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Abstract. A temperature gradient across a thick (≥ .1 mm) film selective emitter will produce a significant reduction in the spectral emittance from the no temperature gradient case. Thick film selective emitters of rare earth doped host materials such as yttrium-aluminum-garnet (YAG) are examples where temperature gradient effects are important. In this paper a model is developed for the spectral emittance assuming a linear temperature gradient across the film. Results of the model indicate that temperature gradients will result in reductions the order of 20% or more in the spectral emittance.

INTRODUCTION

Emission from thick films is not a surface phenomenon as is usually assumed when discussing emissive materials. It depends on the geometry of the material, which for the film emitters means the film thickness. Thus radiation leaving the film originates at various depths within the film.

To model these film emitters we use a macroscopic approach. That is we solve the radiative transfer equation that applies for Boltzmann equilibrium of excited state densities and includes stimulated emission and absorption, as well as, spontaneous emission and scattering of radiation. These atomic processes manifest themselves on the macroscopic scale through the extinction coefficient, α_λ.

The product of the extinction coefficient, α_λ, and the film thickness, d, $\alpha_\lambda d = K_d$, which is usually called the optical depth, will determine the spectral emittance if the temperature is a constant through the film. However, for thick films (≥ .1 mm) the temperature gradients are not negligible (> 100°K) so the emittance model must include a variable temperature through the film. In the analysis to follow we assume a linear temperature variation across the film. This is the result that will occur if thermal conduction dominates radiative energy transfer. In the case where $d \leq 1$ mm this is a good assumption for the rare-earth selective emitters we are considering (3).

In the following section the emittance model will be developed. Following that, two approximate expressions for the spectral emittance, ε_λ, that apply when scattering is neglected and the temperature gradient is small will be presented. The first approximation is applicable for large optical depth, K_d, and the second approximation applies for small optical depth. Both of these approximations are compared to the exact result for ε_λ, neglecting scattering but for any temperature gradient, obtained by a numerical solution of the governing equations. Following that a discussion of the optimum film thickness to obtain maximum emittance will be presented. Finally, spectral emittance results will be compared to experimental results obtained for an erbium oxide (Er_2O_3) selective emitter that has an emission band centered at a photon wavelength, $\lambda = 1.5$ µm.

THICK FILM EMITTANCE MODEL

The emittance model for the thick film emitter has been previously developed for the case of no temperature gradient (1, 4). This model can be extended to include a temperature gradient across the film. The model is based on the radiative transfer equation (5), which is macroscopic in nature. Thus the emissive, absorptive and scattering properties of the material, which depend on the atomic structure, are expressed through the extinction coefficient, α_λ. The key parameter in determining the spectral emittance, ε_λ, is the optical depth, $K = \alpha_\lambda d$.

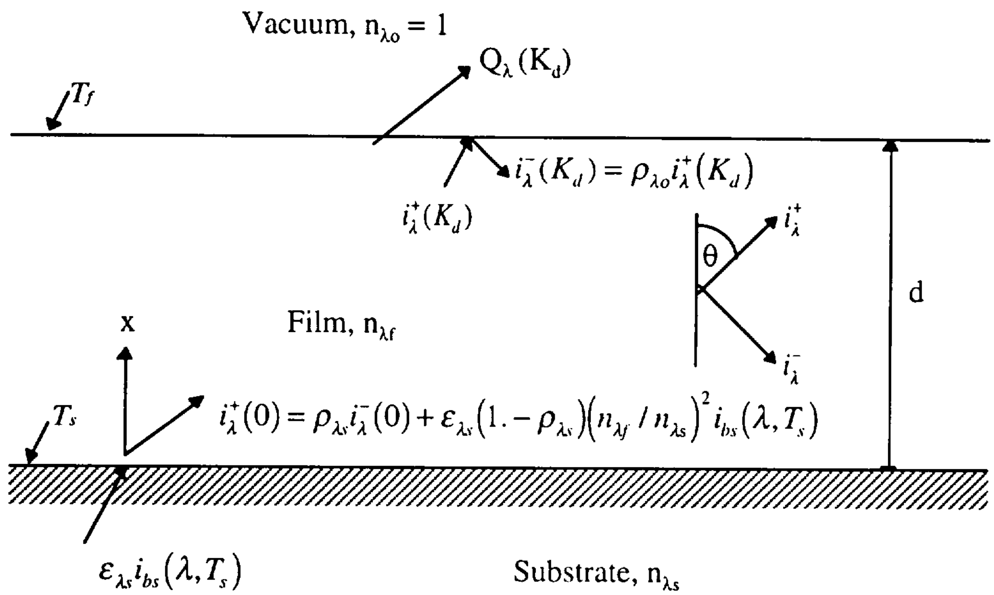
Consider Figure 1 which is a schematic drawing of a thick film emitter. Thermal energy enters through the film substrate. Part or all of the thermal input leaves the film at x = d as radiation flux, $Q_\lambda(K_d)$. To determine ε_λ, $Q_\lambda(K_d)$ must be calculated since ε_λ is defined as follows.

$$\varepsilon_\lambda \equiv \frac{Q_\lambda(K_d)}{e_{bs}(\lambda, T_s)} \tag{1}$$

Where $e_{bs}(\lambda, T_s)$ is the blackbody emissive power and T_s is the substrate temperature.

$$e_{bs} = \pi i_{bs} = \frac{2\pi h c_o^2}{\lambda^5\left[\exp(hc_o/\lambda k T_s) - 1\right]} \tag{2}$$

Here h is Plank's constant, k is Boltzmann's constant, c_o is the vacuum speed of light, and i_{bs}, is the blackbody intensity. Notice that ε_λ has been defined in terms of the substrate temperatures, T_s. The spectral emittance could be defined in terms of the film surface temperature, T_f, or some combination of T_f and T_s. However, defining ε_λ in terms of T_s means $\varepsilon_\lambda \leq 1$ in all cases since $e_{bs}(\lambda,T_s) \geq Q_l(K_d)$. This definition agrees with the usual concept of emittance.



n_λ = index of refraction
$\rho_{\lambda o}$ = reflectance at film-vacuum interface
$\rho_{\lambda s}$ = reflectance at film-substrate interface
$\varepsilon_{\lambda s}$ = emittance of substrate
$i_{bs}(\lambda,T_s)$ = blackbody intensity for $T = T_s$

FIGURE 1. Schematic Diagram of Thick Film Emittance Model

To calculate Q_λ we require the radiative transfer equations for radiation intensity moving in the + x direction, $i_\lambda^+(K,\cos\theta)$, and intensity in the - x direction, $i_\lambda^-(K,\cos\theta)$, (5).

$$i_\lambda^+(K,\mu) = i_\lambda^+(0,\mu)\exp\left[-\frac{K}{\mu}\right] + \int_0^{K_d} S(K^*,\mu)\exp\left[\frac{K-K^*}{\mu}\right]\frac{dK^*}{\mu} \qquad (3)$$

$$0 \leq \mu = \cos\theta \leq 1$$

$$i_\lambda^-(K,\mu) = i_\lambda^-(K_d,\mu)\exp\left[-\frac{K_d-K}{\mu}\right] - \int_0^{K_d} S(K^*,\mu)\exp\left[\frac{K^*-K}{\mu}\right]\frac{dK^*}{\mu} \qquad (4)$$

$$-1 \leq \mu = \cos\theta \leq 0$$

In using these equations we are assuming that y and z variation of intensity can be neglected. Appearing in equations (3) and (4) is the so-called source function, S(K,μ), which in the case of isotopic scattering (S(K,μ) = S(K)) satisfies the following equation (5).

$$S(K)=n_{\lambda f}^{2}(1-\Omega_{\lambda})i_{\lambda b}(T,\lambda)+\frac{\Omega_{\lambda}}{2}\left\{\int_{0}^{1}i_{\lambda}^{+}(0,\mu)\exp\left[-\frac{K}{\mu}\right]d\mu+\int_{0}^{1}i_{\lambda}^{-}(K_{d},-\mu)\exp\left[\frac{K_{d}-K}{-\mu}\right]d\mu\right\}$$

$$+\frac{\Omega_{\lambda}}{2}\int_{0}^{K_{d}}S(K^{*})E_{1}(|K^{*}-K|)dK^{*} \qquad (5)$$

Appearing in equation (5) is the scattering albedo.

$$\Omega_{\lambda}=\frac{\sigma_{\lambda}}{\sigma_{\lambda}+a_{\lambda}}=\frac{\sigma_{\lambda}}{\alpha_{\lambda}} \qquad (6)$$

Where σ_λ is the scattering coefficient and a_λ is the absorption coefficient, which have the dimensions, cm^{-1}. The sum of σ_λ and a_λ is the extinction coefficient, α_λ. Also appearing in equation (5) is the film index of refraction, $n_{\lambda f}$, and the exponential integral, $E_1(x)$.

The general exponential integral, $E_n(x)$, is defined as follows.

$$E_{n}(x)\equiv\int_{0}^{1}v^{n-2}\exp\left[-\frac{x}{v}\right]dv \qquad (7)$$

Note that we are assuming isotropic scattering. As a result, S is independent of $\mu = \cos\theta$. Therefore, assuming diffuse boundary intensities, $i_\lambda^+(0,\mu) = i_\lambda^+(0)$ and $i_\lambda^+(K_d,\mu) = i_\lambda^+(K_d)$ we see from equations (3) and (4) that i_λ^+ and i_λ^- are also independent of μ.

The diffuse (independent of μ) boundary conditions at $K = K_d$ and $K = 0$ are the following.

$$i_{\lambda}^{-}(K_{d})=\rho_{\lambda o}i_{\lambda}^{+}(K_{d}) \qquad \text{at } K=K_d \qquad (8a)$$

$$i_{\lambda}^{+}(0)=\rho_{\lambda s}i_{\lambda}^{-}(0)+(1-\rho_{\lambda s})\varepsilon_{\lambda s}(n_{\lambda f}/n_{\lambda s})^{2}\,i_{bs}(\lambda,T_{s}) \qquad \text{at } K=0 \qquad (8b)$$

Equation (8a) states that the radiation leaving the film-vacuum interface in the -x direction is equal to the reflected radiation at that interface. For the film-substrate interface equation (8b) states that $i_\lambda^+(0)$ is the sum of the reflected radiation and the radiation emitted by the substrate that is transmitted ($1-\rho_{\lambda s}$) through that interface.

The $(n_{\lambda f}/n_{\lambda s})^2$ term accounts for refraction at the interface (5, pg. 738). The reflectance at the film-vacuum interface is $\rho_{\lambda o}$ and the reflectance at the film-substrate interface is $\rho_{\lambda s}$. In the previous studies (1,3,4) the transmittance, $(1-\rho_{\lambda s})$, at the film-substrate interface was assumed to be 1 and the refraction term $(n_{\lambda f}/n_{\lambda s})^2$ was neglected. We approximate $\rho_{\lambda o}$ and $\rho_{\lambda s}$ by the reflectance for normal incidence, (5)

$$\rho_{\lambda o} = \left(\frac{n_{\lambda f} - 1}{n_{\lambda f} + 1} \right)^2 \tag{9}$$

$$\rho_{\lambda s} = \left(\frac{n_{\lambda s} - n_{\lambda f}}{n_{\lambda s} + n_{\lambda f}} \right)^2 \tag{10}$$

Where, $n_{\lambda s}$ is the substrate index of refraction.

At the film-substrate and film-vacuum interfaces there is the possibility of total reflection occurring. At an interface between a material with an index of refraction, n_ℓ, and a material with index of refraction n_m, where $n_\ell > n_m$, radiation moving from ℓ into m with an angle of incidence, $\theta > \theta_{\ell m}$, where $\theta_{\ell m}$ is given by Snell's law will be totally reflected. This will be taken into account when calculating $Q_\lambda(K_d)$. At the film-substrate interface refraction has been taken into account by including the $(n_{\lambda f}/n_{\lambda s})^2$ term in equation (8b). However, the possibility of total reflection is not included. Therefore, by using equation (8b) as the boundary condition we are assuming that $n_{\lambda f} > n_{\lambda s}$ so that total reflection does not occur for radiation entering the film from the substrate.

Now consider $Q_\lambda(K_d)$, which is the radiation flux leaving the film. Since $n_{\lambda f} > 1$ the radiation leaving the film will be refracted and some of the radiation that reaches the film-vacuum interface will be totally reflected at the interface. Therefore,

$$Q_\lambda\left(K_d\right) = 2\pi \int_{\theta=0}^{\theta_M} \left[i_\lambda^+\left(K_d, \cos\theta\right) - i_\lambda^-\left(K_d, \cos\theta\right) \right] \cos\theta \, \sin\theta \, d\theta \tag{11a}$$

and using equation (8a) and letting $\mu = \cos\theta$ this becomes the following.

$$Q_\lambda\left(K_d\right) = 2\pi \left(1 - \rho_{\lambda o}\right) \int_{\mu_M}^{1} i_\lambda^+\left(K_d, \mu\right) \mu \, d\mu \tag{11b}$$

Where μ_M is given by Snell's Law.

$$\mu_M^2 = \cos^2 \mu_M = 1 - n_{\lambda f}^{-2} \tag{12}$$

Substituting (3) in (11b) yields the following.

$$Q_\lambda\left(K_d\right) = \left(1-\rho_{\lambda o}\right)\left[2\pi i_\lambda^+(0) h_- + \Phi_+ - \Phi_M\right] \tag{13}$$

Where,

$$h_- = E_3\left(K_d\right) - \mu_M^2 E_3\left(\frac{K_d}{\mu_M}\right) \tag{14}$$

$$\Phi_+ = 2\pi \int_0^{K_d} S(K)\ E_2\ (K_d - K)\ dK \tag{15}$$

$$\Phi_M = 2\pi\mu_M \int_0^{K_d} S(K)\ E_2\left(\frac{K_d - K}{\mu_M}\right) dK \tag{16}$$

Equation (13) gives $Q_\lambda(K_d)$ in terms of the source function S(K) and $i_\lambda^+(0)$. The $i_\lambda^+(0)$ intensity is obtained by using equations (3) and (4) to get two simultaneous equations for $i_\lambda^-(K_d)$ and $i_\lambda^-(0)$. These can then be solved for $i_\lambda^-(0)$ and the result used in equation (8b) to obtain $i_\lambda^+(0)$ (4).

$$\pi i_\lambda^+(0) = q^+(0) = \frac{1}{D}\left[\left(\frac{n_{\lambda f}}{n_{\lambda s}}\right)^2 \left(1-\rho_{\lambda s}\right)\varepsilon_{\lambda s} e_{bs}(\lambda, T_s) + 2\rho_{\lambda 0}\rho_{\lambda s} \mathrm{E}_3\left(K_d\right)\Phi_+ + \rho_{\lambda s}\Phi_-\right] \tag{17}$$

Where,

$$D = 1 - 4\rho_{\lambda 0}\rho_{\lambda s}\mathrm{E}_3^2\left(K_d\right) \tag{18}$$

$$\Phi_- = 2\pi \int_0^{K_d} S(K)\,\mathrm{E}_3(K)\,dK \tag{19}$$

Now substitute equation (17) in (13).

$$Q_\lambda(K_d)=\frac{1-\rho_{\lambda 0}}{D}\left\{2\left[\varepsilon_{\lambda s}\left(\frac{n_{\lambda f}}{n_{\lambda s}}\right)^2(1-\rho_{\lambda s})e_{bs}(\lambda,T_s)+\rho_{\lambda s}\Phi_-\right]h_-+\Phi_+h_+-\Phi_M D\right\} \quad (20)$$

Where,

$$h_+=1-4\rho_{\lambda 0}\rho_{\lambda s}\mu_M^2 \mathrm{E}_3(K_d)\mathrm{E}_3\left(\frac{K_d}{\mu_M}\right) \quad (21)$$

Equation (20) can be substituted in equation (1) to obtain the spectral emittance, ε_λ, in terms of the source function, S(K). In the general case where scattering exists the source function must be obtained by solving equation (5). In the case of no scattering, $\Omega_\lambda = 0$, and equation (5) reduces to the following.

$$S(K)=n_{\lambda f}^2 i_b(\lambda,T) \quad (22)$$

If we also assume T is a constant through the film, $T = T_s$, then the integrations in Φ_+, Φ_-, and Φ_M, can be carried out to yield the following.

$$\varepsilon_{\lambda o}=\frac{n_{\lambda f}^2(1-\rho_{\lambda 0})}{D}\left\{2h_-\left[\frac{\varepsilon_{\lambda s}(1-\rho_{\lambda s})}{n_{\lambda s}^2}+\rho_{\lambda s}(1-2\mathrm{E}_3(K_d))\right]+h_+[1-2\mathrm{E}_3(K_d)]\right.$$
$$\left.-\mu_M^2 D\left[1-2\mathrm{E}_3\left(\frac{K_d}{\mu_M}\right)\right]\right\}$$

constant temperature, no scattering (23)

Thus ε_λ is determined by the optical depth, K_d, the indices of refraction, $n_{\lambda f}$ and $n_{\lambda s}$ and the substrate emittance, $\varepsilon_{\lambda s}$. In the case when scattering is important ε_λ will also be a function of the scattering albedo, Ω_λ.

Now consider the case where a temperature gradient exists. To demonstrate the temperature gradient effects in the simplest manner we consider the no scattering case since in that case the source function has the simple solution given by equation (22). We also assume a linear temperature gradient across the film. As discussed in the introduction this is a good approximation for the rare earth selective emitters. As a result, the temperature across the film is given by the following expression.

$$\frac{T}{T_s}=1-\Delta T\left(\frac{x}{d}\right)=1-\Delta T\left(\frac{K}{K_d}\right) \quad (24)$$

Where, the temperature gradient is defined as follows.

$$\Delta T \equiv \frac{T_s - T_f}{T_s} \tag{25}$$

Using equations (24), (22) and (2) in the expressions for Φ_+, Φ_-, and Φ_M, yields the following.

$$\Phi'_+ = \frac{\Phi_+}{2n_{\lambda f}^2 e_{bs}(\lambda, T_s)} = \left(e^u - 1\right)K_d \int_{\upsilon=0}^{1} \frac{E_2\left[K_d(1-\upsilon)\right]}{\exp\left[\frac{u}{1-\upsilon\Delta T}\right]-1} d\upsilon \tag{26}$$

$$\Phi'_- = \frac{\Phi_-}{2n_{\lambda f}^2 e_{bs}(\lambda, T_s)} = \left(e^u - 1\right)K_d \int_{\upsilon=0}^{1} \frac{E_2\left(K_d\upsilon\right)}{\exp\left[\frac{u}{1-\upsilon\Delta T}\right]-1} d\upsilon \tag{27}$$

$$\Phi'_M = \frac{\Phi_M}{2n_{\lambda f}^2 e_{bs}(\lambda, T_s)} = \mu_M\left(e^u - 1\right)K_d \int_{\upsilon=0}^{1} \frac{E_2\left[\frac{K_d}{\mu_M}(1-\upsilon)\right]}{\exp\left[\frac{u}{1-\upsilon\Delta T}\right]-1} d\upsilon \tag{28}$$

Where,

$$u = \frac{hc_0}{\lambda k T_s} \tag{29}$$

$$\upsilon = \frac{K}{K_d} \tag{30}$$

Equations (26) - (28) can be used in equations (20) and (1) to obtain ε_λ.

$$\varepsilon_\lambda = \frac{2n_{\lambda f}^2\left(1-\rho_{\lambda o}\right)}{D}\left\{\left[\frac{\varepsilon_{\lambda s}\left(1-\rho_{\lambda s}\right)}{n_{\lambda s}^2} + 2\rho_{\lambda s}\Phi'_-\right]h_- + \Phi'_+ h_+ - \Phi'_M D\right\} \tag{31}$$

no scattering, with temperature gradient

As equations (26) - (28) indicate Φ'_+, Φ'_-, and Φ'_M, are functions of ΔT. The integrations in equations (26) - (28) must be carried out numerically. However, for small ΔT approximations to the integrals can be made. In most cases of interest for selective emitters, $(\lambda \le 7\mu m, T_s \le 2000K)$, the dimensionless photon energy, u, is greater than 1. Therefore, the following approximations can be made.

$$\left[\exp\left(\frac{u}{1-\upsilon\Delta T}\right)-1\right]^{-1} \approx \exp\left[\frac{-u}{1-\upsilon\Delta T}\right] \qquad e^{u} >> 1 \qquad (32a)$$

$$e^{u} - 1 \approx e^{u} \qquad e^{u} >> 1 \qquad (32b)$$

In addition for $\Delta T << 1$ and $0 \leq \upsilon \leq 1$;

$$\exp\left[\frac{-u}{1-\upsilon\Delta T}\right] \approx e^{-u} e^{-u\Delta T\upsilon} \qquad e^{u} >> 1,\ \Delta T << 1 \qquad (33)$$

With the approximations given by equations (32) and (33) equations (26) - (28) become the following after a change in the integration variables.

$$\Phi'_{+} \approx e^{-u\Delta T} \int_0^{K_d} \exp\left[\frac{K u \Delta T}{K_d}\right] \mathrm{E}_2(K)dK \qquad (34)$$

$$\Phi'_{-} \approx \int_0^{K_d} \exp\left[\frac{-K u \Delta T}{K_d}\right] \mathrm{E}_2(K)dK \qquad (35)$$

$$\Phi'_{M} \approx \mu_M^2 e^{-u\Delta T} \int_0^{\frac{K_d}{\mu_M}} \exp\left[\frac{K \mu_M u \Delta T}{K_d}\right] \mathrm{E}_2(K)dK \qquad (36)$$

For a selective emitter the optical depth, K_d, will be large (K_d>1) in the emission band and small (K_d<<1) outside the emission band. Therefore, consider the two limiting cases; $\frac{u\Delta T}{K_d} << 1$ and $\frac{K_d}{u\Delta T} << 1$. For the case where $\frac{u\Delta T}{K_d} << 1$, integration by parts using

$$\mathrm{E}_{n-1}(x) = -\frac{d\mathrm{E}_n(x)}{dx} \qquad (37)$$

results in the following to first order in $\frac{u\Delta T}{K_d}$.

$$\Phi'_{+} \approx \frac{1}{2}e^{-u\Delta T} - \mathrm{E}_3(K_d) - \left[\mathrm{E}_4(K_d) - \frac{1}{3}e^{-u\Delta T}\right]\frac{u\Delta T}{K_d} \qquad (38)$$

$$\Phi'_{-} \approx \frac{1}{2} - e^{-u\Delta T}\mathrm{E}_3(K_d) + \left[e^{-u\Delta T}\mathrm{E}_4(K_d) - \frac{1}{3}\right]\frac{u\Delta T}{K_d} \qquad \frac{u\Delta T}{K_d} << 1 \qquad (39)$$

$$\Phi'_M \approx \mu_M^2 \left\{ \frac{1}{2} e^{-u\Delta T} - \mathrm{E}_3\left(\frac{K_d}{\mu_M}\right) - \left[\mathrm{E}_4\left(\frac{K_d}{\mu_M}\right) - \frac{1}{3} e^{-u\Delta T}\right] \frac{u\Delta T}{K_d} \right\} \tag{40}$$

Since we are interested in showing the effect of temperature gradient on ε_λ we define the following quantity.

$$\Delta\varepsilon_\lambda \equiv \varepsilon_{\lambda 0} - \varepsilon_\lambda \tag{41}$$

Where $\varepsilon_{\lambda 0}$ is the emittance for no temperature gradient and is given by equation (23). By using $\Delta\varepsilon_\lambda$to demonstrate the temperature gradient effect the dependence on substrate emittance, $\varepsilon_{\lambda s}$, is removed. Therefore, using equations (38) - (40) in (31) and equation (23) for $\varepsilon_{\lambda 0}$ results in the following.

$$\Delta\varepsilon_\lambda \approx \frac{2n_M^2(1-\rho_{\lambda 0})}{D} \Bigg\{ \frac{1}{2}\left[h_+ - 4\rho_{\lambda s} h_- \mathrm{E}_3(K_d) - \mu_M^2 D\right]\left[1 - e^{-u\Delta T}\right]$$

$$+ \left[2\rho_{\lambda s}\left(\frac{1}{3} - e^{-u\Delta T}\mathrm{E}_4(K_d)\right)h_- - \left(\frac{e^{-u\Delta T}}{3} - \mathrm{E}_4(K_d)\right)h_+ \right.$$

$$\left. - \mu_M^3\left(\frac{e^{-u\Delta T}}{3} - \mathrm{E}_4\left(\frac{K_d}{\mu_M}\right)D\right)\right]\frac{u\Delta T}{K_d} \Bigg\}$$

$$\text{no scattering, } \Delta T \ll 1,\ e^u \gg 1,\ \frac{u\Delta T}{K_d} \ll 1 \tag{42}$$

Notice that if uΔT<<1 then $\Delta\varepsilon$ as given by equation (42) will be a linear function of ΔT. However, if uΔT is not small then $\Delta\varepsilon_\lambda \sim \left(1 - e^{-u\Delta T}\right)$ provided $\frac{u\Delta T}{K_d} \ll 1$. Also, note by looking at equation (31) that $\Delta\varepsilon_\lambda$ is independent of the substrate emittance.

Now consider $\Delta\varepsilon_\lambda$ for the case where K_d<<1. In that case $\mathrm{E}_2(K)$ can be expanded in a power series and the integrations in equations (34) - (36) performed. To first order in $\frac{K_d}{u\Delta T}$ the results are the following.

$$\Phi'_+ = \Phi'_- \approx \left(1 - e^{-u\Delta T}\right)\frac{K_d}{u\Delta T} \tag{43}$$

$$\frac{K_d}{u\Delta T} \ll 1$$

$$\Phi'_M \approx \mu_M^2\left(1 - e^{-u\Delta T}\right)\frac{K_d}{\mu_M u\Delta T} \tag{44}$$

If equations (43) and (44) are used in (31) and equation (23) for $\varepsilon_{\lambda 0}$ then $\Delta\varepsilon_\lambda$ becomes the following when the approximation $E_3(K_d) \approx 1/2 - K_d$ is used.

$$\Delta\varepsilon_\lambda = \frac{2n_{\lambda f}^2(1-\rho_{\lambda 0})}{1-\rho_{\lambda 0}\rho_{\lambda s}}\left\{1+(1-\mu_M^2)\rho_{\lambda f} - \mu_M\left[1-\rho_{\lambda 0}\rho_{\lambda s}(1-\mu_M)\right]\right\}K_d\left[1-\frac{1-e^{-u\Delta T}}{u\Delta T}\right]$$

$$\text{no scattering, } \Delta T \ll 1, e^u \gg 1, \frac{K_d}{\mu\Delta T} \ll 1. \qquad (45)$$

Again, if $u\Delta T \ll 1$ then $\Delta\varepsilon_\lambda$ will be approximately a linear function of ΔT just as in the case of $\frac{u\Delta T}{K_d} \ll 1$. Also note that $\Delta\varepsilon_\lambda$ is a linear function of K_d and that $\varepsilon_{\lambda s}$ has no effect on $\Delta\varepsilon_\lambda$.

TEMPERATURE GRADIENT EFFECT ON SPECTRAL EMITTANCE FOR NO SCATTERING

Comparison of Exact and Approximate Solutions for Spectral Emittance

With the results developed in the previous section we can now illustrate the effect of ΔT on ε_λ. In Figure 2 $\Delta\varepsilon_\lambda$ is shown as a function of ΔT for large optical depth $(K_d = 2)$ at several values of u. The exact result for $\Delta\varepsilon_\lambda$ is obtained using

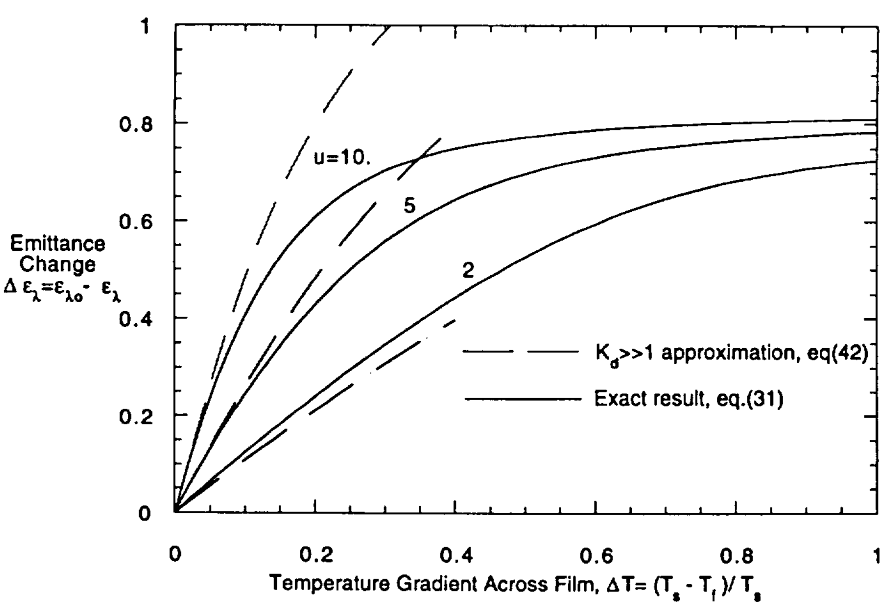


FIGURE 2. Emittance change as a function of temperature gradient at large optical depth, $K_d = 2$, for several dimensionless photon energies, $u=hc_0/\lambda kT_s$ with $n_{\lambda s}=10$ and $n_{\lambda f}= 1.9$.

equation (31) for ε_λ and numerical integration to obtain Φ'_+, Φ'_- and Φ'_M. Also, the $\frac{u\Delta T}{K_d} << 1$ result for $\Delta\varepsilon_\lambda$ (equation (42)) is shown in Figure 2.

As Figure 2 indicates $\Delta\varepsilon_\lambda$ changes rapidly at small ΔT with the slope increasing for increasing u. Thus even for $\Delta T \leq .1$ there will be a significant reduction in the spectral emittance for $u \geq 5$. In most cases, for the emission bands of rare earth selective emitters where $K_d > 1$ the dimensionless photon energy, $u > 5$. Therefore, even a small temperature gradient will result in a significant reduction in the spectral emittance in the emittance band of the rare earth selective emitters. Obviously, making the emitter as thin as possible will reduce ΔT. However, the optical depth will also be reduced, if the thickness, d, is reduced, resulting in decreased ε_λ. As a result, there will be an optimum thickness, d, to obtain maximum ε_λ. This will be discussed in the next section. Note also that the approximate solution (equation (42)) is in close agreement with the exact results when $\Delta T < .1$.

Results in Figure 2 are for large optical depth ($K_d = 2$). However, similar results occur for small optical depth and are illustrated in Figure 3 where $K_d = .1$. Again there is good agreement between the approximate solution (equation (45))

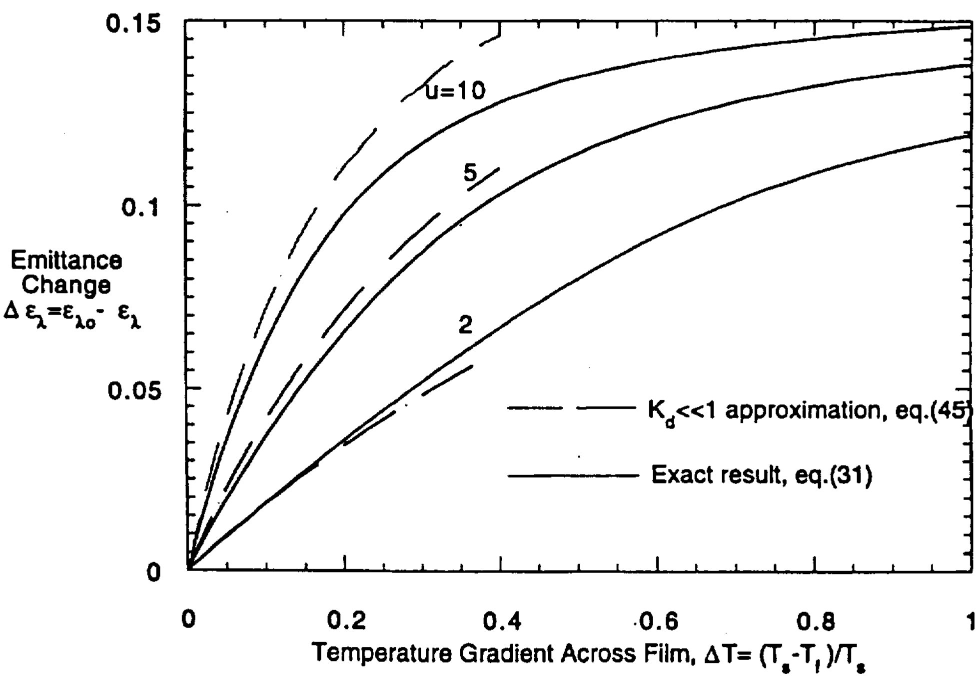


FIGURE 3. Emittance change as a function of temperature gradient at an optical depth, $K_d = .1$, for several dimensionless photon energies, $u = hc_0/\lambda kT_s$ with $n_{\lambda f} = 1.9$ and $n_{\lambda s} = 10$.

and the exact solution when $\Delta T<.1$. The range of values for $\Delta\varepsilon_\lambda$ is much smaller for the case where $K_d<<1$ than for $K_d>1$. Thus the temperature gradient has only a small effect on ε_λ when $K_d<<1$. Therefore, for a selective emitter the emittance outside the emission band will not be greatly effected by ΔT.

Optimum Thickness for Maximum Spectral Emittance

As already stated, the counteracting effects of increasing spectral emittance with optical depth and decreasing spectral emittance with increasing temperature gradient will result in an optimum film thickness for maximum spectral emittance. This can be demonstrated as follows. Neglecting any conductive or convective heat transfer at the film surface (which will occur if a vacuum exists at the film surface) then the total power/area leaving the film is the following.

$$Q_{out} = \int_0^\infty Q_\lambda(K_d)d\lambda \tag{46}$$

This same power/area must be supplied by thermal conduction and radiation at the film-substrate interface to maintain a steady state. Therefore, at $x = 0$, assuming conduction is much greater than radiation,

$$Q_{out} = -\beta_f \left.\frac{dT}{dx}\right|_{x=0} \tag{47}$$

Where β_f is the film thermal conductivity. As stated earlier, energy transfer through the film is dominated by thermal conduction so that, equation (24) applies and $-\left.\frac{dT}{dx}\right|_{x=0} = \left(\frac{T_s - T_f}{d}\right)$. Therefore, from equations (46) and (47) the following is obtained.

$$\Delta T = \frac{T_s - T_f}{T_s} = \frac{Q_{out}}{\beta_f T_s} d \tag{48}$$

To calculate Q_{out}, equation (31) for ε_λ, which is a function of ΔT must be used to determine $Q_\lambda(K_d)$ (equation (1)). However, since ε_λ is a function of ΔT, equations (46) and (48) must be solved simultaneously in order to obtain ΔT as a function of Q_{out}. This has been done in ref. 3. But to illustrate how an optimum thickness occurs we can write Q_{out} as follows.

$$Q_{out} = \varepsilon_T \sigma_{sb} T_s^4 \tag{49}$$

Where ε_T is the total emittance of the film and will be a function of T_s and σ_{sb} is the Stefan-Boltzmann constant (5.67 x 10^{-12} w/ cm^2 K^4). By using equation (49) in equation (48) the following results.

$$\Delta T = \tau_f d \tag{50}$$

Where,

$$\tau_f = \frac{\varepsilon_T \sigma_{sb} T_s^3}{\beta_f} \; cm^{-1} \tag{51}$$

The quantity $\tau_f d$ is the ratio of radiation to thermal conduction (3). Thus equation (50) shows that ΔT will be small as long as thermal conduction dominates.

For selective emitters of interest, $\varepsilon_T < .2$, $\beta_f > .02$ w/cmK and T_s< 2000K, so that $0 < \tau_f < 5mm^{-1}$. If equation (50) is used for ΔT in equation (31) and since $K_d = \alpha_\lambda d$ the results for ε_λ when $\alpha_\lambda = 100$ cm^{-1} shown in figure 4 are obtained. An extinction coefficient $\alpha_\lambda = 100$ cm^{-1} is representative of the emission band of a selective emitter. The first thing to note from figure 4 is that for $\Delta T > 0(\tau_f > 0)$ there is an optimum thickness for maximum ε_λ. For the case of no temperature gradient $(\tau_f = 0)$ there is no optimum d. The larger the temperature gradient the more pronounced the optimum d becomes. For small τ_f large values of ε_λ occur over a broad range of thicknesses. Note that the curve for τ_f = 2 mm^{-1} and τ_f = 5 mm^{-1} have been truncated at d = .5 mm and d = .2 mm since $\Delta T = \tau_f d \le 1$. Also notice that the optimum d becomes smaller as τ_f increases (larger ΔT). Based on the results of figure 4 it appears that the optimum selective emitter thickness to obtain maximum emittance in the emission band for α_λ= 100 cm^{-1} is in the range .15≤ d ≤ .4 mm. For α_λ= 100 cm^{-1} this corresponds to an optical depth range, $1.5 \le K_d \le 4$.

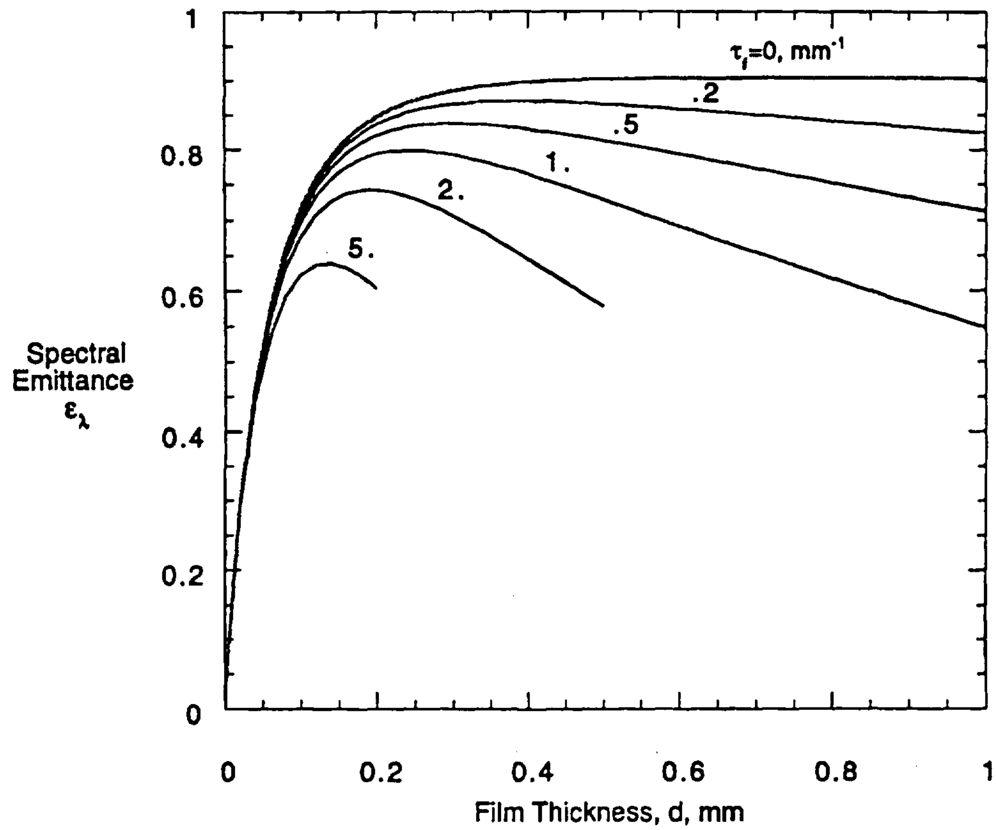


FIGURE 4. Effect of temperature gradient on spectral emittance for large extinction coefficient, α_λ=100 cm^{-1}, at several values of the temperature gradient parameter, τ_f. Also, u=5., $\varepsilon_{\lambda s}$=.1, $n_{\lambda f}$=1.9, $n_{\lambda s}$=10.

Now consider the case of small extinction coefficient, which is representative of the wavelength region outside the emission band of a selective emitter. Spectral emittance results for α_λ= 1 cm^{-1} are shown in figure 5. In this case, ε_λ does not attain a maximum value even for thicknesses over 1 mm. Because α_λ is small much larger thicknesses (1 cm to obtain K_d=1) are required before ε_λ will approach its maximum value. For d<.4 mm, the region where maximum ε_λ occurs for large α_λ, the spectral emittance is nearly independent of τ_f.

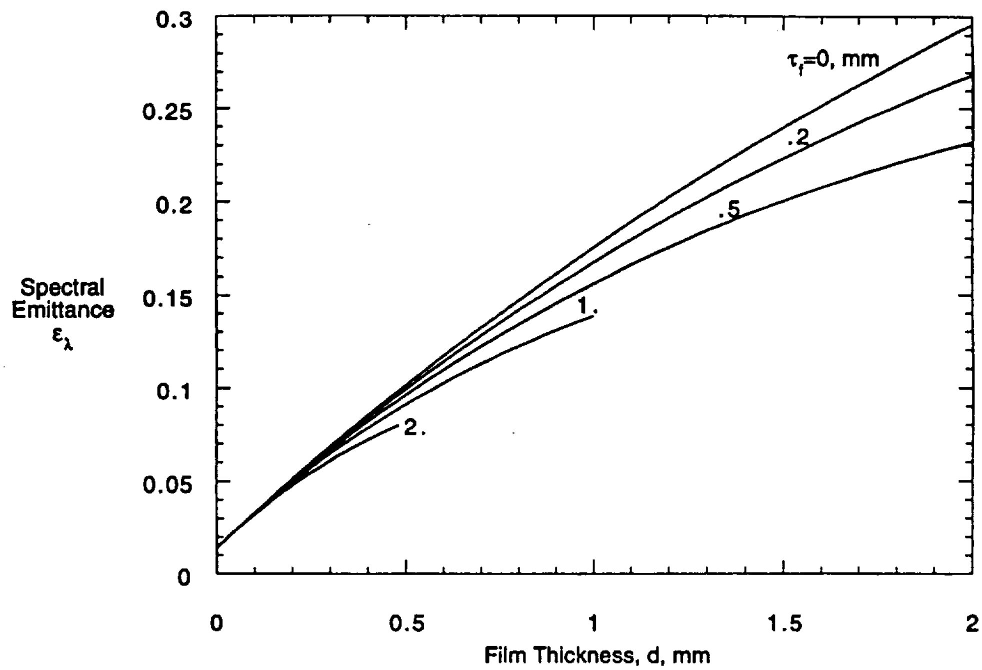
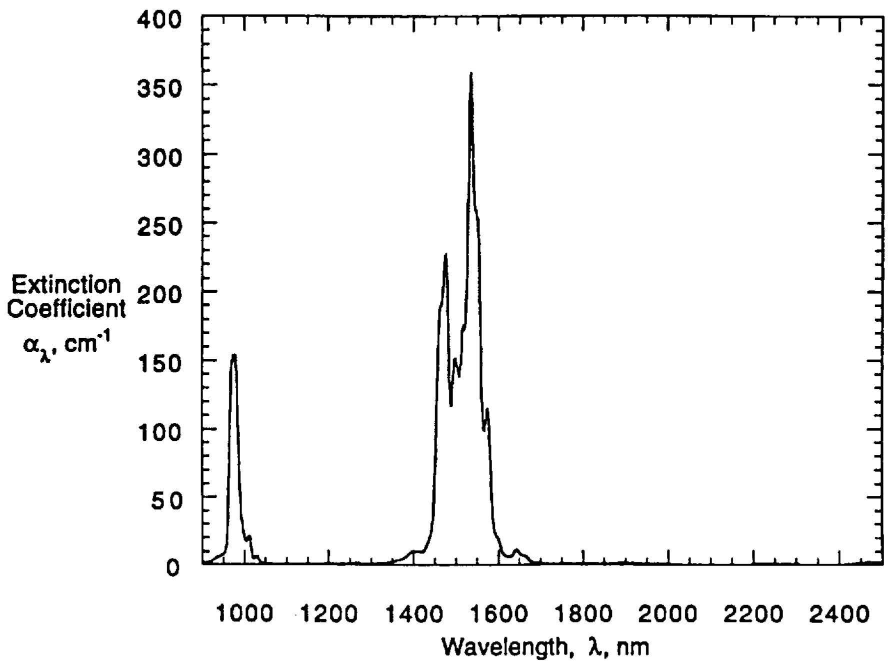
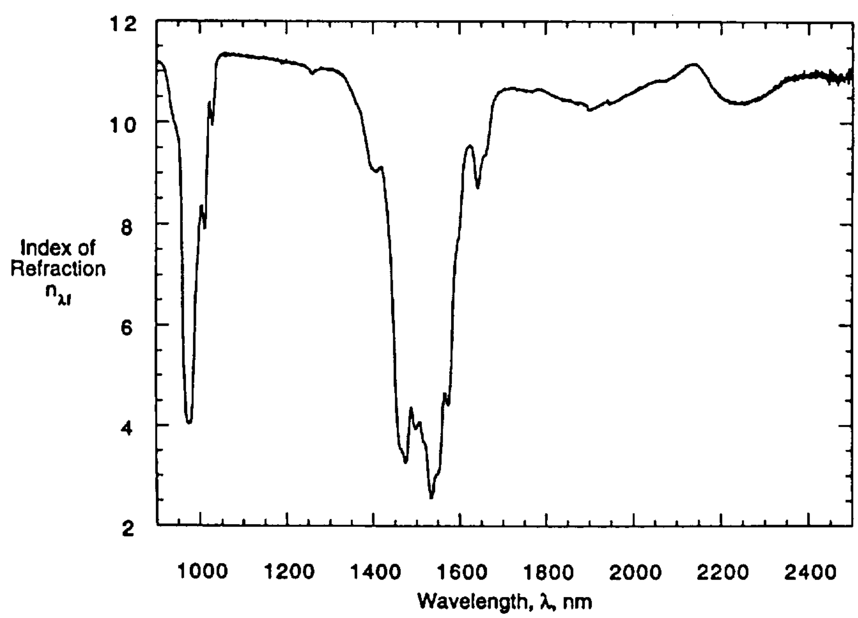


FIGURE 5. Effect of temperature gradient on spectral emittance for small extinction coefficient, α_λ =1 cm^{-1}, at several values of the temperature gradient parameter, τ_f. Also, u=5., $\varepsilon_{\lambda s}$=.1, $n_{\lambda f}$=1.9, $n_{\lambda s}$=10.

Based on the results displayed in figures 4 and 5 several conclusions can be made about the efficiency of a thick film selective emitter. The emitter efficiency (1,3,4) depends on the ratio of the emittance within the emission band ε_b to the emittance outside the emission band, ε_ℓ. Obviously it is desirable for $\varepsilon_b/\varepsilon_\ell$ to be as large as possible. For the emission band, where α_λ is large, there will be an optimum thickness, d_{opt}, (corresponding to $1.5 \le K_d \le 4.$) to maximize ε_b. Outside the emission band, where α_λ is small, the spectral emittance increases at a much slower rate with d than for the emission band for d < d_{opt}. For d<.4mm figure 5 shows that ε_λ increases nearly at the same linear rate regardless of the temperature gradient. Thus it appears that maximum emitter efficiency will occur for the thickness, d_{opt}, corresponding to maximum emittance within the emission band. As stated earlier this thickness corresponds to $1.5 \le K_d \le 4.0$ when α_λ = 100 cm^{-1}.



6a. Extinction Coefficient



6b. Index of refraction, $n_{\lambda f}$

FIGURE 6. Extinction coefficient and index of refraction for Er_2O_3-Al_2O_3 selective emitter

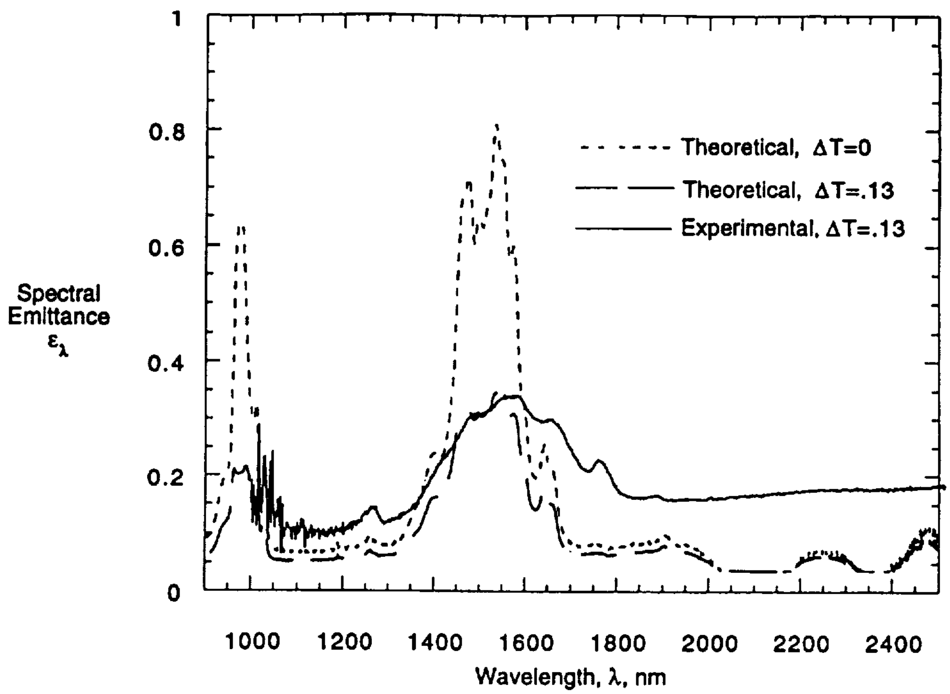


FIGURE 7. Comparison of theoretical and experimental spectral emittance for Er_2O_3-Al_2O_3 thick film selective emitter. Film thickness, d = .36 mm, $n_{\lambda s}$= 1., $\varepsilon_{\lambda s}$ = .2, T_s=1500K.

Comparison of Experimental and Theoretical Spectral Emittance

To complete this study we compare the measured spectral emittance of a selective emitter made of erbia (Er_2O_3) reinforced with alumina (Al_2O_3) with the spectral emittance calculated using equation (31). This emitter was fabricated at the Auburn Space Power Institute (6). The calculated ε_λ is based on the extinction coefficient, α_λ, and index of refraction, $n_{\lambda f}$, shown in figure 6. These quantities were obtained using measured transmittance and reflectance data (6).

Figure 7 shows the experimental and theoretical ε_λ for an emitter of thickness, d = .36 mm. This emitter had a platinum foil substrate. A constant substrate emittance $\varepsilon_{\lambda s}$ = .2 was used for the platinum foil. However, since there is an air gap between the foil and the film the appropriate index of refraction for the film-substrate interface is $n_{\lambda s}$ = 1.0, which was used in the calculation. The measured temperature gradient was ΔT = .13 and the platinum foil substrate temperature was T_s = 1500K.

The first thing to notice is the considerable reduction in ε_λ within the emission bands centered at $\lambda \approx$ 1000 nm and $\lambda \approx$ 1500 nm as a result of the temperature gradient. In the main emission band at $\lambda \approx 1500$ nm the theoretical maximum goes from $\varepsilon_\lambda \approx .8$ when $\Delta T = 0$ to $\varepsilon_\lambda \approx .35$ when $\Delta T = .13$. As discussed earlier (fig. 3), the spectral emittance outside the emission bands is not greatly affected by ΔT.

The measured emission band is broader than the theoretical emission band. This occurs because the theoretical result is based on the extinction coefficient that was measured at room temperature. At high temperature broadening of the emission band will occur which will therefore not be accounted for in the theoretical results. Part of the difference between the theoretical and experimental ε_λ for radiation outside the emission band is caused by experimental error. Outside the emission band where ε_λ is small, background radiation coming from sources other than the emitting film result in the measured ε_λ being larger than the actual value, (2).

CONCLUSION

The no scattering theoretical spectral emittance model shows the importance of even small ($\Delta T \approx .1$) temperature gradients on ε_λ. For both small (K_d<<1) and large (K_d>>1) optical depths, approximations for ε_λ were developed that give good agreement with the exact results as long as $\Delta T \leq .1$.

Because of the opposite dependence of ε_λ on temperature gradient and optical depth there will an optimum film thickness for maximum, ε_λ. The model predicts that the optimum optical depth has the range, $1.5 \leq K_d \leq 4.0$, depending on the temperature gradient.

Finally, there is good agreement between the theoretical spectral emittance and experimental spectral emittance for a Er_2O_3-Al_2O_3 selective emitter fabricated at the Auburn Space Power Institute.

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A temperature gradient across a thick (≥ .1 mm) film selective emitter will produce a significant reduction in the spectral emittance from the no temperature gradient case. Thick film selective emitters of rare earth doped host materials such as yttrium-aluminum-garnet (YAG) are examples where temperature gradient effects are important. In this paper a model is developed for the spectral emittance assuming a linear temperature gradient across the film. Results of the model indicate that temperature gradients will result in reductions the order of 20% or more in the spectral emittance.

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