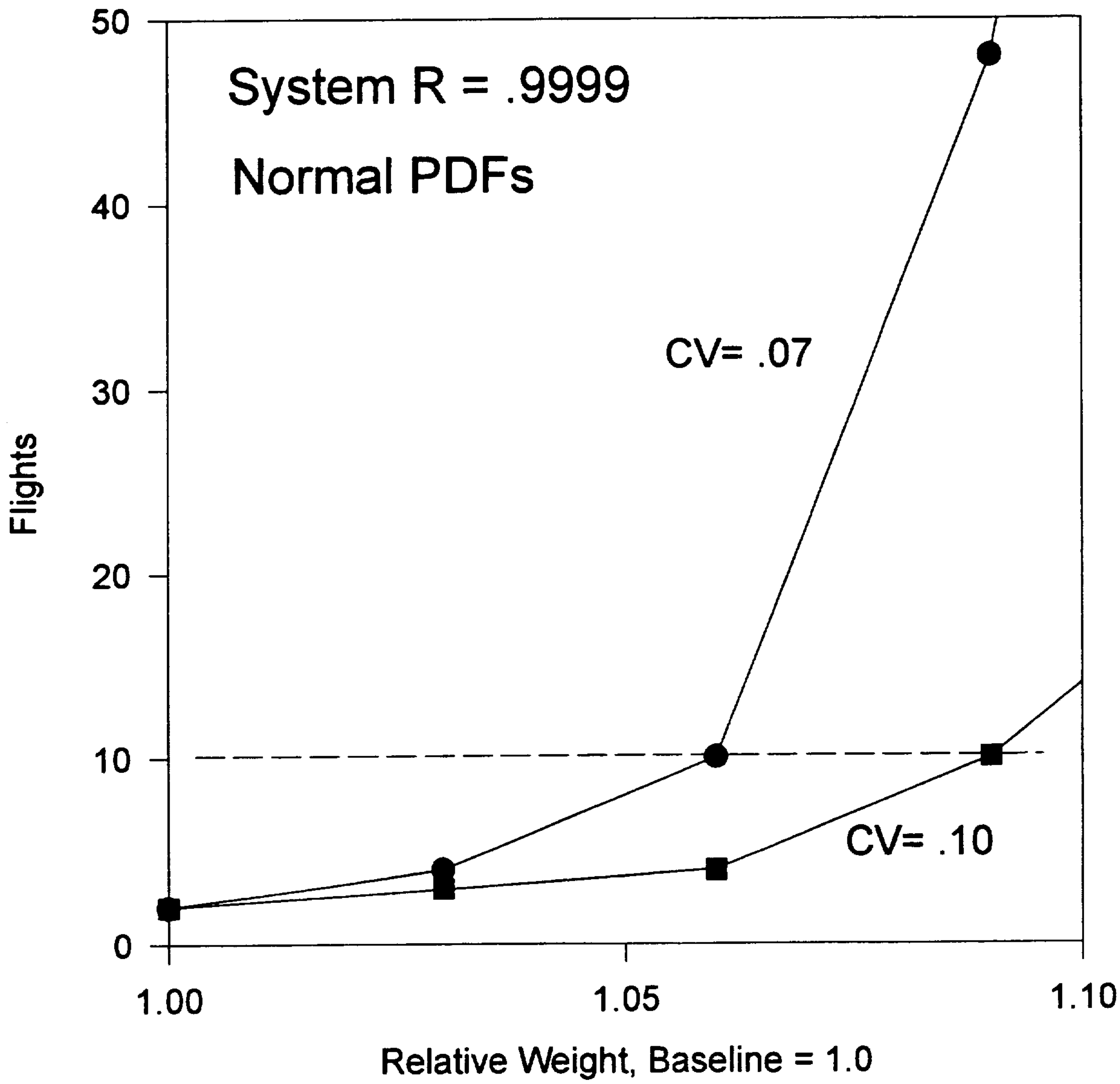


Figure 11. Biased dual path residual life vs. relative weight for CV equal to .07 and .10

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
March 1997

3. REPORT TYPE AND DATES COVERED
Technical Memorandum

4. TITLE AND SUBTITLE
A Statistical Simulation Approach to Safe Life Fatigue Analaysis of Redundant Metallic Components

5. FUNDING NUMBERS
538-02-10-01

6. AUTHOR(S)
William T. Matthews (Army - Retired)
Donald M. Neal (Formerly, U.S. Army Research Laboratory)

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Langley Research Center
Hampton, VA 23681-0001

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
NASA TM-110328
ARL-TR-1340

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Unclassified-Unlimited
Subject Category 24

Availability: NASA CASI, (301)621-0390

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This paper introduces a dual active load path fail-safe fatigue design concept analyzed by Monte Carlo simulation. The concept utilizes the inherent fatigue life differences between selected pairs of components for an active dual path system, enhanced by a stress level bias in one component. The design is applied to a baseline design; a safe life fatigue problem studied in an Amercian Helicopter Society (AHS) round robin. The dual active path design is compared with a two-element standby fail-safe system and the baseline design for life at specified reliability levels and weight. The sensitivity of life estimates for both the baseline and fail-safe designs was examined by considering normal and Weibull distribution laws and coefficient of variation levels.

Results showed that the biased dual path system lifetimes, for both the first element failure and residual life, were much greater than for standby systems. The sensitivity of the residual life-weight relationship was not excessive at reliability levels up to R=0.9999 and the weight penalty was small. The sensitivity of life estimates increases dramatically at higher reliability levels.

14. SUBJECT TERMS
reliability, fatigue life, metals

15. NUMBER OF PAGES
33

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UNLIMITED