model that generates primordial isocurvature baryon (PIB) fluctuations. They conclude that the mean skewness vanishes and that its cosmic variance is smaller than in adiabatic power-law models by orders of magnitude. Assuming that this result applies to the full three-point function, as they suggest, this would imply a level of cosmic variance far less than our current level of instrument noise. We consider the implications of our current results for this model in the discussion below.

In Paper I we computed the three-point correlation function of the first-year DMR maps and found evidence for non-vanishing three-point correlations in our sky, albeit with only moderate statistical significance. The amplitude of the correlations was consistent with the level of cosmic variance expected from Gaussian initial fluctuations, and thus could not be deemed intrinsic. In this Letter we compute the three-point function of the 2 year DMR sky maps. Because of the very significant reduction of noise in the two-year data (by a factor of $\sim 2^{3/2} = 2.8$ relative to the first-year data), we now detect three-point correlations in our sky with high signal-to-noise ratio. However, the amplitude still appears to be consistent with the level of cosmic variance expected from adiabatic, Gaussian power-law models. To minimize cosmic variance, we have applied a series of high-pass filters to the data to remove the low order-multipole moments. In particular, we subtract best-fit multipole expansions up to order $\ell_{\rm fit} = 9$ to see whether the observed signal persists above the diminishing cosmic variance. In most cases the residual signal is statistically significant relative to the instrument noise, but does not exceed the expected level of cosmic variance. Thus we cannot unambiguously interpret the observed three-point correlations as evidence for intrinsic non-Gaussian fluctuations in the CMB.

2. METHOD AND RESULTS

The DMR experiment has produced two independent microwave maps (A and B) at each of three frequencies (31.5, 53, and 90 GHz). The results presented here are based only on the relatively sensitive 53 and 90 GHz maps. For this analysis we employ an extended Galactic plane cut to exclude some Galactic features that extend beyond the straight 20° cut previvously considered, and which could be a source of positive skewness in the data. The extended cut additionally excludes a region at positive latitude near $l = 0°$ (the Ophiuchus complex) and a region at negative latitude near $l = 180°$ (the Orion complex). The resulting map has 3885 remaining pixels. We also subtract a best-fit multipole expansion from the data (see below) before evaluating the three-point function. In § 2.1 we compute the equilateral configuration of C_3, $\theta_1 = \theta_2 = \theta_3$, and in § 2.2 we compute the "pseudocollapsed" configuration, $\theta_1 \approx \theta_2$, $\theta_3 \approx 0$, as described in Paper I. Our notation is as follows: latin indices i, j, and k refer to sky pixels, while greek indices α and β refer to bins of angular separation, of width 2°6.

2.1. *The Equilateral Case*

We define the equilateral three-point correlation function to be

$$C_3^{(e)} = \frac{\sum_{i,j,k} w_i w_j w_k T_i T_j T_k}{\sum_{i,j,k} w_i w_j w_k},$$

where the sum is restricted to pixel triples (i, j, k) for which all three pixel separations reside in a single angular separation bin, T_i is the observed temperature in pixel i after multipole subtraction, and w_i is the statistical weight of pixel i. To minimize the effects of cosmic variance, we adopt uniform pixel weights, $w_i = 1$. We compute two forms of the autocorrelation function: the 53 GHz autocorrelation, with $T = 0.5(T_{53A} \pm T_{53B})$, and the 53 + 90 GHz autocorrelation, with $T = 0.5(0.65T_{53A} + 0.35T_{90A}) \pm 0.5(0.65T_{53B} + 0.35T_{90B})$. The weights in the 53 + 90 GHz combination were chosen to maximize sensitivity while retaining equal weight in the A and B channels to permit difference map analysis.

The equilateral three-point functions obtained from the 53 + 90 GHz maps are presented in Figure 1. The data are presented from top to bottom with multipole expansions of increasingly higher order subtracted from the maps before processing. The top panels ($\ell_{\rm min} = 2$) include all moments from quadrupole up, the middle panels ($\ell_{\rm min} = 4$) include all moments from hexadecapole up, and so forth. The error bars on the data points represent the rms uncertainty due to instrument noise as determined by 2000 Monte Carlo simulations, while the gray band represents the rms (i.e., 1 σ) scatter due to a superposition of instrument noise and sky signal, the latter modeled as a scale-invariant ($n = 1$) power law with Gaussian amplitudes and random phases (Bond & Efstathiou 1987), with a mean quadrupole normalization of 20 μK (Górski et al. 1994). Further details on the simulation methodology may be found in Paper I.

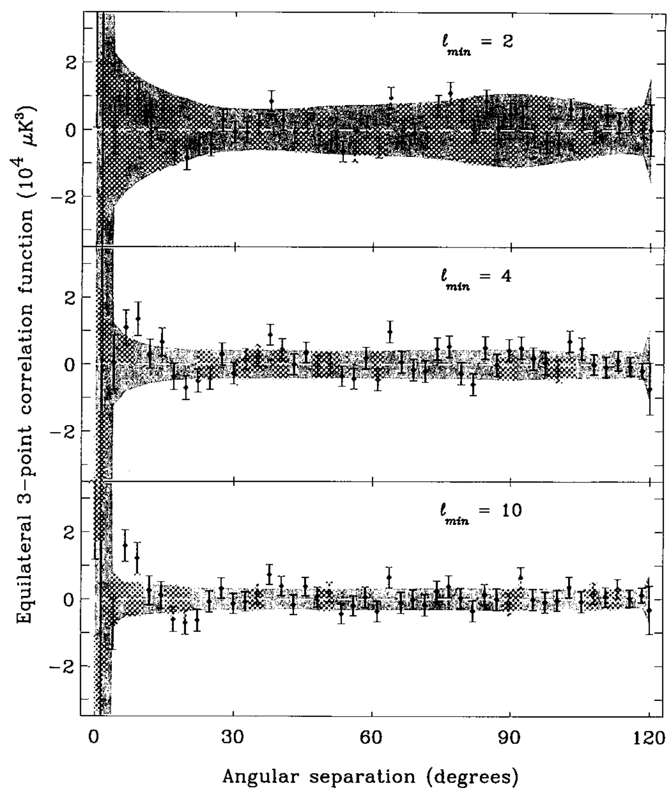

FIG. 1.—*Top*: Equilateral three-point function, in thermodynamic temperature units, evaluated from the 2 year 53 + 90 GHz average map containing power from the quadrupole moment and up ($\ell_{\rm min} = 2$). The error bars represent the rms uncertainty due to instrument noise, while the gray band represents the rms (i.e., 1 σ) range of fluctuations expected from a superposition of instrument noise and Gaussian sky signal (see § 2.1). Due to modest deviations from Gaussianity, the 68% confidence interval is ~10%–30% smaller, depending on angular separation, than the rms range shown. For the filtered cases considered below, $\ell_{\rm min} = 4$, 10, the differences are less than 5%. *Middle*: Same as in top panel for the map with hexadecapole power and up ($\ell_{\rm min} = 4$). *Bottom*: Same as in top panel for the map with power from $\ell = 10$ and up.

The hypothesis that the computed three-point function is consistent with zero is tested by computing $\chi^2 = \sum_{\alpha,\beta} C_\alpha (M^{-1})_{\alpha\beta} C_\beta$, where C_α is the observed three-point function in angular separation bin α and $M_{\alpha\beta}$ is the covariance matrix computed from the simulated three-point functions. (As pointed out by Srednicki 1993, the distribution of C_3 due to cosmic variance is non-Gaussian; thus our χ^2 statistic is only approximately distributed as a true χ^2. However, we compare our data to a distribution obtained from simulations, so this effect is calibrated out. For completeness we note that deviations from Gaussianity are modest: a typical bin of C_3 has a distribution whose kurtosis is $\sim$2.8, 0.2, and 0.1 for the cases $\ell_{\rm min} = 2$, 4, and 10, respectively.) The observed values of χ^2 are given in Table 1; the numbers given in parentheses are the percentage of simulations for which χ^2 exceeded the observed value. Consider first the hypothesis that the observed three-point functions are consistent with *no* sky signal, i.e., that the observed fluctuations are due only to instrument noise. In this case we evaluate χ^2 using the three-point functions observed in the sum maps and the covariance matrix derived from the ensemble of difference (i.e., noise) maps, denoted $\chi^2_{\rm sn}$ in Table 1. For the most sensitive 53 + 90 GHz autocorrelation, we find $\chi^2_{\rm sn} = 101.6$, 87.2, and 65.9 for the $\ell_{\rm min} = 2$, 4, and 10 data, respectively, with 48 degrees of freedom. Typically less than 1% of the difference (i.e., noise) simulations had higher values of χ^2; thus we conclude that there exist nonvanishing equilateral three-point correlations in the observed CMB sky at greater than a 99% confidence level. As a check on the noise levels in our simulations, we evaluate χ^2 using the three-point functions observed in the difference maps, denoted $\chi^2_{\rm nn}$ in Table 1. We find excellent consistency with the modeled noise (see col. [4] of Table 1).

Next we test the hypothesis that the fluctuations seen in the three-point function are consistent with the level expected to arise from a superposition of instrument noise and Gaussian sky signal. In this case χ^2 is evaluated using the covariance matrix computed from the ensemble of sum maps, denoted $\chi^2_{\rm ss}$; for the 53 + 90 GHz autocorrelation we find $\chi^2_{\rm ss} = 31.4$, 45.6, and 57.7 for the $\ell_{\rm min} = 2$, 4, and 10 data, respectively. As shown in Table 1, these values are within the range seen in the simulations, which indicates that the fluctuations observed in the three-point function are consistent with a superposition of instrument noise and Gaussian CMB fluctuations. However, it is interesting to note that for all three map combinations considered, $\chi^2_{\rm ss}$ is an increasing function of $\ell_{\rm min}$ when compared to the level of cosmic variance. Indeed, for the 53 GHz autocorrelation with $\ell_{\rm min} = 10$, only 2% of our signal + noise simulations had a higher χ^2 than seen in the data. Moreover, the relatively more sensitive 53 + 90 GHz autocorrelation appears to have a form reminiscent of the two-point correlation function, as expected in some inflationary models (see, e.g., Falk et al. 1993). However, the Monte Carlo simulations of the 53 + 90 GHz autocorrelation indicate that this result is not statistically exceptional for Guassian models. In all likelihood, the high χ^2 value seen in the 53 GHz autocorrelation is a manifestation of a slight excess noise ($\sim 2\,\sigma$) in the 53 GHz (A + B)/2 map; see, e.g., Figure 1 of Hinshaw (1995) and Table 2 of Banday et al. (1994). Taken as a whole, we do not feel these results present compelling evidence for intrinsic three-point correlations in the CMB.

It is important to note that these data are still noise dominated at moderately high multipole orders: compare, for examples, the relative size of the error bars (which indicate noise) with the gray band (which indicates signal plus noise) in Figure 1. The 4 year data will be $\sim$2.8 times more sensitive than the 2 year data, which will permit a significant clarification of the present results.

TABLE 1

χ^2 VALUES FOR THE EQUILATERAL CONFIGURATION[a]

$\ell_{\rm min}$[b]	$\chi^2_{\rm ss}$[c]	$\chi^2_{\rm sn}$[d]	$\chi^2_{\rm nn}$[e]
	53 GHz Autocorrelation		
2	34.2 (83%)	74.2 (1.5%)	44.7 (59%)
4	55.5 (23%)	81.5 (0.3%)	48.5 (45%)
10	71.1 (2%)	76.8 (0.6%)	43.9 (63%)
	53 + 90 GHz Autocorrelation		
2	31.4 (88%)	101.6 (0%)	41.7 (70%)
4	45.6 (53%)	87.2 (0.2%)	40.2 (76%)
10	57.7 (18%)	65.9 (5%)	42.1 (69%)

[a] There are 48 bins in the equilateral configuration.

[b] $\ell_{\rm min}$ is the lowest order multipole remaining in the map after subtracting a best-fit multipole of order $\ell_{\rm min} - 1$.

[c] $\chi^2_{\rm ss}$ is the χ^2 computed with the covariance derived from the ensemble of signal plus noise maps.

[d] $\chi^2_{\rm sn}$ is the χ^2 computed with the covariance derived from the ensemble of noise maps.

[e] $\chi^2_{\rm nn}$ is the χ^2 computed from the difference map data, (A − B)/2, with the covariance derived from the ensemble of noise maps.

2.2. *The Pseudocollapsed Case*

The pseudocollapsed three-point function, $C_3^{(\rm pc)}$, is defined as above, but now the sum on j is over all pixels that are nearest neighbors to i, and the sum on k is over all pixels (except j) within a given angular separation bin of i. This nearest-neighbors configuration averages over many more distinct pixel triples than the equilateral configuration does, making it more sensitive than the former with respect to instrument noise.

The pseudocollapsed three-point functions obtained from the 53 + 90 GHz DMR maps are presented in Figure 2, which has the same format as Figure 1. As with the equilateral case, we test the hypothesis that the data are consistent with vanishing three-point correlations by computing $\chi^2 = \sum_{\alpha,\beta} C_\alpha (M^{-1})_{\alpha\beta} C_\beta$, where C_α is now the observed pseudocollapsed function in angular separation bin α and $M_{\alpha\beta}$ is the covariance matrix computed from the ensemble of simulated, pseudocollapsed functions. The values of χ^2 are given in Table 2, which has the same format as Table 1. For the 53 + 90 GHz

TABLE 2

χ^2 VALUES FOR THE PSEUDOCOLLAPSED CONFIGURATION[a]

$\ell_{\rm min}$	$\chi^2_{\rm ss}$	$\chi^2_{\rm sn}$	$\chi^2_{\rm nn}$
	53 GHz Autocorrelation		
2	58.3 (67%)	464.5 (0%)	69.6 (49%)
4	70.8 (43%)	290.5 (0%)	71.2 (43%)
10	83.4 (17%)	109.7 (0.5%)	61.9 (71%)
	53 + 90 GHz Autocorrelation		
2	47.5 (86%)	820.1 (0%)	62.2 (69%)
4	76.2 (30%)	594.9 (0%)	58.0 (80%)
10	63.3 (68%)	91.3 (7%)	78.4 (25%)

[a] There are 71 bins in the pseudocollapsed configuration. See notes to Table 1 for further definitions.

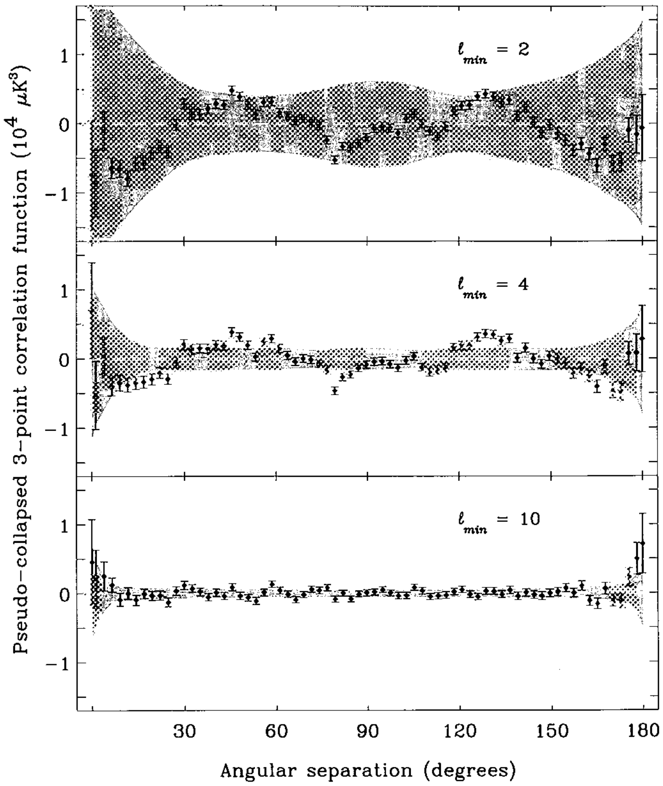

FIG. 2.—*Top*: Pseudocollapsed three-point correlation function, in thermodynamic temperature units, evaluated from the 2 year 53 + 90 GHz average map containing power from the quadrupole moment and up ($\ell_{\rm min} = 2$). The error bars represent the rms uncertainty due to instrument noise, while the gray band represents the rms (i.e., 1 σ) range of fluctuations expected from a superposition of instrument noise and Gaussian sky signal. Note that the datum at zero angular separation is *not* $C_3(0, 0, 0)$, i.e., the skewness, because of the nearest-neighbor averaging employed to circumvent the noise bias (see § 2.2). *Middle*: same as in top panel for the map with hexadecapole power and up ($\ell_{\rm min} = 4$). *Bottom*: Same as in top panel for the map with power from $\ell = 10$ and up.

autocorrelation, the χ^2 values for the hypothesis that the three-point correlations in the sum maps are consistent with instrument noise alone are 820.1, 594.9, and 91.3 for the $\ell_{\rm min} = 2$, 4, and 10 filters, respectively, with 71 dof. None of the difference (i.e., noise) map simulations, with $\ell_{\rm min} = 2$ or 4, had such large χ^2 values, which reinforces our conclusion from the first-year data analysis that we observe nonvanishing three-point correlations in our sky that cannot be attributed to instrument noise. When we include the effects of cosmic variance, the χ^2 values diminish considerably: we find 47.5, 76.2, and 63.3 for the $\ell_{\rm min} = 2$, 4, and 10 filters, respectively. These values are well within the range seen in the Gaussian simulations, indicating that cosmic variance can comfortably explain the observed pseudocollapsed three-point fluctuations. As with the equilateral configuration, the $\chi^2_{\rm ss}$ values for the 53 GHz autocorrelation increase with increasing $\ell_{\rm min}$, but not so in the 53 + 90 GHz combination. The apparent lack of intrinsic three-point correlations in the relatively sensitive pseudocollapsed configuration is further evidence that the high χ^2 seen in the 53 GHz equilateral function is probably spurious.

3. CONCLUSIONS

We have evaluated two configurations of the three-point temperature correlation function for the 2 year *COBE* DMR sky maps and find evidence for nonzero three-point correlations in the data, even at relatively high multipole order ($\ell \gtrsim 10$). This would appear to rule out the particular model for generating PIB fluctuations considered by Yamamoto & Sasaki (1994). However, we demonstrate that the observed level of fluctuations is consistent with the level expected to result from the superposition of instrument noise and a CMB sky signal arising from a Gaussian, scale-invariant power-law model of initial fluctuations, with a quadrupole normalized amplitude of 20 μK. Given that most conventional models of structure formation predict nearly Gaussian fluctuations on large angular scales, it is important to determine whether non-Gaussian features exist in the DMR data. The fact that we find no evidence for large, intrinsic three-point correlations implies that standard structure formation models pass an important consistency test.

In the 4 year DMR maps, the instrument noise will be ~ 2.8 times smaller than in the 2 year maps. The added sensitivity will be particularly important for studying correlations in the regime of relatively high multipole order, which is still dominated by instrument noise in the 2 year maps.

We gratefully acknowledge the many people who made this Letter possible: the NASA Office of Space Sciences, the *COBE* flight operations team, and all of those who helped process and analyze the data. We thank Charley Lineweaver for his useful comments on an earlier version of this Letter. This work was supported in part by NASA ADP Program under grant NAS 5-32648.

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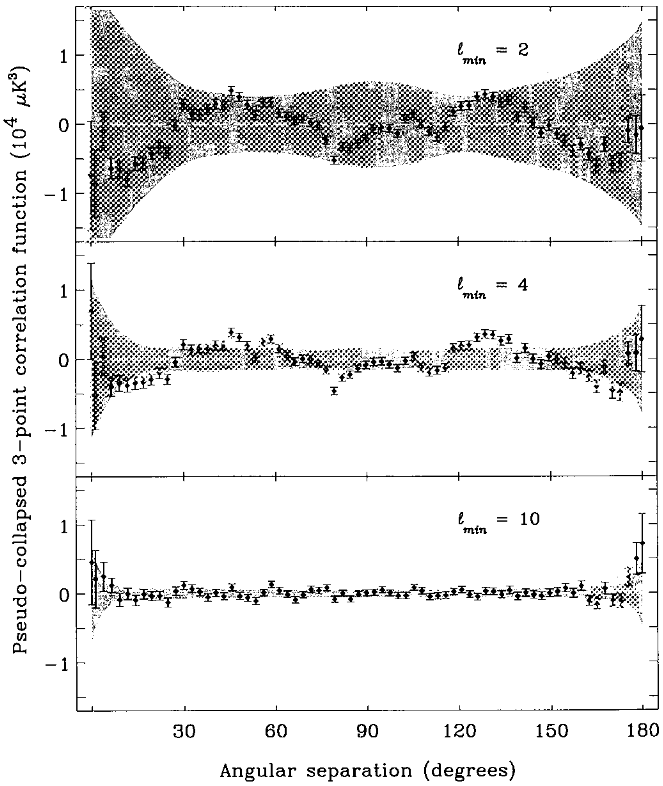

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